
**Problem 1** Given a 1-form $\omega$ on an open set $U$, prove that the following are equivalent (i) $d\omega = 0$; (ii) $\int_{\partial R} \omega = 0$ for all closed rectangles $R$ contained in $U$; (iii) every point in $U$ has a neighborhood such that $\int_{\partial R} \omega = 0$ for all closed rectangles contained in the neighborhood. Is the same true if closed rectangles are replaced by disks?

**Problem 2** Let $H : U \to \mathbb{R}^2$ be a smooth function. Show that $dH^* = H^* d$.

**Problem 3** Let $R$ be a region in the plane between two concentric circles $\gamma_1$ and $\gamma_2$ of radius $r_1 < r_2$. Prove that if $U$ is an open set containing $R$ and $\omega$ is a 1-form on $U$, then
\[ \int_R d\omega = \int_{\partial R} \omega, \]
where $\partial R = \gamma_2 - \gamma_1$.

**Problem 4** Prove that the relation of being homotopic relative to endpoints, or homotopic as closed paths, is an equivalence relation.

**Problem 5** Let $\gamma : [a, b] \to \mathbb{R}^2 \setminus \{P\}$ be a continuous path, and let $v$ be a vector in the plane. Let $\gamma + v$ denote the path in $\mathbb{R}^2 \setminus \{P + v\}$ defined by $(\gamma + v)(t) = \gamma(t) + v$. Prove that
\[ W(\gamma + v, P + v) = W(\gamma, P). \]