Chapter 1 - Introduction

- Derivative securities
- Futures contracts
- Forward contracts
- Futures and forward markets
- Comparison of futures and forward contracts
- Options contracts
- Options markets
- Comparison of futures and options
- Types of traders
- Applications

- Derivative securities

  Securities whose values are derived from the values of other underlying assets
  Futures and forward contracts
  Options
  Swaps
  Others

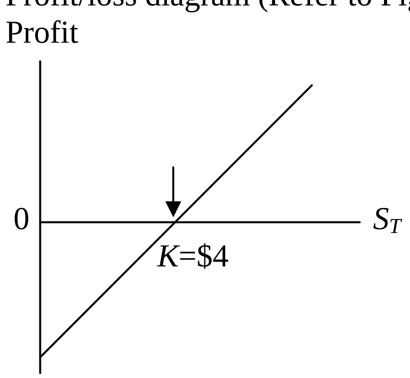
- Futures contracts

  An agreement between two parties to either buy or sell an asset at a certain time in the future for a certain price

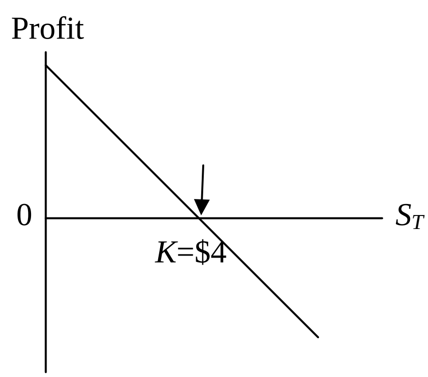
  For example, in March a trader buys a July futures contract on corn at 400 cents (or \$4) per bushel

  March: a trader buys a July futures contract on corn at 400 cents/bushel → July: the trader must buy 5,000 bushels of corn for \$20,000 →
  - If S_T > \$4 (gain)
  - If S_T = \$4 (no gain/loss)
  - If S_T < \$4 (lose)

  Profit/loss diagram (Refer to Figure 2.3)



Long position (buy futures)



Short position (sell futures)

K = delivery price = \$4/bushel and S_T = the spot price at maturity (can be greater than, equal to, or less than \$4/bushel)

If S_T is greater than K, the person with a long position gains (S_T - K) and the person with a short position loses (K - S_T) - zero sum game (someone's gain is someone else's loss)

If S_T is less than K, the person with a long position loses (S_T - K) and the person with a short position gains (K - S_T) - zero sum game

If S_T is equal to K, there is no gain or loss on both sides - zero sum game

Corn: underlying asset - commodity (commodity futures contract)
Buy a futures contract: long position - promise to buy
400 cents/bushel: futures price/delivery price
5,000 bushels: contract size - standardized
July: delivery month
Spot price: actual price in the market for immediate delivery

More examples
(1) Long futures positions: agree to buy or call for delivery
On February 1, you buy a June gold futures contract at 1,100: you agree to buy (or call for delivery) 100 ounces of gold in June at 1,100 dollars per troy ounce

Contract details
1,100 dollars per ounce - futures price of gold on February 1 for June delivery, also called delivery price (Note: the futures price of gold on February 2 for June delivery may be different)
100 ounces - contract size
June - delivery month
Underlying asset: gold - commodity (commodity futures contract)
Position: long position
Actual market price of gold - spot price which can be different from the futures price

(2) Short futures positions: agree to sell or promise to deliver
On February 1, you sell a June Yen futures contract at 1.1250: you agree to sell (or promise to deliver) 12,500,000 Yen in June at \$1.1250 per 100 Yen (\$1 for 88.8889 Yen)

Contract details
\$1.1250 per 100 Yen - futures price (exchange rate), also called delivery price
12,500,000 Yen - contract size
June - delivery month
Underlying asset: foreign currency - financial asset (financial futures contract)
Position: short position
Actual market exchange rate - spot exchange rate which can be different from the futures exchange rate

Futures contracts can be written on different assets (underling assets):
Commodities - commodity futures, for example, grains, livestock, meat, metals, and oil
Financial assets - financial futures, for example, stock indices, bonds, currencies

Technical details (covered in later chapters)
Margin requirements
Daily settlement procedures - marking to market
Bid-offer spreads
Clearinghouses
Delivery

- Forward contracts

  A forward contract is similar to a futures contract in that it is an agreement between two parties to either buy or sell an asset at a certain time in the future for a certain price. But forward contracts are less formal, traded only in OTC markets, and contract sizes are not standardized.

- Futures and forward markets

  (1) Exchange-traded markets
  Chicago Board of Trade (CBOT): futures contracts
  Chicago Mercantile Exchange (CME): futures contracts

  Open-outcry system: traders physically meet on the floor of the exchange and use a complicated set of hand signals to trade

  Electronic trading: increasingly replacing the open-outcry system to match buyers and sellers

  (2) Over-the-counter (OTC) markets
  Telephone- and computer-linked network of dealers

  Flexibility - tailor your needs

  Credit risk - the risk that your contract will not be honored

- Comparison of futures and forward contracts (Refer to Table 2.3)

	Exchange trading	Standardized contract size	Marking to market	Delivery	Delivery time	Default risk
Forward	No	No	No	Yes or cash settlement	One date	Some credit risk
Futures	Yes	Yes	Yes	Usually closed out	Range of dates	Virtually no risk

- Options contracts

  Rights to buy or sell an asset by a certain date for a certain price

  American options vs. European options
  American options can be exercised at any time before the expiration date
  European options can only be exercised on the maturity date

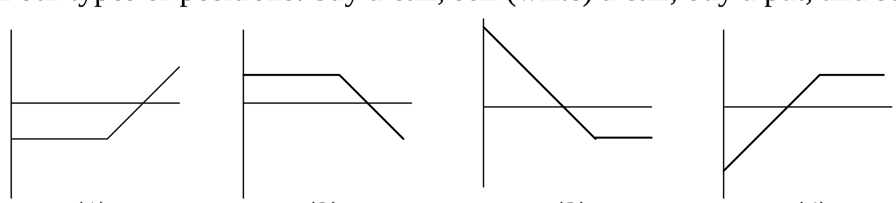
  Keeping other things the same, which type of options should be worth more and why?
  Answer: American options because they can be exercised at any time before expiration

  Two types of options: call options vs. put options

  A call option gives the right to buy an asset for a certain price by a certain date
  For example, you buy an IBM June 110 call option for $3.00
  Option type: call option - the right to buy
  The underlying asset - IBM stock
  Exercise (strike) price - $110 per share
  Expiration date - the third Friday in June
  Contract size: 100 shares
  Option premium (price of the option): $300.00

  A put option gives the right to sell an asset for a certain price by a certain date
  For example, you buy a GE June 16 put option for $2.00
  Option type: put option - the right to sell
  The underlying asset - GE stock
  Exercise (strike) price - $16 per share
  Expiration date - the third Friday in June
  Contract size: 100 shares
  Option premium: $200.00

  Four types of positions: buy a call, sell (write) a call, buy a put, and sell (write) a put



(1) (2) (3) (4)
(1) Buy a call (2) Sell a call (3) Buy a put (4) Sell a put

  Options can be written on different assets:
  Stocks - stock options
  Stock indices - index options
  Currencies - currency options
  Futures - futures options
  Others

- Options markets

  Exchange traded markets

  (1) Chicago Board Options Exchange (CBOE): options

  (2) Over-the-counter (OTC) markets

- Comparison of futures and options

  Rights (options) vs. obligations (futures)

  Initial outlay (buying options requires an initial outlay while buying futures doesn't)

  Both need a margin account (futures are subject to marking to market daily)

  Both use leverage

- Types of traders

  (1) Hedgers: use options and futures markets to reduce price uncertainty (risk) in the future

  For example, a farmer can sell corn futures contracts to lock in a price and an investor can buy a put option to protect a potential downward movement of a particular stock

  More details: a company can use forward contracts for hedging currency risk
  Import Co. purchased goods from a British supplier in June and needs to pay 10 million British pounds in September. A local financial institution offers forward contracts for British pounds. The quotes are shown below:

	Bid	Offer
Spot	1.6382	1.6386
1-month forward	1.6380	1.6385
3-month forward	1.6378	1.6384
6-month forward	1.6376	1.6383

  How should Import Co. hedge the exchange rate risk?

