Chapter 3

1 For the function \( f(x) = 10x^2 - 7x + 3 \) calculate the following values (simplify completely):

(i) \( f(a) \)

(ii) \( f(a + h) \)

(iii) \( \frac{f(a + h) - f(a)}{h} \)

2 Let \( f(x) = \frac{x}{\sqrt{1 - x}} \).

(i) Evaluate \( f(3), f(5), \) and \( f(a - 1) \).

(ii) Find the domain of \( f \).

3 Determine the average rate of change of the function \( f(x) = x + x^4 \) between \( x = -1 \) and \( x = 3 \).

4 The function \( f \) has the following graph.

(v) Find the average rate of change of \( f \) between 3 and -2.

5 For the piecewise function \( f \) defined by

\[
\begin{align*}
    f(x) &= \begin{cases} 
        1 - x^2, & x \leq 2 \\
        2x - 1, & x > 2. 
    \end{cases}
\end{align*}
\]

(i) Find the values \( f(0), f(-2) \) and \( f(2) \).

(ii) Sketch the graph of \( f \).

6 Use basic shapes and transformations to sketch the graph of \( f(x) = 1 - \sqrt{2 - x} \).

7 For the functions \( f(x) = \frac{1}{x + 3} \) and \( g(x) = \frac{1}{x - 2} \), find:

(i) the functions \( f + g, f - g, f \cdot g, \) and \( f/g \);

(ii) the domains of the functions in Part (i).

(i) What are the values \( f(-2), f(-1), \) and \( f(3) \)?

(ii) Determine the domain and the range of \( f \). (Use interval notation.)

(iii) Determine the intervals on which \( f \) is increasing and the intervals on which \( f \) is decreasing.

(iv) Find the local maxima and the local minima of \( f \).
8 Sketch the graph of \( f(x) = -2x^2 + 8x + 3 \). Label the vertex and intercepts. What is the maximum value of this function?

9 Odyssey Travel Agency’s monthly profit \( P \) (in thousands of dollars) depends on the amount of money \( x \) (in thousands of dollars) spent on advertising per month according to the rule \( P(x) = -x^2 + 8x + 20 \).

(i) What would Odyssey’s monthly advertising be in order to maximize its monthly profit?

(ii) What is this maximum monthly profit?

10 A farmer with 2000 meters of fencing wants to enclose a rectangular plot that borders on a straight highway. If the farmer does not fence along the highway, what is the largest area that can be enclosed?

11 The following curve is the graph of a polynomial \( P \).

(i) How many real zeros does \( P \) have? Mark them in the graph.

(ii) How many local maxima and how many local minima does \( P \) have? Mark them in the graph.

12 For the polynomial \( P(x) = x(x + 1)^2(x + 3) \)

(i) Find \( x \) and \( y \)-intercepts.

(ii) Determine its end behavior.

(iii) Sketch its graph. (Make sure that your graph shows all intercepts and exhibits the proper end behavior.)

13 The following curve is the graph of a polynomial of degree 3. Find the polynomial.

14 Find the intercepts and asymptotes of the rational function \( r(x) = \frac{1 - 2x}{2x + 3} \), and then sketch its graph.

15 For the rational function \( R(x) = \frac{x + 1}{x^2 + 2x - 3} \), find the horizontal and vertical asymptotes, then sketch its graph.