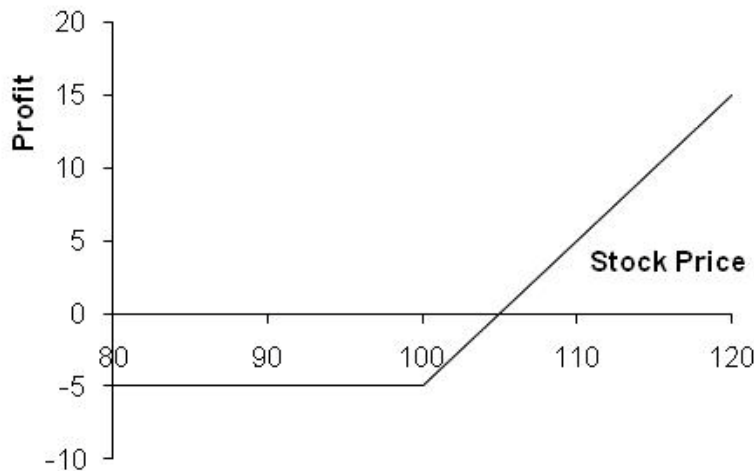**Problem 9.9**
*Suppose that a European call option to buy a share for $100.00 costs $5.00 and is held until maturity. Under what circumstances will the holder of the option make a profit? Under what circumstances will the option be exercised? Draw a diagram illustrating how the profit from a long position in the option depends on the stock price at maturity of the option.*

Ignoring the time value of money, the holder of the option will make a profit if the stock price at maturity of the option is greater than $105. This is because the payoff to the holder of the option is, in these circumstances, greater than the $5 paid for the option. The option will be exercised if the stock price at maturity is greater than $100. Note that if the stock price is between $100 and $105 the option is exercised, but the holder of the option takes a loss overall. The profit from a long position is as shown in Figure S9.1.
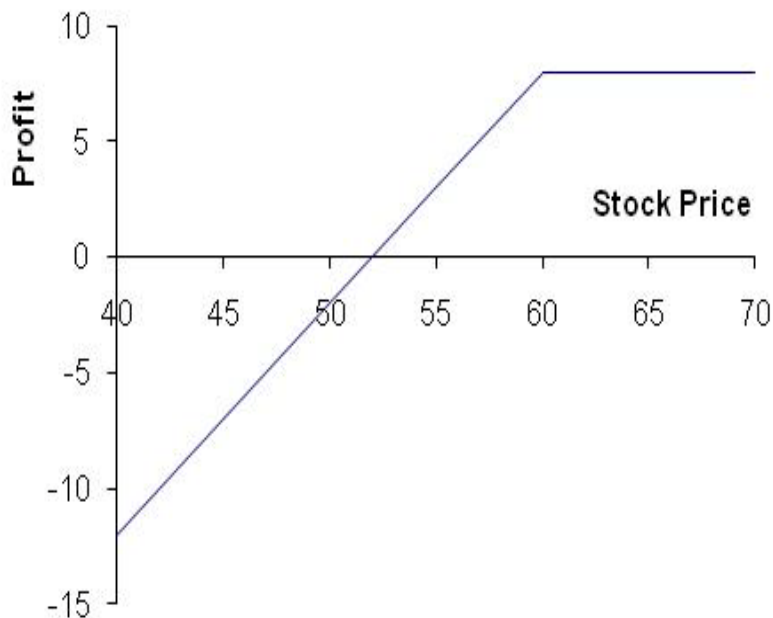


**Figure S9.1**    Profit from long position in Problem 9.9

**Problem 9.10**
*Suppose that a European put option to sell a share for $60 costs $8 and is held until maturity. Under what circumstances will the seller of the option (the party with the short position) make a profit? Under what circumstances will the option be exercised? Draw a diagram illustrating how the profit from a short position in the option depends on the stock price at maturity of the option.*

Ignoring the time value of money, the seller of the option will make a profit if the stock price at maturity is greater than $52.00. This is because the cost to the seller of the option is in these circumstances less than the price received for the option. The option will be exercised if the stock price at maturity is less than $60.00. Note that if the stock price is between $52.00 and $60.00 the seller of the option makes a profit even though the option is exercised. The profit from the short position is as shown in Figure S9.2.