  Answer: Import Co. should buy 10 million British pounds in the three-month forward market to lock in the exchange rate of 1.6384 (or 16.384 million dollars for 10 million pounds for September delivery)

  When you sell pounds to the financial institution you get the bid price
  When you buy pounds from the financial institution you pay the offer price
  The difference between bid and offer is called bid-offer spread which is the profit for the institution

(2) Speculators: bet on price movement

For example, you buy gold futures contracts because you bet that the price of gold will go up in the future, or you buy a put option on Intel if you bet that Intel stock price will drop

More details: a trader uses options for speculation (leverage effect)
A trader with $2,000 to invest bets that the price of ORCL will increase in the near future and has the following quotes:
Current stock price: $20.00
ORCL June call option with exercise price of $22.50 sells for $1.00

Alternative 1: buy 100 shares of ORCL
Alternative 2: buy 20 ORCL June call options with exercise price of $22.50

Possible outcomes:
If ORCL stock price rises to $27
Alternative 1: profit of $700 = 100*(27 - 20)
Alternative 2: profit of $7,000 = 20*100(27 - 22.5 - 1)

If ORCL stock price falls to $17
Alternative 1: loss of $300 = 100*(17 - 20)
Alternative 2: loss of $2,000 (the options are worthless)

If ORCL stock price rises to $23.5
Alternative 1: profit of $350 = 100*(23.5 - 20)
Alternative 2: break-even (23.5 - 22.5 - 1)

(3) Arbitrageurs: look for risk-free profits by taking the advantage of mispricing in two or more markets simultaneously

Conditions for arbitrage:
Zero net cost
No risk
Positive profit

Example: use stocks and exchange rates for arbitrage
A stock is traded on both NYSE and London Stock Exchange. The following quotes are obtained at a particular time:
Stock price at NYSE: $20/share
Stock price at London Stock Exchange: 13 pounds/share
Spot exchange rate: 1 pound = $1.60

Detailed arbitrage process:
Borrow $2,000 to buy 100 shares at NYSE
Sell the shares at London Stock Exchange for 1,300 pounds
Covert the sale proceeds from pounds to dollars at the spot exchange rate to receive $2,080 = 1,300*1.60
Repay the loan of $2,000
Arbitrage profit: 2,080 – 2,000 = $80 (ignoring transaction costs)

(4) Dangers: start from hedgers or arbitrageurs and consciously or unconsciously become speculators

- Applications

  Market completeness - any and all identifiable payoffs can be obtained by trading the derivative securities available in the market (Financial Engineering)

  Risk management - reduce risk, hedging

  Trading efficiency - increase market efficiency

  Price discovery - futures price is the best expected spot price in the future

  Speculation - bet on prices

- Assignments

  Quiz (required)
  Practice Questions: 1.11, 1.13, 1.14, and 1.20

Chapter 2 - Mechanics of Futures Markets

- Specification of a futures contract
- Convergence of futures price to spot price
- Operation of margins
- Quotes and prices
- Delivery
- Types of orders
- Regulation
- Accounting and taxes

- Specification of futures contracts

  Opening a futures position vs. closing a futures position

  Opening a futures position can be either a long position or a short position
  (i.e., the opening position can be either to buy or to sell a futures contract)
  Closing a futures position involves entering an opposite trade to the original one

  The underlying asset or commodity: must be clearly specified

  The contract size: standardized
  Corn and wheat: 5,000 bushels per contract
  Live cattle: 40,000 pounds per contract
  Cotton: 50,000 pounds per contract
  Gold: 100 troy ounces per contract
  DJIA: $10*index
  Mini DJIA: $5*index
  S&P 500: $250*index
  Mini S&P 500: $50*index

  Delivery month: set by the exchange

  Delivery place: specified by the exchange

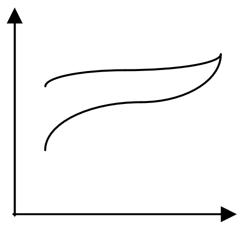
  Daily price limits: usually specified by the exchange, it is the restriction on the day-to-day price change of an underlying commodity. For example, the price of corn can change by no more than 10 cents per bushel from one day to the next and the price of wheat can change by no more than 20 cents per bushel from the preceding closing price

  Position limits: the maximum number of contracts that a speculator may hold
  For example, less than 1,000 contracts total with no more than 300 contracts in any one delivery month for random-length lumber contract in CME

  Tick: the minimum price fluctuation, for example, ¼ cent per bushel for wheat

- Convergence of futures price to spot price

  As the delivery period approaches, the futures price converges to the spot price of the underlying asset



Why? Show that the arbitrage opportunity exists if it doesn't happen

If the futures price is above the spot price as the delivery period is reached

(1) Short a futures contract
(2) Buy the asset at the spot price
(3) Make the delivery

If the futures price is below the spot price as the delivery period is reached

(1) Buy a futures contract
(2) Short sell the asset and deposit the proceeds
(3) Take the delivery and return the asset

- Operation of margins

  Margin account: an account maintained by an investor with a brokerage firm in which borrowing is allowed

  Initial margin: minimum initial deposit

  Maintenance margin: the minimum actual margin that a brokerage firm will permit investors to keep their margin accounts

  Margin call: a demand on an investor by a brokerage firm to increase the equity in the margin account

  Variation margin: the extra fund needs to be deposited by an investor

  Marking to market: the procedure that the margin account is adjusted to reflect the gain or loss at the end of each trading day

Example: suppose an investor buys two gold futures contracts. The initial margin is \$2,000 per contract (or \$4,000 for two contracts) and the maintenance margin is \$1,500 per contract (or \$3,000 for two contracts). The contract is entered into on June 5 at \$850 and closed out on June 18 at \$840.50. (Gold is trading around \$1,200 per ounce now)

Day	Futures price (settlement)	Daily gain (loss)	Cumulative gain (loss)	Margin balance	Margin call (variation margin)
	850.00			4,000	
June 5	848.00	(400)	(400)	3,600	
June 6	847.50	(100)	(500)	3,500	
June 9	848.50	200	(300)	3,700	
June 10	846.00	(500)	(800)	3,200	
June 11	844.50	(300)	(1,100)	2,900	Yes (1,100)
June 12	845.00	100	(1,000)	4,100	
June 13	846.50	300	(700)	4,400	
June 16	842.50	(800)	(1,500)	3,600	
June 17	838.00	(900)	(2,400)	2,700	Yes (1,300)
June 18	840.50	500	(1,900)	4,500	

Note: the investor earns interest on the balance in the margin account. The investor can also use other assets, such as T-bills to serve as collateral (at a discount)

Clearinghouse: an intermediary to guarantee in futures transactions

- Quotes and prices

  Opening price, highest and lowest prices, and settlement price

  Bid price vs. offer price

  Bid price: the price an institution is prepared to buy
  Offer price: the price an institution is prepared to sell

  Settlement price: the average of the prices immediately before the closing bell and marking to market is based on the settlement price

  Open interest: the total number of contracts outstanding at a particular time

  Open interest: an example
  Given the following trading activities, what is the open interest at the end of each day?

