

## Chapter 5 - Determination of Forward and Futures Prices

- Investment assets vs. consumption assets
  - Short selling
  - Assumptions and notations
  - Forward price for an investment asset that provides no income
  - Forward price for an investment asset that provides a known cash income
  - Forward price for an investment asset that provides a known dividend yield
  - Valuing forward contracts
  - Forward prices and futures prices
  - Stock index futures
  - Currency futures
  - Commodity futures
  - Cost of carry
- 
- Investment assets vs. consumption assets
    - An investment asset is an asset that is held mainly for investment purpose, for example, stocks, bonds, gold, and silver
  
    - A consumption asset is an asset that is held primarily for consumption purpose, for example, oil, meat, and corn
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- Short selling
    - Selling an asset that is not owned – Table 5.1, cash flows from short sale and purchase of shares, a review
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- Assumptions and notations
    - Assumptions
      - Perfect capital markets: transaction costs are ignored, borrowing and lending rates are the same, taxes are ignored (or subject to the same tax rate), and arbitrage profits are exploited away
  
    - Arbitrage profit is the profit from a portfolio that involves
      1. Zero net cost
      2. No risk in terminal portfolio value
      3. Positive profit
  
    - Notations
      - $T$ : time until delivery date (years)
      - $S_0$ : spot price of the underlying asset today
      - $F_0$ : forward price today = delivery price  $K$  if the contract were negotiated today
      - $r$ : zero coupon risk-free interest rate with continuous compounding for  $T$  years to maturity

- Forward price for an investment asset that provides no income  
Consider a forward contract negotiated today

$T = 3 \text{ months} = 1/4 \text{ year}, S_0 = \$40, r = 5\%, F_0 = ?$

General solution:  $F_0 = S_0 e^{rT}$  (5.1)

So,  $F_0 = \$40.50$

If equation (5.1) does not hold, an arbitrage opportunity exists

For example, if  $F_0 = 43 > 40.50$ , an arbitrage profit =  $43 - 40.50 = \$2.50$

Strategy: (Forward price is too high relative to spot price)

Today:

- (1) Borrow \$40.00 at 5% for 3 months and buy one unit of the asset
- (2) Sell a 3-month forward contract for one unit of the asset at \$43

After 3 months:

- (1) Make the delivery and collect \$43
- (2) Repay the loan of  $\$40.50 = 40e^{0.05 \cdot 3/12}$
- (3) Count for profit =  $43.00 - 40.50 = \$2.50$

If  $F_0 = 39 < 40.50$ , an arbitrage profit =  $40.50 - 39 = \$1.50$

Show the proof by yourself as an exercise

Application: stocks, bonds, and any other securities that do not pay current income during the specified period

- Forward price for an investment asset that provides a known cash income  
Consider a forward contract negotiated today

$T = 3 \text{ months} = 1/4 \text{ year}, S_0 = \$40, r = 5\%$

In addition, the asset provides a known income in the future (dividends, coupon payments, etc.) with a PV of  $I = \$4, F_0 = ?$

General solution,  $F_0 = (S_0 - I)e^{rT}$  (5.2)

So  $F_0 = \$36.45$

If  $F_0 = 38 > 36.45$ , an arbitrage profit =  $38 - 36.45 = \$1.55$

Show the proof by yourself as an exercise

If  $F_0 = 35 < 36.45$ , an arbitrage profit =  $36.45 - 35 = \$1.45$   
Strategy: (Forward price is too low relative to spot price)

Today:

- (1) Short sell one unit of asset at the spot price for \$40.00
- (2) Deposit \$40.00 at 5% for 3 months
- (3) Buy a 3-month forward contract for one unit of the asset at \$35

After 3 months:

- (1) Take money out of the bank (\$40.50)
- (2) Take the delivery by paying \$35.00 and return the asset plus income (\$4.05)
- (3) Count for profit =  $40.50 - 35.00 - 4.05 = \$1.45$

Application: stocks, bonds, and any other securities that pay a known cash income during the specified period

