Math 102. Fall 2006. Practice 3rd Midterm

1 For the parabola defined by the equation \(x^2 - 4x = 8y - 28\), determine the vertex, focus, and directrix and sketch the graph.

2 Write an equation for the parabola whose focus is \((3, -1)\) and whose directrix is the line \(x = 1\).

3 For the ellipse defined by the following equations, determine their centers, major axes, vertices, and foci and sketch their graphs.
   (a) \(x^2 + 9y^2 - 3 = 0\)
   (b) \(x^2 + 3y^2 - 12y + 9 = 0\)

4 Find an equation for the ellipse with foci \((0, \pm 4)\), center at the origin, and minor axis with length of 6.

5 For the hyperbolas defined by the following equations determine their centers, vertices, transverse axes, asymptotes and sketch their graphs.
   (a) \(5y^2 - 7x^2 = 70\),
   (b) \(y^2 - x^2 + 4x - 4y - 1 = 0\)

6 Find an equation for the hyperbola with foci \((0, \pm 7)\) and vertices \((0, \pm 5)\).

7 Find an equation for the hyperbola with foci \((0, \pm 7)\) and asymptotes \(y = \pm 2\sqrt{2}x\).

8 Solve the system \[
\begin{align*}
0.5x + 0.3y &= 2.7 \\
0.7x - 0.2y &= 1.3
\end{align*}
\]

9 Use row operation to transform the following augmented matrix to reduced row echelon form:
\[
\begin{bmatrix}
2 & 1 & 8 & -1 \\
1 & -1 & 1 & -2 \\
3 & -2 & -2 & 2
\end{bmatrix}
\]

10 Solve the following system of equations by transforming its augmented matrix to reduced row echelon form.
\[
\begin{align*}
x - y + 2z &= -1 \\
3x + 2y - 6z &= 1 \\
2x + 3y + 4z &= 8
\end{align*}
\]

11 The amount of money awarded in medical malpractice suits has been rising in the past decade, as shown in the following table:

<table>
<thead>
<tr>
<th>Year</th>
<th>1997</th>
<th>2001</th>
<th>2005</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ (thousands)</td>
<td>253</td>
<td>287</td>
<td>431</td>
</tr>
</tbody>
</table>

It can be modeled with a quadratic function \(y = at^2 + bt + c\), where \(t\) is the time in years since 1995 and \(y\) is the amount of dollars (in thousands). Find \(a\), \(b\), and \(c\).

12 Solve the system \[
\begin{align*}
y - 3x + 5 &= 0 \\
x^2 + y^2 - 5 &= 0
\end{align*}
\]

13 Solve the system \[
\begin{align*}
\ln x &= 5 \ln y \\
\log_2 x &= 3 + 2 \log_2 y
\end{align*}
\]

14 Graph the system of inequalities
\[
\begin{align*}
y &\leq x + 2 \\
y &\geq |x| \\
y &> -3 < 0
\end{align*}
\]

15 Graph the system of inequalities \[
\begin{align*}
x^2 + 3y^2 &< 9 \\
y &\leq -x^2 - 1
\end{align*}
\]