Time	Actions	Open Interest
t = 0	Trading opens for gold contract	0
t = 1	Trader A buys 2 and trader B sells 2 gold contracts	2
t = 2	Trader A sells 1 and trader C buys 1 gold contract	2
t = 3	Trader D sells 2 and trader C buys 2 gold contracts	4

- Delivery

  Very few futures contracts lead to actual delivery and most of futures contracts are closed out prior to the delivery month

  If delivery takes place, the decision on when to deliver is made by the party with the short position

  Cash settlement: some financial futures, for example, stock index futures are cash settled (No actual delivery)

- Types of orders

  Market order: the best price currently available in the market

  Limit order: specifies a price (limit price) and your order will be executed only at that price or a more favorable price to you

  More specifically, you will buy at or below a specified price (limit price) or you will sell at or above a specified price (limit price)

  For example, current price of gold = $1,100, you can specify to buy a futures contract on gold if the futures price $\leq$ $1,080 or you can specify to sell a futures contract on gold if the futures price $\geq$ $1,150

  Stop (stop-loss) order: specifies a price (stop price) and your order will become a market order if the stop price is reached

  For example, current price of gold = $1,100, you can specify to sell a futures contract if the futures price $\leq$ $1,060 or you can specify to buy a futures contract if the futures price $\geq$ $1,150

  The main difference between a limit order and a stop order:
  With a limit order you buy when the price drops and you sell when the price rises
  With a stop order you buy when the price rises and you sell when the price drops

  Stop-limit order: a combination of a stop order and a limit order
  For example, current price of gold = $1,100
  A stop-limit order to sell at a stop price $1,050 with a limit price $1,020
  A stop-limit order to buy at a stop price $1,120 with a limit price $1,150

  Market-if-touched order: executed at the best available price after a specified price is reached
  For example, current price of gold = $1,100
  A market-if-touched order to sell if price $\geq$ $1,120

Day order: valid for the day

Open order (good-till-canceled): in effect until the end of trading in a particular contract

- Regulation

  Futures markets are mainly regulated by the Commodity Futures Trading Commission (CFTC). Other agencies, for example, National Futures Association (NFA), Securities and Exchange Committee (SEC), the Federal Reserve Board, and U.S. Treasury Department also step in from time to time.

  The Commodities Futures Trading Commission (CFTC): to approve new contracts, set up daily maximum price fluctuation, minimum price movements, and certain features of delivery process

  The National Futures Association (NFA): to prevent fraudulent and manipulative acts and practices

  Trading irregularities: corner the market - take a huge long futures position and also try to exercise some control of the underlying commodity
  Example: Hunt brothers' price manipulation in the silver market in 1979-1980

- Accounting and taxes

  Changes in the market value of a futures contract are recognized when they occur unless the contract is qualified as a hedge

  For speculators, all paper gains or losses on futures contracts are treated as though they were realized at the end of the tax year - marking to market at the end of the year

  40% of any gains or losses are to be treated as short-term and
  60% of any gains or losses are to be treated as long-term

  For hedgers, all paper gains or losses are realized when the contracts are closed out

  The 40% short-term and 60% long-term rule doesn't apply for hedgers

- Assignments

  Quiz (required)
  Practice Questions: 2.11, 2.15, 2.16, and 2.23

Chapter 3 - Hedging Strategies Using Futures

- Hedging principles
- Arguments for and against hedging
- Basis risk
- Cross hedging
- Stock index futures
- Rolling hedging

- Hedging principles

  Hedging: to reduce risk
  Complete hedging: to eliminate risk

  Short hedges: use short positions in futures contracts to reduce or eliminate risk

  For example, an oil producer can sell oil futures contracts to reduce oil price uncertainty in the future

  Long hedge: use long positions in futures contracts to reduce or eliminate risk

  For example, a brewer buys wheat futures contracts to reduce future price uncertainty in wheat

- Arguments for and against hedging

  Hedging can reduce risk but it has a cost

  (1) Firms don't need to hedge because shareholders can hedge by themselves

  (2) Hedging may cause profit margin to fluctuate

  (3) Hedging may offset potential gains

- Basis risk

  Hedging usually can not be perfect for the following reasons:

  The asset whose price is to be hedged may not be exactly the same as the asset underlying the futures contract (e.g., stock index futures and your stock portfolio)

  The hedger may be uncertain as to the exact date when the asset will be bought or sold

  The hedge may require the futures contract to be closed out well before its expiration date

Basis

Basis (b) = spot price (S) - futures price (F)

Basis risk

Let S_1, F_1, and b_1 be the spot price, futures price, and basis at time t_1 and S_2, F_2, and b_2 be the spot price, futures price, and basis at time t_2, then $b_1 = S_1 - F_1$ at time t_1 and $b_2 = S_2 - F_2$ at time t_2.

Consider a hedger who knows that the asset will be sold at time t_2 and takes a short position at time t_1. The spot price at time t_2 is S_2 and the payoff on the futures position is ($F_1 - F_2$) at time t_2. The effective price is

$S_2 + F_1 - F_2 = F_1 + b_2$, where b_2 refers to the basis risk

Basis risk in a short hedge

Suppose it is March 1 (t_1). You expect to receive 50 million yen at the end of July (t_2). The September futures price (exchange rate) is currently 0.8900 dollar for 100 yen.

Hedging strategy:

(1) Sell four September yen futures contracts on March 1 (Since the contract size is 12.5 million yen 4 contracts will cover 50 million yen)

(2) Close out the contracts when yen arrives at the end of July

Basis risk: arises from the uncertainty as to the difference between the spot price and September futures price of yen at the end of July ($S_2 - F_2 = b_2$)

The outcome: at the end of July, suppose the spot price was 0.9150 and the September futures price was 0.9180, then the basis b_2 = 0.9150 - 0.9180 = - 0.0030

Gain on futures contract $F_1 - F_2$ = 0.8900 - 0.9180 = -0.0280

Effective price $F_1 + b_2$ = 0.8900 - 0.0030 = 0.8870 or

Effective price $S_2 + F_1 - F_2$ = 0.9150 - 0.0280 = 0.8870

Detailed illustration:

Sell 4 Sept. yen futures contracts	Receive 50 million yen, exchange yen to \$ at S_2, close Sept. contracts	Delivery month
↓	↓	↓

March 1 (t_1)	End of July (t_2)	September
(Know S_1, F_1, and b_1)	(Receive 50 million yen)	(F_1 = 0.8900, known in t_1)
(Don't know S_2, F_2, and b_2)	(Know S_2, F_2, and b_2)	(F_2 = 0.9180, known in t_2)
	(Effective price = $S_2 + F_1 - F_2$)	

Choice of contract for hedging:

(1) Same underlying asset (or closely related)

(2) Delivery month (usually a later delivery month)

- Cross hedging

  If the asset underlying the futures contract is the same as the asset whose price is being hedged, the hedge ratio usually is 1.0. For example, if a farmer expects to harvest 10,000 bushels of corn, the farmer should sell 2 corn futures contracts.

  Cross hedging: two different assets (but closely related)

  Minimum variance hedge ratio: the ratio of the size of the position taken in futures contract to the size of the exposure to minimize the variance of the hedged position

  Define
  ΔS : change in spot price, S
  ΔF : change in futures price, F
  σ_S : standard deviation of ΔS
  σ_F : standard deviation of ΔF
  ρ: correlation coefficient between ΔS and ΔF

  When the hedger is long the asset and short futures, the change in value of the hedged position is $\Delta S - h\Delta F$

  When the hedger is short the asset and long futures, the change in value of the hedged position is $h\Delta F - \Delta S$

  Taking variance of the hedged position, both positions yield

  $$v = \sigma_S^2 + h^2\sigma_F^2 - 2h\rho\sigma_S\sigma_F$$

  Taking the first order derivative of v with respect to h in order to minimize the risk

  Minimum variance hedge ratio, $h^* = \rho \dfrac{\sigma_S}{\sigma_F}$, where ρ is the correlation coefficient

  Note: h^* is the estimated slope coefficient in a linear regression of ΔF on ΔS and ρ^2 is the R^2 from the regression and it is called hedge effectiveness

  Optional number of contracts is given by:

  $N^* = h^* N_A/Q_F$, where N_A is the size of position being hedged, and Q_F is the size of futures contract

  For example, if an airline wants to purchase 2 million gallons of jet fuel and decides to use heating oil futures to hedge, and if h^* is 0.78 and the contract size for heating oil is 42,000 gallons, then the airline should buy 37 contracts to hedge (futures contracts on jet fuel are not available in the market).