- Forward price for an investment asset that provides a known yield  
Consider a forward contract negotiated today

$T = 3 \text{ months} = \frac{1}{4} \text{ year}$ ,  $S_0 = \$40$ ,  $r = 5\%$

In addition, a constant yield which is paid continuously as the percentage of the current asset price is  $q = 4\%$  per year. Then, in a smaller time interval, for example, return on one day = current asset price \*  $0.04/365$ ,  $F_0 = ?$

General solution:  $F_0 = S_0 e^{(r-q)T}$  (5.3)

So,  $F_0 = \$40.10$

If  $F_0 = 42 > 40.10$ , an arbitrage profit =  $42 - 40.10 = \$1.90$   
Strategy: (forward price is too high relative to spot price)

Today:

- (1) Borrow \$40 at 5% for 3 months and buy one unit of the asset at \$40
- (2) Sell a 3-month forward contract for one unit of the asset at \$42

After 3 months:

- (1) Make the delivery and collect \$42.00
- (2) Pay off the loan in the amount of \$40.50 ( $40e^{0.05*3/12}$ )
- (3) Receive a known yield for three months of \$0.40 ( $3/12$  of  $40*0.04$ )
- (4) Count for profit =  $42 - 40.50 + 0.40 = \$1.90$

If  $F_0 = 39 < 40.10$ , an arbitrage profit =  $40.10 - 39 = \$1.10$   
Show the proof by yourself as an exercise

Application: stock indexes

- Valuing forward contracts

$K$ : delivery price

$f$ : value of the forward contract today,  $f = 0$  at the time when the contract is first entered into the market ( $F_0 = K$ )

In general:  $f = (F_0 - K) e^{-rT}$  for a long position, where  $F_0$  is the current forward price

For example, you entered a long forward contract on a non-dividend-paying stock some time ago. The contract currently has 6 months to maturity. The risk-free rate is 10%, the delivery price is \$24, and the current market price of the stock is \$25.

Using (5.1),  $F_0 = 25e^{0.1*6/12} = \$26.28$ ,  $f = (26.28 - 24) e^{-0.1*6/12} = \$2.17$

Similarly,  $f = (K - F_0) e^{-rT}$  for a short position

- Forward prices and futures prices

Under the assumption that the risk-free interest rate is constant and the same for all maturities, the forward price for a contract with a certain delivery date is the same as the futures price for a contract with the same delivery date.

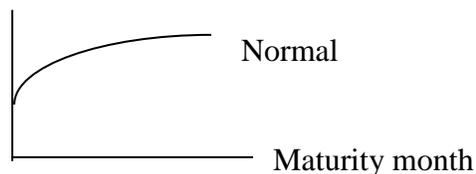
Futures price = delivery price determined as if the contract were negotiated today

The formulas for forward prices apply to futures prices after daily settlement

Patterns of futures prices

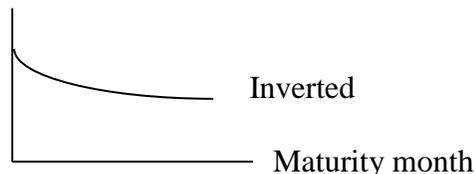
It increases as the time to maturity increases - normal market

Futures price



It decreases as the time to maturity increases - inverted market

Futures price



Futures prices and expected future spot prices

Keynes and Hicks: hedgers tend to hold short futures positions and speculators tend to hold long futures position, futures price < expected spot price because speculators ask for compensation for bearing the risk (or hedgers are willing to pay a premium to reduce the risk)

Risk and return explanation: if the return from the asset is not correlated with the market (beta is zero),  $k = r$ ,  $F_0 = E(S_T)$ ; if the return from the asset is positively correlated with the market (beta is positive),  $k > r$ ,  $F_0 < E(S_T)$ ; if the return from the asset is negatively correlated with the market (beta is negative)  $k < r$ ,  $F_0 > E(S_T)$