**Figure S9.2**    Profit from short position in Problem 9.10


**Problem 9.12**
*A trader buys a call option with a strike price of $45 and a put option with a strike price of $40. Both options have the same maturity. The call costs $3 and the put costs $4. Draw a diagram showing the variation of the trader's profit with the asset price.*

Figure S9.4 shows the variation of the trader's position with the asset price. We can divide the alternative asset prices into three ranges:
a)  When the asset price less than $40, the put option provides a payoff of $40 - S_T$ and the call option provides no payoff. The options cost $7 and so the total profit is $33 - S_T$.
b)  When the asset price is between $40 and $45, neither option provides a payoff. There is a net loss of $7.
c)  When the asset price greater than $45, the call option provides a payoff of $S_T - 45$ and the put option provides no payoff. Taking into account the $7 cost of the options, the total profit is $S_T - 52$.

The trader makes a profit (ignoring the time value of money) if the stock price is less than $33 or greater than $52. This type of trading strategy is known as a strangle and is discussed in Chapter 11.
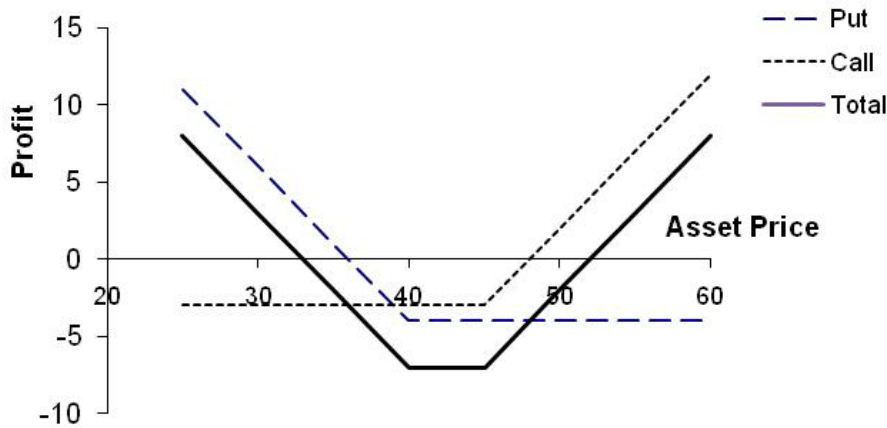
**Figure S9.4**   Profit from trading strategy in Problem 9.12

## Problem 10.10
*What is a lower bound for the price of a two-month European put option on a non-dividend-paying stock when the stock price is $58, the strike price is $65, and the risk-free interest rate is 5% per annum?*

The lower bound is
$$65e^{-0.05\times2/12} - 58 = \$6.46$$

## Problem 10.11
*A four-month European call option on a dividend-paying stock is currently selling for $5. The stock price is $64, the strike price is $60, and a dividend of $0.80 is expected in one month. The risk-free interest rate is 12% per annum for all maturities. What opportunities are there for an arbitrageur?*

The present value of the strike price is $60e^{-0.12\times4/12} = \$57.65$. The present value of the dividend is $0.80e^{-0.12\times1/12} = 0.79$. Because
$$5 < 64 - 57.65 - 0.79$$
the condition in equation (10.8) is violated. An arbitrageur should buy the option and short the stock. This generates $64 - 5 = \$59$. The arbitrageur invests $0.79 of this at 12% for one month to pay the dividend of $0.80 in one month. The remaining $58.21 is invested for four months at 12%. Regardless of what happens a profit will materialize.
If the stock price declines below $60 in four months, the arbitrageur loses the $5 spent on the option but gains on the short position. The arbitrageur shorts when the stock price is $64, has to pay dividends with a present value of $0.79, and closes out the short position when the stock price is $60 or less. Because $57.65 is the present value of $60, the short position generates at least $64 - 57.65 - 0.79 = \$5.56$ in present value terms. The present value of the arbitrageur's gain is therefore at least $5.56 - 5.00 = \$0.56$.

If the stock price is above $60 at the expiration of the option, the option is exercised. The arbitrageur buys the stock for $60 in four months and closes out the short position. The present value of the $60 paid for the stock is $57.65 and as before the dividend has a present value of $0.79. The gain from the short position and the exercise of the option is therefore exactly equal to $64 - 57.65 - 0.79 = \$5.56$. The arbitrageur's gain in present value terms is exactly equal to $5.56 - 5.00 = \$0.56$.

## Problem 10.12

*A one-month European put option on a non-dividend-paying stock is currently selling for $2.50. The stock price is $47, the strike price is $50, and the risk-free interest rate is 6% per annum. What opportunities are there for an arbitrageur?*

In this case the present value of the strike price is $50e^{-0.06 \times 1/12} = 49.75$. Because
$$2.5 < 49.75 - 47.00$$
the condition in equation (10.5) is violated. An arbitrageur should borrow $49.50 at 6% for one month, buy the stock, and buy the put option. This generates a profit in all circumstances.
If the stock price is above $50 in one month, the option expires worthless, but the stock can be sold for at least $50. A sum of $50 received in one month has a present value of $49.75 today. The strategy therefore generates profit with a present value of at least $0.25.
If the stock price is below $50 in one month the put option is exercised and the stock owned is sold for exactly $50 (or $49.75 in present value terms). The trading strategy therefore generates a profit of exactly $0.25 in present value terms.