- Stock index futures

  Stock indices: price weighted vs. value weighted

  Price weighted indices: for example, DJIA

  Value weighted indices: for example, S&P 500

  For stock index futures, the optional number of contracts is given by:

  $N^* = \beta P/A$, where P is the current value of the portfolio, A is the current value of the stocks underlying one futures contract, and β is the beta of the portfolio

  For example, if you want to hedge a stock portfolio using the S&P 500 futures contract, if P = \$5,000,000, β = 1.5, S&P 500 index = 1,000, A = 250*1,000 = 250,000, then

  N^* = 1.5*5,000,000/250,000 = 30 contracts (short)

  Changing beta of a portfolio from β to β^* ($\beta > \beta^*$)

  $N^* = (\beta - \beta^*)\, P/A$

  For example, if you want to reduce the portfolio beta to 0.75

  N^* = 15 contracts (short)

- Rolling hedging

  Rolling the hedge forward multiple times (n times)

Short futures contract 1	Close out future contract 1 and short futures contract 2	Close out futures contract 2 and short futures contract 3		Close out futures contract *n*
Time 1 (t_1)	Time 2 (t_2)	Time 3 (t_3)	- - - - - -	Time T (t_n)

- Assignments

  Quiz (required)

  Practice Questions: 3.12, 3.16, and 3.18

Chapter 4 - Interest Rates

- Types of interest rates
- Measuring interest rates
- Zero rates
- Bond pricing
- Forward rates
- Term structure theories

- Interest rates

  Treasury rates - risk-free rates

  T-bill rates vs. T-bond rates

  LIBOR (London Interbank Offer Rate): used between large international banks

  LIBID (London Interbank Bid Rate): used between large international banks

  LIBOR > LIBID

  Repo rate: an investment dealer sells its securities to another company and agrees to buy them back later at a slightly higher price - the percentage change in prices is the repo rate

- Measuring interest rates

  Nominal rate vs. effective rate

  Effective rate $= (1 + R/m)^m - 1$, where *R* is the annual nominal rate and *m* is the number of compounding within a year

  If *m* goes to infinity, we have effective rate under continues compounding, e^R - 1

  In general, the FV of \$A compounded continuously for *n* years at a nominal annual rate of *R* is FV = Ae^{Rn}

  In the same way, the PV of \$A discounted continuously at a nominal rate of *R* for *n* years is PV = Ae^{-Rn}

  Relationship between continues compounding and compounding *m* times per year:

  $e^{R_C} = (1 + R_m/m)^m$, where R_c is a rate of interest with continues compounding and R_m is the equivalent rate with compounding *m* times per year, or $R_C = m * \ln(1 + R_m/m)$

  For example, for a 10% annual rate with semiannual compounding, the equivalent rate with continuous compounding is R_c = 2*ln (1 + 0.1/2) = 9.758%

- Zero rates

  Zero rates: n-year zero-coupon interest rate is the rate of interest earned on an investment that starts today and lasts for n years

- Bond pricing

  When pricing a bond, you should use different zero rates to discount all expected future cash flows (coupon payments and face value) to the present.

  Determining Treasury zero rates using T-bond price quotes

Bond principal ($)	Time to maturity (years)	Annual coupon ($)	Bond price (S)
100	0.25	0	97.5
100	0.50	0	94.9
100	1.00	0	90.0
100	1.50	8	96.0
100	2.00	12	101.6

  For a 3 month T-bond, the price is 97.5 for $100 face value. With quarterly compounding, the 3 month T-bond has a zero rate of 4*(2.5)/97.5 = 10.256%. Convert that rate to continuous compounding

  $R_c = m$*ln($1 + R_m/m$) = 0.10127 = 10.127% (with $m = 4$ and R_m is 10.256%)

  Similarly, we can obtain the 6 month and 1 year T-bond zero rates of 10.469% and 10.536% for continuous compounding.

  To determine the 1.5 year zero-rate, we solve

  $4e^{-0.10469*0.5} + 4e^{-0.10536*1} + 104e^{-R*1.5} = 96$ (semiannual coupon payments)

  R = 10.681%

  In a similar way, we can obtain the 2 year zero-rate of 10.808%

- Forward rates

  Spot rate vs. forward rate

  An n-year spot rate is the interest rate on an investment that starts today and lasts for n years

  A forward rate is an interest rate that is implied by the current spot rates for periods of time in the future

  In general, $R_F = (R_2T_2 - R_1T_1) / (T_2 - T_1)$

  For example, if $T_1 = 3$, $T_2 = 4$, $R_1 = 0.046$ = 4.6%, and $R_2 = 0.05$ = 5%, then R_F = 6.2%

- Term structure theories

    Term structure of interest rates and yield curves

    Term structure of interest rates: relationship between interest rates (yields) and time to maturity

    Yield curve: a graph showing the term structure of interest rates

    Expectation theory: long-tern interest rates should reflect expected futures short-term interest rates

    For example, if a 1-year bond yields 5% and a 2-year bond yields 5.5%, then the 1-year forward rate between year 1 and year 2 is expected to be 6%.

    $R_F = (R_2T_2 - R_1T_1) / (T_2 - T_1) = (5.5\%*2 - 5\%*1) / (2 - 1) = 6\%$

    If a yield curve is upward sloping, it indicates that the short term interest rates in the future will rise.

    Market segmentation theory: interest rates are determined by the demand and supply in each of different markets (short-, medium-, and long-term markets, for example)

    Liquidity preference theory: investors prefer to invest in short-term funds while firms prefer to borrow for long periods - yield curves tend to be upward sloping

- Assignments

    Quiz (required)
    Practice Questions: 4.10 and 4.14

Chapter 5 - Determination of Forward and Futures Prices

- Investment assets vs. consumption assets
- Short selling
- Assumptions and notations
- Forward price for an investment asset that provides no income
- Forward price for an investment asset that provides a known cash income
- Forward price for an investment asset that provides a known dividend yield
- Valuing forward contracts
- Forward prices and futures prices
- Stock index futures
- Currency futures
- Commodity futures
- Cost of carry

- Investment assets vs. consumption assets

  An investment asset is an asset that is held mainly for investment purpose, for example, stocks, bonds, gold, and silver

  A consumption asset is an asset that is held primarily for consumption purpose, for example, oil, meat, and corn

- Short selling

  Selling an asset that is not owned

- Assumptions and notations

  Assumptions

  Perfect capital markets: transaction costs are ignored, borrowing and lending rates are the same, taxes are ignored (or subject to the same tax rate), and arbitrage profits are exploited away

  Arbitrage profit is the profit from a portfolio that involves

  1. Zero net cost
  2. No risk in terminal portfolio value
  3. Positive profit

  Notations

  T: time until delivery date (years)

  S_0: spot price of the underlying asset today

  F_0: forward price today = delivery price K if the contract were negotiated today

  r: zero coupon risk-free interest rate with continuous compounding for T years to maturity

- Forward price for an investment asset that provides no income

  Consider a forward contract negotiated today

  T = 3 months = ¼ year, S_0 = \$40, r = 5%, F_0 = ?

