Math 102.
Key Concepts for Chapter 3 Quiz

A. Basic Simplification
Simplify the difference quotient \( \frac{f(x + h) - f(x)}{h} \) for a polynomial function.

**Problem 1** For the function \( H(x) = 3 - 4x - 4x^2 \), find \( \frac{H(x + h) - H(x)}{h} \) and simplify completely.

B. Graphs of Functions
Sketch the graph of a function using basic shapes combined with transformations.

**Problem 2** Sketch the graph of each function, not by plotting points, but by applying transformations to basic shapes
(a) \( f(x) = 2 - \sqrt{x + 1} \)
(b) \( g(x) = 2 - 2\sqrt{1 - 2x} \)
(c) \( h(x) = \frac{1}{2} |x| + 3 \)

C. Increasing and decreasing. Average rate of Change
Find the average rate of change of a function on a specified interval. Identify the intervals where a function is increasing and the intervals where a function is decreasing.

**Problem 3** The graph of \( f \) is given in the figure below.

(a) Find the average value of \( f \) between the points \( x = -3 \) and \( x = 1 \).
(b) Determine the intervals on which \( f \) is increasing and the intervals on which \( f \) is decreasing.

D. Quadratic functions
Identify the extreme value of a quadratic function by changing the form to standard form. Find \( x \)- and \( y \)-intercepts and sketch the graph.

**Problem 4** For \( f(x) = 7 - 2x(x - 4) \),
(a) Express \( f \) in standard form \( f(x) = a(x - h)^2 + k \).
(b) Find the extreme value of \( f \). What type of extreme value is it?
(c) Sketch the graph of \( f \), clearly labeling its \( x \) and \( y \)-intercepts.

E. Combining functions
Find the composite function and its domain.

**Problem 5** For the functions \( f(x) = \frac{1}{x + 3} \) and \( g(x) = \frac{1}{x - 2} \),
(a) Find \((f \circ g)(x)\), \((g \circ f)(x)\), \((f \circ f)(x)\) and \((g \circ g)(x)\)
(b) Find the domains of \( f \circ g \), \( g \circ f \), \( f \circ f \), and \( g \circ g \).

F. Inverse Functions
Find the inverse function, and sketch its graph.

**Problem 6** Find the inverse function
(a) Find the inverse function of \( f(x) = 1 + \sqrt{1 + x} \).
(b) Find the inverse function of \( g(x) = \frac{2 + 5x}{3 - 4x} \).