- Stock index futures

Stock index futures: futures contracts written on stock indexes

Futures contracts can be written on many indices, such as

DJIA: price weighted, \$10 times the index

Nikkei: price weighted, \$5 times the index

S&P 500 index: value weighted, \$250 times the index

NASDAQ 100 index: value weighted, \$20 times the index

Stock index futures prices, recall (5.3)

General formula:  $F_0 = S_0 e^{(r-q)T}$ , where  $q$  is the continuous dividend yield

If this relationship is violated, you can arbitrage - index arbitrage

Speculating with stock index futures

If you bet that the general stock market is going to fall, you should short (sell) stock index futures

If you bet that the general stock market is going to rise, you should long (buy) stock index futures

Hedging with stock index futures

Short hedging: take a short position in stock index futures to reduce downward risk in portfolio value

Long hedging: take a long position in stock index futures to not miss rising stock market

- Currency futures

Exchange rate and exchange rate risk

Direct quotes vs. indirect quotes

1 pound / \$1.60 (direct) vs. 0.625 pound / \$1.00 (indirect)

Exchange rate risk: risk caused by fluctuation of exchange rates

Currency futures prices, recall (5.3)

General formula:  $F_0 = S_0 e^{(r-r_f)T}$ , where  $r_f$  is the foreign risk-free rate

For example, if the 2-year risk-free interest rate in Australia and the US are 5% and 7% respectively, and the spot exchange rate is 0.6200 USD per AUD, then the 2-year forward exchange rate should be 0.6453. If the 2-year forward rate is 0.6300, arbitrage opportunity exists. To arbitrage:

- (1) Borrow 1,000 AUD at 5%, convert it to 620 USD and invest it in the U.S. at 7% for 2 years (713.17 USD in 2 years)
- (2) Enter a 2-year forward contract to buy AUD at 0.6300
- (3) After 2 years, collect 713.17 USD and convert it to 1,132.02 AUD
- (4) Repay the loan plus interest of 1,105.17 AUD
- (5) Net profit of 26.85 AUD (or 16.91 USD)

Speculation using foreign exchange futures

If you bet that the British pound is going to depreciate against US dollar you should sell British pound futures contracts

If you bet that the British pound is going to appreciate against US dollar you should buy British pound futures contracts

Hedging with foreign exchange futures to reduce exchange rate risk – Table 5.4 Quotes

- Commodity futures

Commodities: consumption assets with no investment value, for example, wheat, corn, crude oil, etc.

For commodity futures prices, recall (5.1), (5.2), and (5.3)

For commodities with no storage cost:

$$F_0 = S_0 e^{rT}$$

For commodities with storage cost:

$$F_0 = (S_0 + U)e^{rT}, \text{ where } U \text{ is the present value of all storage costs}$$

$$F_0 = S_0 e^{(r+u)T}, \text{ where } u \text{ is the storage costs per year as a percentage of the spot price}$$

Example 5.8: consider a one-year futures contract on gold. We assume no income and that it costs \$2 per ounce per year to store gold, with the payment being made at the end of the year. The gold spot price is \$1,600 and the risk-free rate is 5% per year for all maturities.

$$U = 2e^{-rT} = 2 * e^{-0.05 * 1} = 1.90$$

$$F_0 = (S_0 + U)e^{rT} = (1,600 + 1.90)e^{0.05*1} = \$1,684.03$$

If the futures price is too high, say \$1,700, an arbitrageur can

- (1) Borrow \$160,000 at the risk-free rate of 5% for one year and buy 100 ounces of gold
- (2) Store gold for one year
- (3) Short one gold futures contract at 1,700 for delivery in one year

After one year, the arbitrageur can

- (1) Get out gold from storage and pay \$200 storage fee
- (2) Deliver gold for \$170,000
- (3) Payout the loan (principal plus interest of \$168,203 =  $160,000e^{0.05*1}$ )
- (4) Count for profit of  $170,000 - 168,203 - 200 = \$1,587$

If the futures price is too low, say \$1,650, an arbitrageur can reverse the above steps to make risk-free profit – an exercise for students