  General solution: $F_0 = S_0\, e^{rT}$ (5.1)

  So, F_0 = \$40.50

  If equation (5.1) does not hold, an arbitrage opportunity exists

  For example, if $F_0 = 43 > 40.50$, an arbitrage profit = 43 - 40.50 = \$2.50
  Strategy: (Forward price is too high relative to spot price)
  Today:
  (1) Borrow \$40.00 at 5% for 3 months and buy one unit of the asset
  (2) Sell a 3-month forward contract for one unit of the asset at \$43

  After 3 months:
  (1) Make the delivery and collect \$43
  (2) Reply the loan of \$40.50 = $40e^{0.05*3/12}$
  (3) Count for profit = 43.00 - 40.50 = \$2.50

  If $F_0 = 39 < 40.50$, an arbitrage profit = 40.50 - 39 = \$1.50

  Show the proof by yourself as an exercise

  Application: stocks, bonds, and any other securities that do not pay current income during the specified period

- Forward price for an investment asset that provides a known cash income

  Consider a forward contract negotiated today

  T = 3 months = ¼ year, S_0 = \$40, r = 5%
  In addition, the asset provides a known income in the future (dividends, coupon payments, etc.) with a PV of I = \$4, F_0 = ?

  General solution, $F_0 = (S_0 - I)e^{rT}$ (5.2)

  So F_0 = \$36.45

  If $F_0 = 38 > 36.45$, an arbitrage profit = 38 - 36.45 = \$1.55

  Show the proof by yourself as an exercise

If F_0 = 35 < 36.45, an arbitrage profit = 36.45 - 35 = \$1.45
Strategy: (Forward price is too low relative to spot price)

Today:
(1) Short sell one unit of asset at the spot price for \$40.00
(2) Deposit \$40.00 at 5% for 3 months
(3) Buy a 3-month forward contract for one unit of the asset at \$35

After 3 months:
(1) Take money out of the bank (\$40.50)
(2) Take the delivery by paying \$35.00 and return the asset plus income (\$4.05)
(3) Count for profit = 40.50 - 35.00 - 4.05 = \$1.45

Application: stocks, bonds, and any other securities that pay a known cash income during the specified period

- Forward price for an investment asset that provides a known yield

Consider a forward contract negotiated today

T = 3 months = ¼ year, S_0 = \$40, r = 5%
In addition, a constant yield which is paid continuously as the percentage of the current asset price is q = 4% per year. Then, in a smaller time interval, for example, return on one day = current asset price * 0.04/365, F_0 = ?

General solution: $F_0 = S_0\, e^{(r-q)T}$ (5.3)

So, F_0 = \$40.10

If F_0 = 42 > 40.10, an arbitrage profit = 42 - 40.10 = \$1.90
Strategy: (forward price is too high relative to spot price)
Today:
(1) Borrow \$40 at 5% for 3 months and buy one unit of the asset at \$40
(2) Sell a 3-month forward contract for one unit of the asset at \$42

After 3 months:
(1) Make the delivery and collect \$42.00
(2) Pay off the loan in the amount of \$40.50 ($40e^{0.05*3/12}$)
(3) Receive a known yield for three months of \$0.40 (3/12 of 40*0.04)
(4) Count for profit = 42 - 40.50 + 0.40 = \$1.90

If F_0 = 39 < 40.10, an arbitrage profit = 40.10 - 39 = \$1.10
Show the proof by yourself as an exercise

Application: stock indexes

- Valuing forward contracts

  K: delivery price

  f: value of the forward contract today, $f = 0$ at the time when the contract is first entered into the market ($F_0 = K$)

  In general: $f = (F_0 - K)\, e^{-rT}$ for a long position, where F_0 is the current forward price

  For example, you entered a long forward contract on a non-dividend-paying stock some time ago. The contract currently has 6 months to maturity. The risk-free rate is 10%, the delivery price is $24, and the current market price of the stock is $25.

  Using (5.1), $F_0 = 25e^{0.1*6/12} = \$26.28$, $f = (26.28 - 24)\, e^{-0.1*6/12} = \$2.17$

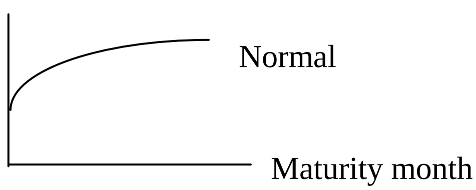
  Similarly, $f = (K - F_0)\, e^{-rT}$ for a short position

- Forward prices and futures prices

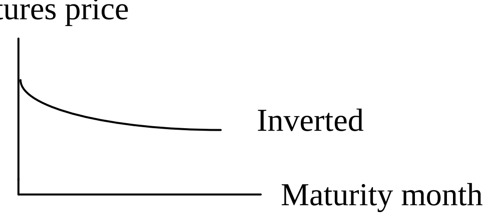
  Under the assumption that the risk-free interest rate is constant and the same for all maturities, the forward price for a contract with a certain delivery date is the same as the futures price for a contract with the same delivery date.

  Futures price = delivery price determined as if the contract were negotiated today
  The formulas for forward prices apply to futures prices after daily settlement

  Patterns of futures prices
  It increases as the time to maturity increases - normal market
  Futures price



  It decreases as the time to maturity increases - inverted market
  Futures price



  Futures prices and expected future spot prices
  Keynes and Hicks: hedgers tend to hold short futures positions and speculators tend to hold long futures position, futures price < expected spot price because speculators ask for compensation for bearing the risk (or hedgers are willing to pay a premium to reduce the risk)

Risk and return explanation: if the return from the asset is not correlated with the market (beta is zero), $k = r$, $F_0 = E(S_T)$; if the return from the asset is positively correlated with the market (beta is positive), $k > r$, $F_0 < E(S_T)$; if the return from the asset is negatively correlated with the market (beta is negative) $k < r$, $F_0 > E(S_T)$

- Stock index futures

Stock index futures: futures contracts written on stock indexes

Futures contracts can be written on many indices, such as
DJIA: price weighted, $10 time the index
Nikkei: price weighted, $5 times the index
S&P 500 index: value weighted, $250 times the index
NASDAQ 100 index: value weighted, $20 times the index

Stock index futures prices, recall (5.3)

General formula: $F_0 = S_0\, e^{(r-q)T}$, where q is the continuous dividend yield

If this relationship is violated, you can arbitrage - index arbitrage

Speculating with stock index futures

If you bet that the general stock market is going to fall, you should short (sell) stock index futures

If you bet that the general stock market is going to rise, you should long (buy) stock index futures

Hedging with stock index futures

Short hedging: take a short position in stock index futures to reduce downward risk in portfolio value

Long hedging: take a long position in stock index futures to not miss rising stock market

- Currency futures

Exchange rate and exchange rate risk

Direct quotes vs. indirect quotes
1 pound / $1.60 (direct) vs. 0.625 pound / $1.00 (indirect)

Exchange rate risk: risk caused by fluctuation of exchange rates

Currency futures prices, recall (5.3)

General formula: $F_0 = S_0\, e^{(r-rf)T}$, where r_f is the foreign risk-free rate

For example, if the 2-year risk-free interest rate in Australia and the US are 5% and 7% respectively, and the spot exchange rate is 0.6200 USD per AUD, then the 2-year forward exchange rate should be 0.6453. If the 2-year forward rate is 0.6300, arbitrage opportunity exists. To arbitrage:
(1) Borrow 1,000 AUD at 5%, convert it to 620 USD and invest it in the U.S. at 7% for 2 years (713.17 USD in 2 years)
(2) Enter a 2-year forward contract to buy AUD at 0.6300
(3) After 2 years, collect 713.17 USD and convert it to 1,132.02 AUD
(4) Repay the loan plus interest of 1,105.17 AUD
(5) Net profit of 26.85 AUD (or 16.91 USD)

Speculation using foreign exchange futures
If you bet that the British pound is going to depreciate against US dollar you should sell British pound futures contracts
If you bet that the British pound is going to appreciate against US dollar you should buy British pound futures contracts

Hedging with foreign exchange futures to reduce exchange rate risk

- Commodity futures

  Commodities: consumption assets with no investment value, for example, wheat, corn, crude oil, etc.