## Problem 11.10

*Suppose that put options on a stock with strike prices $30 and $35 cost $4 and $7, respectively. How can the options be used to create (a) a bull spread and (b) a bear spread? Construct a table that shows the profit and payoff for both spreads.*

A bull spread is created by buying the $30 put and selling the $35 put. This strategy gives rise to an initial cash inflow of $3. The outcome is as follows:

| Stock Price | Payoff | Profit |
|---|---|---|
| $S_T \geq 35$ | 0 | 3 |
| $30 \leq S_T < 35$ | $S_T - 35$ | $S_T - 32$ |
| $S_T < 30$ | −5 | −2 |

A bear spread is created by selling the $30 put and buying the $35 put. This strategy costs $3 initially. The outcome is as follows:

| Stock Price | Payoff | Profit |
|---|---|---|
| $S_T \geq 35$ | 0 | −3 |
| $30 \leq S_T < 35$ | $35 - S_T$ | $32 - S_T$ |
| $S_T < 30$ | 5 | 2 |

**Problem 11.12**
*A call with a strike price of $60 costs $6. A put with the same strike price and expiration date costs $4. Construct a table that shows the profit from a straddle. For what range of stock prices would the straddle lead to a loss?*

A straddle is created by buying both the call and the put. This strategy costs $10. The profit/loss is shown in the following table:

| Stock Price | Payoff | Profit |
|---|---|---|
| $S_T > 60$ | $S_T - 60$ | $S_T - 70$ |
| $S_T \leq 60$ | $60 - S_T$ | $50 - S_T$ |

This shows that the straddle will lead to a loss if the final stock price is between $50 and $70.


**Problem 12.9**
*A stock price is currently $50. It is known that at the end of two months it will be either $53 or $48. The risk-free interest rate is 10% per annum with continuous compounding. What is the value of a two-month European call option with a strikeprice of $49? Use no-arbitrage arguments.*

At the end of two months the value of the option will be either $4 (if the stock price is $53) or $0 (if the stock price is $48). Consider a portfolio consisting of:

$$+\Delta \quad : \quad \text{shares}$$
$$-1 \quad : \quad \text{option}$$

The value of the portfolio is either $48\Delta$ or $53\Delta - 4$ in two months. If
$$48\Delta = 53\Delta - 4$$
i.e.,
$$\Delta = 0.8$$
the value of the portfolio is certain to be 38.4. For this value of $\Delta$ the portfolio is therefore riskless. The current value of the portfolio is:
$$0.8 \times 50 - f$$
where $f$ is the value of the option. Since the portfolio must earn the risk-free rate of interest
$$(0.8 \times 50 - f)e^{0.10 \times 2/12} = 38.4$$
i.e.,
$$f = 2.23$$
The value of the option is therefore $2.23.
This can also be calculated directly from equations (12.2) and (12.3). $u = 1.06$, $d = 0.96$ so that
$$p = \frac{e^{0.10 \times 2/12} - 0.96}{1.06 - 0.96} = 0.5681$$
and
$$f = e^{-0.10 \times 2/12} \times 0.5681 \times 4 = 2.23$$

**Problem 12.10**

*A stock price is currently $80. It is known that at the end of four months it will be either $75 or $85. The risk-free interest rate is 5% per annum with continuous compounding. What is the value of a four-month European put option with a strike price of $80? Use no-arbitrage arguments.*

At the end of four months the value of the option will be either $5 (if the stock price is $75) or $0 (if the stock price is $85). Consider a portfolio consisting of:

$$-\Delta \quad : \quad \text{shares}$$

$$+1 \quad : \quad \text{option}$$

(Note: The delta, $\Delta$ of a put option is negative. We have constructed the portfolio so that it is +1 option and $-\Delta$ shares rather than $-1$ option and $+\Delta$ shares so that the initial investment is positive.)

The value of the portfolio is either $-85\Delta$ or $-75\Delta+5$ in four months. If

$$-85\Delta = -75\Delta + 5$$

i.e.,

$$\Delta = -0.5$$

the value of the portfolio is certain to be 42.5. For this value of $\Delta$ the portfolio is therefore riskless. The current value of the portfolio is:

$$0.5 \times 80 + f$$

where $f$ is the value of the option. Since the portfolio is riskless

$$(0.5 \times 80 + f)e^{0.05 \times 4/12} = 42.5$$

i.e.,

$$f = 1.80$$

The value of the option is therefore $1.80.