Consumption purpose: reluctant to sell commodities and buy forward contracts

$$F_0 \leq S_0 e^{rT}, \text{ with no storage cost}$$

$$F_0 \leq (S_0 + U)e^{rT}, \text{ where } U \text{ is the present value of all storage costs}$$

$$F_0 \leq S_0 e^{(r+u)T}, \text{ where } u \text{ is the storage costs per year as a percentage of the spot price}$$

- Cost of carry

It measures the storage cost plus the interest that is paid to finance the asset minus the income earned on the asset

Asset	Futures price	Cost of carry
Stock without dividend	$F_0 = S_0 e^{rT}$	$r$
Stock index with dividend yield $q$	$F_0 = S_0 e^{(r-q)T}$	$r-q$
Currency with interest rate $r_f$	$F_0 = S_0 e^{(r-r_f)T}$	$r-r_f$
Commodity with storage cost $u$	$F_0 = S_0 e^{(r+u)T}$	$r+u$

- Assignments

Quiz (required)

Practice Questions: 5.9, 5.10, 5.14 and 5.15

## Chapter 6 - Interest Rate Futures

- Day count and quotation conventions
- T-bond futures
- T-bill futures
- Duration
- Duration based hedging
- Speculation and hedging with interest rate futures

- Day count and quotation conventions
  - Three day counts are used in the U.S.

Actual/actual: T-bonds

For example, the coupon payment for a T-bond with 8% coupon rate (semiannual payments on March 1 and September 1) between March 1 and July 3 is  
 $(124/184) * 4 = \$2.6957$

30/360: corporate and municipal bonds

For example, for a corporate bond with the same coupon rate and same time span mentioned above, the coupon payment is  
 $(122/180) * 4 = \$2.7111$

Actual/360: T-bills and other money market instruments

The reference period is 360 days but the interest earned in a year is 365/360 times the quoted rate

Quotes for T-bonds: dollars and thirty-seconds for a face value of \$100

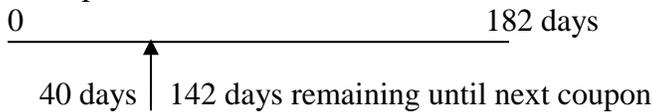
For example, 95-05 indicates that it is \$95  $\frac{5}{32}$  (\$95.15625) for \$100 face value or \$95,156.25 for \$100,000 (contract size)

Minimum tick =  $\frac{1}{32}$

Daily price limit is 3 full points (96 of  $\frac{1}{32}$ , 3% of the face value, equivalent to \$3,000)

For T-bonds, cash price = quoted cash price + accrued interest

Example



Suppose annual coupon is \$8 and the quoted cash price is 99-00 (or \$99 for a face value of \$100) then the cash price =  $99 + (4/182) * 40 = \$99.8791$

T-bills: sold at a discount with no interest payment, the denomination usually is \$1,000

Price quotes are for a face value of \$100

The cash price and quoted price are different

For example, if  $y$  is the cash price of a T-bill that will mature in  $n$  days, then the quoted price ( $P$ ) is given by

$$P = (360/n) * (100 - y), \text{ known as the discount rate}$$

If  $y = 98$ ,  $n = 91$  days, then the quoted price = 7.91

$$\text{Return on the T-bill} = (2/98) * (365/91) = 8.186\% \quad (2/98 = 2.04\% \text{ for 91 days})$$

- T-bond futures

T-bond futures are quoted as T-bonds and the price received for each \$100 face value with a short position upon delivery is

Invoice amount = quoted futures price\*(CF) + AI,  
where CF = conversion factor and AI = accrued interest

For example, if the quoted futures price = 90-00, CF = 1.38, and AI = \$3.00, then  
Invoice amount =  $90(1.38) + 3.00 = \$127.20$

Conversion factor: equal to the quoted price the bond would have per dollar of principal on the first day of the delivery month on the assumption that the interest rate for all maturities equals 6% per year (with semiannual compounding)