  For commodity futures prices, recall (5.1), (5.2), and (5.3)

  For commodities with no storage cost:

  $F_0 = S_0 e^{rT}$

  For commodities with storage cost:

  $F_0 = (S_0 + U)e^{rT}$, where U is the present value of all storage costs
  $F_0 = S_0\, e^{(r+u)T}$, where u is the storage costs per year as a percentage of the spot price

  Consumption purpose: reluctant to sell commodities and buy forward contracts

  $F_0 \leq S_0\, e^{rT}$, with no storage cost
  $F_0 \leq (S_0 + U)e^{rT}$, where U is the present value of all storage costs
  $F_0 \leq S_0\, e^{(r+u)T}$, where u is the storage costs per year as a percentage of the spot price

- Cost of carry

  It measures the storage cost plus the interest that is paid to finance the asset minus the income earned on the asset

Asset	Futures price	Cost of carry
Stock without dividend	$F_0 = S_0\, e^{rT}$	r
Stock index with dividend yield q	$F_0 = S_0\, e^{(r-q)T}$	r-q
Currency with interest rate r_f	$F_0 = S_0\, e^{(r-rf)T}$	r-r_f
Commodity with storage cost u	$F_0 = S_0\, e^{(r+u)T}$	$r+u$

- Assignments

  Quiz (required)

  Practice Questions: 5.9, 5.10, 5.14, and 5.15

Chapter 6 - Interest Rate Futures

- Day count and quotation conventions
- T-bond futures
- T-bill futures
- Duration
- Duration based hedging
- Speculation and hedging with interest rate futures

- Day count and quotation conventions

  Three day counts are used in the U.S.

  Actual/actual: T-bonds
  For example, the coupon payment for a T-bond with 8% coupon rate (semiannual payments on March 1 and September 1) between March 1 and July 3 is
  (124/184)*4 = $2.6957

  30/360: corporate and municipal bonds
  For example, for a corporate bond with the same coupon rate and same time span mentioned above, the coupon payment is
  (122/180)*4 = $2.7111

  Actual/360: T-bills and other money market instruments
  The reference period is 360 days but the interest earned in a year is 365/360 times the quoted rate

  Quotes for T-bonds: dollars and thirty-seconds for a face value of $100

  For example, 95-05 indicates that it is $95 5/32 ($95.15625) for $100 face value or $95,156.25 for $100,000 (contract size)

  Minimum tick = 1/32

  Daily price limit is 3 full points (96 of 1/32, 3% of the face value, equivalent to $3,000)

  For T-bonds, cash price = quoted cash price + accrued interest

  Example

  0 182 days

  40 days 142 days remaining until next coupon

  Suppose annual coupon is $8 and the quoted cash price is 99-00 (or $99 for a face value of $100) then the cash price = 99 + (4/182)*40 = $99.8791

T-bills: sold at a discount with no interest payment, the face value is $1,000,000

Price quotes are for a face value of $100

The cash price and quoted price are different

For example, if y is the cash price of a T-bill that will mature in n days, then the quoted price (P) is given by

P = (360/n)*(100 - y), known as the discount rate

If y = 98, n = 91 days, then the quoted price = 7.91

Return on the T-bill = (2/98)*(365/91) = 8.186% (2/98 = 2.04% for 91 days)

- T-bond futures

T-bond futures are quoted as T-bonds and the price received for each $100 face value with a short position upon delivery is

Invoice amount = quoted futures price*(CF) + AI,
where CF = conversion factor and AI = accrued interest

For example, if the quoted futures price = 90-00, CF = 1.38, and AI = $3.00, then
Invoice amount = 90(1.38) + 3.00 = $127.20

Conversion factor: equal to the quoted price the bond would have per dollar of principal on the first day of the delivery month on the assumption that the interest rate for all maturities equals 6% per year (with semiannual compounding)

For example, let's consider a 10% coupon bond with 20 years and 2 months to maturity. For the purpose of calculating the CF, the bond is assumed to mature in 20 years (round down to the nearest 3 months). The value of the bond is

$\sum_{i=1}^{40} \frac{5}{(1+0.03)^i} + \frac{100}{(1+0.03)^{40}} = \$146.23$. Dividing by the face value gives the CF 1.4623.

(Or you can use a financial calculator to figure out the CF: PMT = 5, FV = 100, N = 40, i/y = 3%, solve for PV = 146.23, CF = 146.23/100 = 1.4623)

Cheapest-to-deliver issues
Since the person with a short position can deliver any T-bond that has more than 15 years to maturity and that it is not callable within 15 years, which bond is the cheapest to deliver?

Recall: invoice price = quoted futures price*(CF) + AI, where CF = conversion factor, and AI = accrued interest

The cost of purchasing a bond is
Cash price = quoted cash price + accrued interest

The net cost is the difference between the cash price and the invoice price

Net cost to deliver = quoted cash price - (quoted futures price*CF)
(Note, AI is cancelled out)

Choose the bond that minimizes the net cost to deliver

Determining the quoted T-bond futures price, use $F_0 = (S_0 - I)e^{rT}$

Example 6.1: choose a bond from below to deliver, assuming the most recent settlement price (quoted futures price) is 93-08 (or 93.25)

Bond	Quoted cash price	CF	Net cost to deliver
1	99.50	1.0382	99.50 – (93.25*1.0382) = $2.69
2	143.50	1.5188	143.50 – (93.25*1.5188) = $1.87
3	119.75	1.2615	119.75 – (93.25*1.2615) = $2.12

The cheapest-to-deliver is bond 2

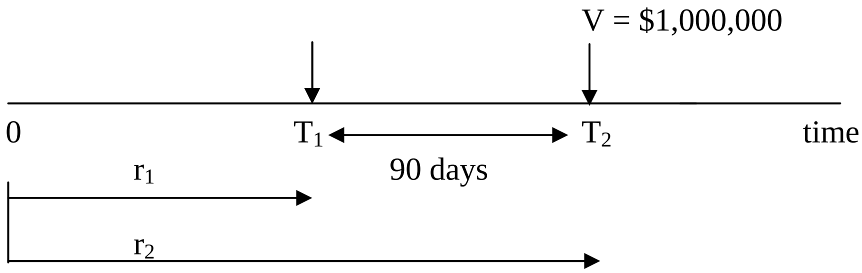
- T-bill futures

  Call for delivery of T-bills with a face value of $1,000,000 and a time to maturity of 90 days

  The contract size is $1,000,000 with delivery months set in March, June, September, and December

  No daily price limit

  Minimum tick is 0.01% of discount yield (one basis point, equivalent to $25.00; or when the interest rate changes by 1 basis point, 0.01%, the interest earned on $1,000,000 face value for 3 months changes by $25.00)



Determining the current (present) value, use $PV = \mathrm{V}\, e^{-r_2 T_2}$

Determining the futures price, use $F = PV e^{r_1 T_1}$

Why buy a futures contract on T-bills? To lock in a short-term interest rate

Eurodollar futures are similar to T-bill futures

- Duration

  A measure of how long on average the bondholder has to wait before receiving cash payments

  Suppose a bond provides cash flows c_t at time t. The bond price B and bond yield y (with continuous compounding) are related by:

$$B = \sum_{i=1}^{n} c_i e^{-y t_i}$$

The duration of the bond (D) is defined as: $D = \dfrac{\sum_{i=1}^{n} t_i c_i e^{-y t_i}}{B} \;=\; \sum_{i=1}^{n} t_i \left[\dfrac{c_i e^{-y t_i}}{B}\right]$

For a small change in yield (Δy) the change in bond price is approximately:

$$\Delta B = \frac{dB}{dy}\Delta y = -\Delta y \sum_{i=1}^{n} c_i t_i e^{-y t_i} \;= -BD\Delta y \;\text{ or }\; \frac{\Delta B}{B} = -D\Delta y$$

Percentage change in bond price = – duration*the change in yield (where yield is expressed with continuous compounding)

For example, a 3-year bond with coupon rate of 10%, payable semiannually, sells for 94.213 and has a yield to maturity of 12% (continuous compounding). The duration for the bond is 2.653 years. If the yield increases by 0.1% the bond price will drop by 0.250 to 93.963 [-94.213*2.653*(0.001) = -0.25].

