Math 480. Assignment 1

1. Compute the derivatives of the following functions:
   (a) \( J(t) = \int_{\sin t}^{2t} x^3 \cos t \, dx \).
   (b) \( J(t) = \int_{\sin t}^{t} \cos(xt) \, dx \).

2. Sequences and series of functions.
   (a) Let \( f_N(x) = \begin{cases} 1 - Nx & 0 \leq x \leq \frac{1}{N} \\ 0 & \frac{1}{N} < x \leq 1 \end{cases} \).
       Find the limit of this sequence and indicate whether this convergence is uniform or not.
   (b) Let \( f_N(x) = \sin(Nx) \). Does this sequence have a limit? If so compute it.
       Justify your answer.
   (c) Let \( f_N(x) = \sum_{n=0}^{N} \frac{\sin(nx)}{3^n} \) for \( x \in \mathbb{R} \). Show that \( f_N \) converges uniformly.

3. Consider the series
   \( f(x) = - \sum_{n=1}^{\infty} \frac{x^n}{n} \).
   (a) Show that this series converges for every \(-1 \leq x < 1\).
   (b) Show that \( f(x) = \ln(1-x) \). (Hint: What is \( \sum_{n=0}^{\infty} x^n \)?)
   (c) Use this to compute \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \).

4. Let \( G \) be an open bounded set in \( \mathbb{R}^3 \) with boundary \( \partial G \) and let \( u \) be a twice differentiable real valued function on \( G \). Use the divergence theorem to prove:
   \( \int_{G} \Delta u \, dV = \int_{\partial G} \nabla u \cdot \mathbf{n} \, dS \).