For example, let's consider a 10% coupon bond with 20 years and 2 months to maturity. For the purpose of calculating the CF, the bond is assumed to mature in 20 years (round down to the nearest 3 months). The value of the bond is

$$\sum_{i=1}^{40} \frac{5}{(1+0.03)^i} + \frac{100}{(1+0.03)^{40}} = \$146.23. \text{ Dividing by the face value gives the CF } 1.4623.$$

(Or you can use a financial calculator to figure out the CF: PMT = 5, FV = 100, N = 40,  $i/y = 3\%$ , solve for PV = 146.23, CF =  $146.23/100 = 1.4623$ )

For example, consider an 8% coupon bond with 18 years and 4 months to maturity. To calculate CF, we assume that the bond has 18 years and 3 months to maturity.

$$4 + \sum_{i=1}^{36} \frac{4}{(1+0.03)^i} + \frac{100}{(1+0.03)^{36}} = \$125.83$$

The interest rate for a 3-month period is  $(1.03)^{1/2} - 1 = 1.4889\%$ . Hence, the present value of the bond is  $125.83/(1+1.4889\%) = \$123.99$ . Subtracting the accrued interest of \$2.00 we get \$121.99. The conversion factor is therefore 1.2199.

Cheapest-to-deliver issues

Since the person with a short position can deliver any T-bond that has more than 15 years to maturity and that it is not callable within 15 years, which bond is the cheapest to deliver?

Recall: invoice price = quoted futures price\*(CF) + AI, where CF = conversion factor, and AI = accrued interest

The cost of purchasing a bond is

Cash price = quoted cash price + accrued interest

The net cost is the difference between the cash price and the invoice price

Net cost to deliver = quoted cash price - (quoted futures price\*CF)

(Note, AI is cancelled out)

Choose the bond that minimizes the net cost to deliver

Determining the quoted T-bond futures price, use  $F_0 = (S_0 - I)e^{rT}$

Example 6.1: choose a bond from below to deliver, assuming the most recent settlement price (quoted futures price) is 93-08 (or 93.25)

Bond	Quoted cash price	CF	Net cost to deliver
1	99.50	1.0382	$99.50 - (93.25 * 1.0382) = \$2.69$
2	143.50	1.5188	$143.50 - (93.25 * 1.5188) = \$1.87$
3	119.75	1.2615	$119.75 - (93.25 * 1.2615) = \$2.12$

The cheapest-to-deliver is bond 2

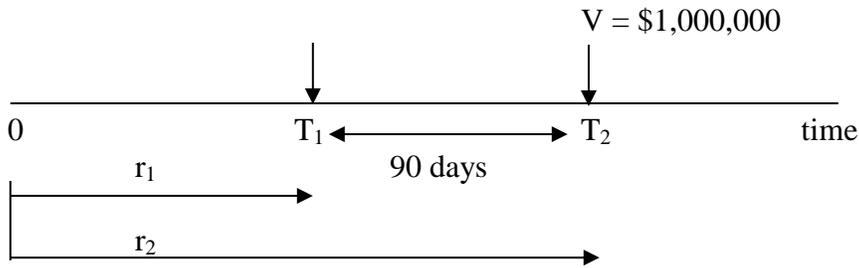
- T-bill futures

Call for delivery of T-bills with a face value of \$1,000,000 and a time to maturity of 90 days

The contract size is \$1,000,000 with delivery months set in March, June, September, and December

No daily price limit

Minimum tick is 0.01% of discount yield (one basis point, equivalent to \$25.00; or when the interest rate changes by 1 basis point, 0.01%, the interest earned on \$1,000,000 face value for 3 months changes by \$25.00)



Determining the current (present) value, use  $PV = V e^{-r_2 T_2}$

Determining the futures price, use  $F_0 = PV e^{r_1 T_1}$

Why buy a futures contract on T-bills? To lock in a short-term interest rate

Eurodollar futures are similar to T-bill futures

- Duration

A measure of how long on average the bondholder has to wait before receiving cash payments