Time (years)	Cash flow ($)	Present value	Weight	Time*weight
0.5	5	4.709	0.050	0.025
1.0	5	4.435	0.047	0.047
1.5	5	4.176	0.044	0.066
2.0	5	3.933	0.042	0.083
2.5	5	3.704	0.039	0.098
3.0	105	73.256	0.778	2.333
Total	130	94.213	1.000	2.653

What if *y* is not continuous (for example, semiannual compounding)?

If *y* is not expressed with continuous compounding then $\Delta B = -BD^* \Delta y$, where D^* is called modified duration

$D^* = \frac{D}{1+y/m} = \frac{2.653}{1+0.12/2} = 2.50$, where D is the duration of the bond and y is the yield with a compounding frequency of m times per year

For a zero-coupon bond, the duration is equal to the time to maturity

- Duration based hedging

Interest rate risk: interest rate price risk vs. interest rate reinvestment risk

Interest rate price risk: risk that the bond value (price) falls when the market interest rates rise

Reinvestment risk: risk that the interest received will be reinvested at a lower rate

Relationship between duration and bond price volatility: $\Delta B = -BD\Delta y$

Define
F_C: contract price for the interest rate futures contract
D_F: duration of the asset underlying the futures contract at the maturity of the futures contract
P: forward value of the portfolio being hedged at the maturity of the hedge (assumed to be the value of the portfolio today)
D_P: duration of the portfolio at the maturity of the hedge

Then we have approximations of $\Delta P = -PD_P \Delta y$ and $\Delta F_C = -F_C D_F \Delta y$, the optimal number of contracts to hedge against an uncertain Δy is

$$N^* = \frac{PD_P}{F_C D_F}$$, called duration-based hedge ratio

For example, on August 2, a fund manager with $10 million invested in government bonds is concerned that interest rates are expected to be volatile over the next 3 months. The manager decides to use December T-bond futures contract to hedge the portfolio. The current futures price is 93-02, or 93.0625 for $100 face value. Since the contract size is $100,000, the futures price is $93,062.50. Further suppose that the duration of the bond portfolio in 3 months is 6.80 years. The cheapest-to-deliver bond in the T-bond contract is expected to be a 20-year 12% coupon bond. The yield on this bond is currently 8.8% and the duration is 9.20 years at maturity of the futures contract. The duration-based hedge ratio is

$$N^* = \frac{10{,}000{,}000}{93{,}062.50} * \frac{6.80}{9.20} = 79.42$$ contracts, short positions

Application of bond immunization: banking management, pension fund management

- Speculation and hedging with interest rate futures

  Speculation with outright positions
  A speculator with a long position is betting that the interest rate is going to fall so the price of interest rate futures is going to rise

  A speculator with a short position is betting that the interest rate is going to rise so the price of interest rate futures is going to fall

  Speculation with spreads
  An intra-commodity T-bill spread: speculating the term structure of interest rates, for example, T-bill futures with different maturities (nearby vs. distant)

  An inter-commodity spread: speculating on shifting risk levels between different instruments, for example, T-bill futures vs. T-bond futures

  Hedging with interest rate futures
  A long hedge: take a long position in futures market to reduce interest rate risk
  A short hedge: take a short position in futures market to reduce interest rate risk

- Assignments

  Quiz (required)
  Practice Questions: 6.8, 6.9, and 6.10

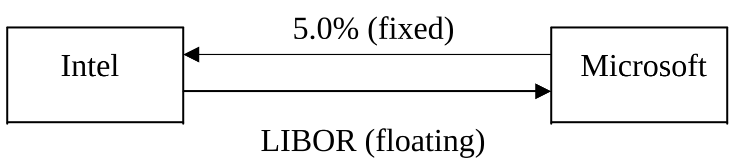
Chapter 7 - Swaps

- Swaps
- Interest-rate swaps
- Role of financial intermediary
- Comparative advantage
- Valuation of interest-rate swaps
- Currency swaps
- Valuation of currency swaps

- Swaps

  A swap is a private agreement between two companies to exchange cash flows in the future according to a prearranged formula: an extension of a forward contract.

  For example, Intel and Microsoft agreed a swap in interest payments on a notional principal of $100 million. Microsoft agreed to pay Intel a fixed rate of 5% per year while Intel agreed to pay Microsoft the 6-month LIBOR.



  Cash flows to Microsoft in a $100 million 3-year interest rate swap when a fixed rate of 5% is paid and LIBOR is received:

Date	6-month LIBOR	Floating cash flow	Fixed cash flow	Net cash flow
3-5-2010	4.20%			
9-5-2010	4.80%	+2.10 million	-2.50 million	-0.40 million
3-5-2011	5.30%	+2.40 million	-2.50 million	-0.10million
9-5-2011	5.50%	+2.65 million	-2.50 million	+0.15 million
3-5-2012	5.60%	+2.75 million	-2.50 million	+0.25 million
9-5-2012	5.90%	+2.80 million	-2.50 million	+0.30 million
3-5-2013		+2.95 million	-2.50 million	+0.45 million

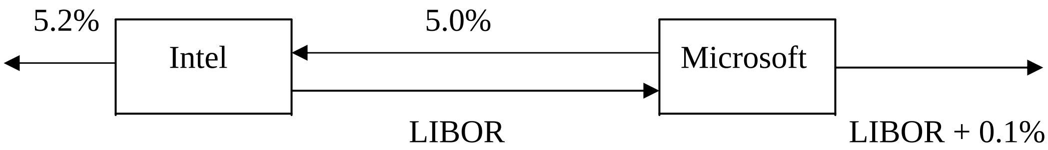
  Cash flows to Intel will be the same in amounts but opposite in signs.

  The floating rate in most interest rate swaps is the London Interbank Offered Rate (LIBOR). It is the rate of interest for deposits between large international banks.

- Interest-rate swaps

  Using swaps to transform a liability

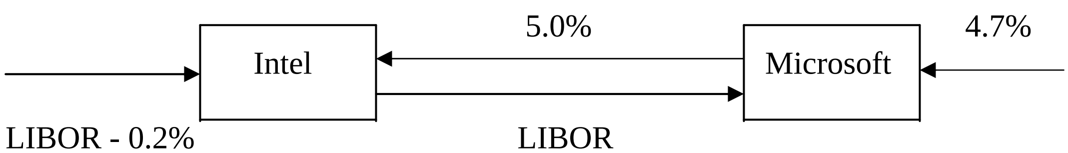
  For example, Microsoft could transform a floating-rate loan to a fixed rate loan while Intel could transform a fixed rate loan to a floating-rate loan



  After swap, Intel pays LIBOR + 0.2% and Microsoft pays 5.1%

  Using swaps to transform an asset

  For example, Microsoft could transform an asset earning fixed-rate to an asset earning floating-rate while Intel could transform an asset that earns floating-rate to an asset that earns fixed-rate



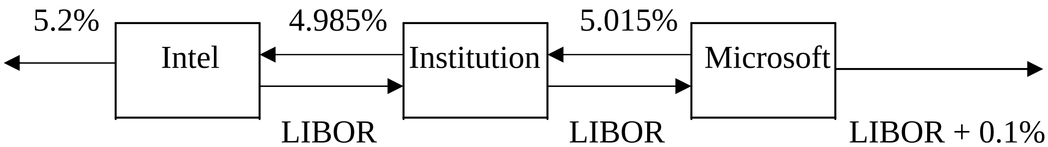
  After swap, Intel earns 4.8% and Microsoft earns LIBOR - 0.3%

  We will ignore the day count convention (LIBOR uses 360 days per year while a fixed rate uses 365 days per year)

- Role of financial intermediary

  Arranging the swaps to earn about 3-4 basis points (0.03-0.04%)

  Refer to the swap between Microsoft and Intel again when a financial institution is involved to earn 0.03% (shared evenly by both companies)



  After swap, Intel pays LIBOR + 0.215%, Microsoft pays 5.115%, and the financial institution earns 0.03%

- Comparative advantage

    Some companies have a comparative advantage when borrowing in fixed rate (U.S dollars) markets, whereas others have a comparative advantage in floating-rate (foreign currency) markets.