This can also be calculated directly from equations (12.2) and (12.3). $u = 1.0625$, $d = 0.9375$ so that

$$p = \frac{e^{0.05 \times 4/12} - 0.9375}{1.0625 - 0.9375} = 0.6345$$

$1 - p = 0.3655$ and

$$f = e^{-0.05 \times 4/12} \times 0.3655 \times 5 = 1.80$$


**Problem 13.9**

*A stock price has an expected return of 16% and a volatility of 35%. The current price is $38.*
    *a) What is the probability that a European call option on the stock with an exercise price of $40 and a maturity date in six months will be exercised?*
    *b) What is the probability that a European put option on the stock with the same exercise price and maturity will be exercised?*

    a) The required probability is the probability of the stock price being above $40 in six months time. Suppose that the stock price in six months is $S_T$

$$\ln S_T \sim \varphi(\ln 38 + (0.16 - \frac{0.35^2}{2})0.5, 0.35\sqrt{0.5})$$

i.e.,

$$\ln S_T \square \varphi(3.687, 0.247)$$

Since $\ln 40 = 3.689$, the required probability is

$$1 - N\left(\frac{3.689 - 3.687}{0.247}\right) = 1 - N(0.008)$$

From normal distribution tables $N(0.008) = 0.5032$ so that the required probability is 0.4968.

b) In this case the required probability is the probability of the stock price being less than $40 in six months time. It is

$$1 - 0.4968 = 0.5032$$

## Problem 13.14

*What is the price of a European put option on a non-dividend-paying stock when the stock price is $69, the strike price is $70, the risk-free interest rate is 5% per annum, the volatility is 35% per annum, and the time to maturity is six months?*

In this case $S_0 = 69$, $K = 70$, $r = 0.05$, $\sigma = 0.35$ and $T = 0.5$.

$$d_1 = \frac{\ln(69/70) + (0.05 + 0.35^2/2) \times 0.5}{0.35\sqrt{0.5}} = 0.1666$$

$$d_2 = d_1 - 0.35\sqrt{0.5} = -0.0809$$

The price of the European put is

$$70e^{-0.05 \times 0.5} N(0.0809) - 69N(-0.1666)$$

$$= 70e^{-0.025} \times 0.5323 - 69 \times 0.4338$$

$$= 6.40$$

or $6.40.

## Problem 15.9

*A foreign currency is currently worth $1.50. The domestic and foreign risk-free interest rates are 5% and 9%, respectively. Calculate a lower bound for the value of a six-month call option on the currency with a strike price of $1.40 if it is (a) European and (b) American.*

Lower bound for European option is

$$S_0 e^{-r_f T} - Ke^{-rT} = 1.5e^{-0.09 \times 0.5} - 1.4e^{-0.05 \times 0.5} = 0.069$$

Lower bound for American option is

$$S_0 - K = 0.10$$

## Problem 15.10
*Consider a stock index currently standing at 250. The dividend yield on the index is 4% per annum, and the risk-free rate is 6% per annum. A three-month European call option on the index with a strike price of 245 is currently worth $10. What is the value of a three-month put option on the index with a strike price of 245?*

In this case $S_0 = 250$, $q = 0.04$, $r = 0.06$, $T = 0.25$, $K = 245$, and $c = 10$. Using put–call parity

$$c + Ke^{-rT} = p + S_0 e^{-qT}$$

or

$$p = c + Ke^{-rT} - S_0 e^{-qT}$$

Substituting:

$$p = 10 + 245e^{-0.25 \times 0.06} - 250e^{-0.25 \times 0.04} = 3.84$$

The put price is 3.84.


## Problem 16.8
*Suppose you buy a put option contract on October gold futures with a strike price of $900 per ounce. Each contract is for the delivery of 100 ounces. What happens if you exercise when the October futures price is $880?*

An amount $(900 - 880) \times 100 = \$2,000$ is added to your margin account and you acquire a short futures position obligating you to sell 100 ounces of gold in October. This position is marked to market in the usual way until you choose to close it out.


## Problem 16.9
*Suppose you sell a call option contract on April live cattle futures with a strike price of 90 cents per pound. Each contract is for the delivery of 40,000 pounds. What happens if the contract is exercised when the futures price is 95 cents?*

In this case an amount $(0.95 - 0.90) \times 40,000 = \$2,000$ is subtracted from your margin account and you acquire a short position in a live cattle futures contract to sell 40,000 pounds of cattle in April. This position is marked to market in the usual way until you choose to close it out.