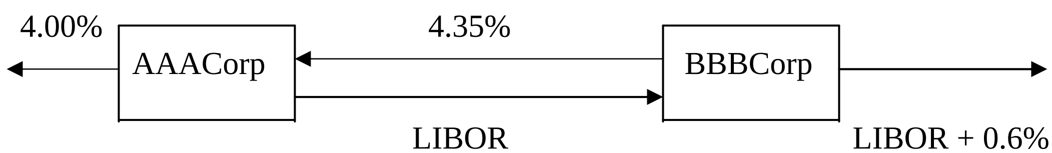
    Let us consider two corporations, AAACorp and BBBCorp. Both companies are going to borrow $10 million and are facing the following rates. Assume that AAACorp wants floating rate and BBBCorp wants fixed rate:

	Fixed	Floating
AAACorp	4.00%	LIBOR - 0.1%
BBBCorp	5.20%	LIBOR + 0.6%

How can a swap benefit both companies?

Answer: AAACorp borrows at the fixed rate and BBBCorp borrows at the floating rate and then two companies engage in a swap

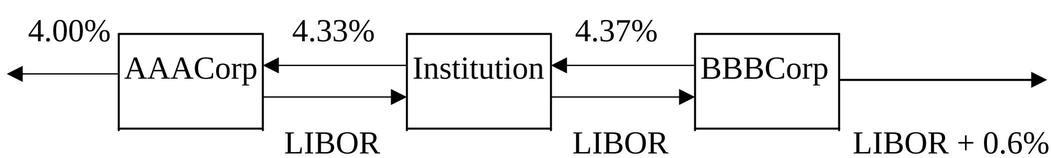
Without a financial institution involved



After swap, AAACorp pays LIBOR - 0.35% and BBBCorp pays 4.95% (both benefit by 0.25%)

The total benefit is equal to a - b (0.5%), where a is the difference between the interest rates in fixed rate markets (1.20%) and b is the difference between the interest rates in floating rate markets (0.7%). The total gain doesn't have to be shared evenly.

When a financial institution is involved and it earns 4 basis points (0.04%):



After swap, AAACorp pays LIBOR - 0.33% and BBBCorp pays 4.97% (both benefit by 0.23%) while the financial institution earns 0.04%. The total gain remains at 0.5%.

- Valuation of interest-rate swaps

  $V_{swap} = B_{fl} - B_{fix}$ (or $V_{swap} = B_{fix} - B_{fl}$), where B_{fix} is the present value of fixed-rate bond underlying the swap and B_{fl} is the preset value of floating-rate bond underlying the swap

  Suppose that a financial institution has agreed to pay 6-month LIBOR and receive 8% fixed rate (semiannual compounding) on a notional principal of $100 million. The swap has a remaining life of 1.25 years. The LIBOR rates with continuous compounding for 3-month, 9-month, and 15-month maturities are 10%, 10.5%, and 11%, respectively. The 6-month LIBOR rate at the last payment date was 10.2%.

Time Years	B_{fix} cash flow	B_{fl} cash flow	Discount factor	Present value B_{fix} cash flow	Present value B_{fl} cash flow
0.25	4.0	105.10	0.9753	3.901	102.505
0.75	4.0		0.9243	3.697	
1.25	104.0		0.8715	90.640	

  $V_{swap} = B_{fix} - B_{fl}$ = (3.901+3.697+90.640) – 102.505 = -4.267 million

  Note: B_{fl} = L (notional principal immediately after an interest payment) and therefore B_{fl} = 100 + 100*0.051 = 105.10 million after 3 months

  If the financial institution pays fixed and receives floating, the value of the swap would be +4.267 million (again a zero-sum game)

- Currency swaps

  A currency swap is an agreement to exchange interest payments and principal in one currency for principal and interest payments in another currency. It can transform a loan in one currency into a loan in another currency.

  Let us consider two corporations, GM and Qantas Airway. Both companies are going to borrow money and are facing the following rates. Assume GM wants to borrow AUD and Qantas Airway wants to borrow USD.

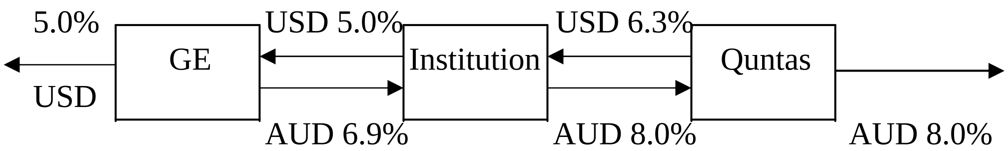
	USD	AUD
GE	5.00%	7.60%
Qantas	7.00%	8.00%

  How can a swap benefit both companies?

  Answer: GE borrows USD and Qantas borrows AUD and then two companies engage in a swap

  Since a = 2.00% and b = 0.4%, therefore the net gain = a – b = 1.6%

Assuming a financial institution arranges the swap and earns 0.2% (by taking the exchange rate risk)



Net outcome:
GE borrows AUD at 6.9% (0.7% better than it would be if it went directly to AUD markets)
Quntas borrows USD at 6.3% (0.7% better than it would be if it went directly to USD markets)
Financial institution receives 1.3% USD and pays 1.1% AUD and has a net gain of 0.2%

- Valuation of currency swaps
  $V_{\text{swap}} = B_D - S_0 B_F$ (or $V_{\text{swap}} = S_0 B_F - B_D$), where S_0 is the spot exchange rate, B_F and B_D are the values of the foreign-denominated bond underlying the swap and the U.S. dollar bond underlying the swap

Swap examples

1. Companies X and Y have been offered the following rates per year on a $5 million ten-year loan:

	Fixed-rate	Floating-rate
Company X	7.0%	LIBOR + 0.5%
Company Y	8.8%	LIBOR + 1.5%

Company X requires a floating-rate loan and Company Y requires a fixed-rate loan. Design a swap that will net a financial institution acting as an intermediary 0.1% per year and it will be equally attractive to X and Y.

Answer: a = 1.8% (8.8% - 7.0%) and b = 1.0% [(LIBOR + 1.5%) – (LIBOR + 0.5%)], swap provides a net gain of 0.8% (1.8% - 1.0%), less 0.1% for the bank, leaving 0.7% net gain to be shared by X and Y (0.35% each)

After the swap, X pays floating at LIBOR + 0.15%, Y pays fixed at 8.45%, and the institution earns 0.1%

2. Companies A and B are facing the following annual interest rates in US and UK:

	Sterling	U.S. Dollar
Company A	8.0%	7.0%
Company B	7.6%	6.2%

A wants to borrow dollar and B wants to borrow sterling. Design a swap that will net a financial institution acting as an intermediary 0.10% per year and it will be equally attractive to both companies.

Answer: a = 0.4% (8.0% - 7.6%) and b = 0.8% (7.0 - 6.2%), a swap provides a total gain of 0.4% (0.8% - 0.4%), less 0.1% for the bank, leaving 0.3% net gain to be shared by A and B (0.15% each)

After the swap, A borrows dollar at 6.85%, B borrows sterling at 7.45%, and the financial institution earns 0.1%.

- Assignments
  Quiz (required)
  Practice Questions: 7.9, 7.10, and 7.11