Suppose a bond provides cash flows  $c_t$  at time  $t$ . The bond price  $B$  and bond yield  $y$  (with continuous compounding) are related by:

$$B = \sum_{i=1}^n c_i e^{-y t_i}$$

The duration of the bond ( $D$ ) is defined as:  $D = \frac{\sum_{i=1}^n t_i c_i e^{-y t_i}}{B} = \sum_{i=1}^n t_i \left[ \frac{c_i e^{-y t_i}}{B} \right]$

For a small change in yield ( $\Delta y$ ) the change in bond price is approximately:

$$\Delta B = \frac{dB}{dy} \Delta y = -\Delta y \sum_{i=1}^n c_i t_i e^{-y t_i} = -BD \Delta y$$

Rearranging, we get  $\frac{\Delta B}{B} = -D \Delta y$

Percentage change in bond price = - duration \* the change in yield (where yield is expressed with continuous compounding)

For example, a 3-year bond with coupon rate of 10%, payable semiannually, sells for 94.213 and has a yield to maturity of 12% (continuous compounding). The duration for the bond is 2.653 years. If the yield increases by 0.1% the bond price will drop by 0.250 to 93.963 [-94.213\*2.653\*(0.001) = -0.25].

Time (years)	Cash flow (\$)	Present value	Weight	Time*weight
0.5	5	4.709	0.050	0.025
1.0	5	4.435	0.047	0.047
1.5	5	4.176	0.044	0.066
2.0	5	3.933	0.042	0.083
2.5	5	3.704	0.039	0.098
3.0	105	73.256	0.778	2.333
Total	130	94.213	1.000	2.653

What if  $y$  is not continuous (for example, semiannual compounding)?

If  $y$  is not expressed with continuous compounding then  $\Delta B = -BD^* \Delta y$ , where  $D^*$  is called modified duration

$$D^* = \frac{D}{1 + y/m} = \frac{2.653}{1 + 0.12/2} = 2.50, \text{ where } D \text{ is the duration of the bond and } y \text{ is the yield}$$

with a compounding frequency of  $m$  times per year

For a zero-coupon bond, the duration is equal to the time to maturity

- Duration based hedging

Interest rate risk: interest rate price risk vs. interest rate reinvestment risk

Interest rate price risk: risk that the bond value (price) falls when the market interest rates rise

Reinvestment risk: risk that the interest received will be reinvested at a lower rate

Relationship between duration and bond price volatility:  $\Delta B = -BD\Delta y$

Define

$V_F$ : contract price for the interest rate futures contract

$D_F$ : duration of the asset underlying the futures contract at the maturity of the futures contract

$P$ : forward value of the portfolio being hedged at the maturity of the hedge (assumed to be the value of the portfolio today)

$D_P$ : duration of the portfolio at the maturity of the hedge

Then we have approximations of  $\Delta P = -PD_p \Delta y$  and  $\Delta V_F = -V_F D_F \Delta y$ , the optimal number of contracts to hedge against an uncertain  $\Delta y$  is

$$N^* = \frac{PD_p}{V_F D_F}, \text{ called duration-based hedge ratio}$$

For example, on August 2, a fund manager with \$10 million invested in government bonds is concerned that interest rates are expected to be volatile over the next 3 months. The manager decides to use December T-bond futures contract to hedge the portfolio. The current futures price is 93-02, or 93.0625 for \$100 face value. Since the contract size is \$100,000, the futures price is \$93,062.50. Further suppose that the duration of the bond portfolio in 3 months is 6.80 years. The cheapest-to-deliver bond in the T-bond contract is expected to be a 20-year 12% coupon bond. The yield on this bond is currently 8.8% and the duration is 9.20 years at maturity of the futures contract. The duration-based hedge ratio is

$$N^* = \frac{10,000,000}{93,062.50} * \frac{6.80}{9.20} = 79.42 \text{ Contracts, short positions}$$

Application of bond immunization: banking management, pension fund management

- Speculation and hedging with interest rate futures

Speculation with outright positions

A speculator with a long position is betting that the interest rate is going to fall so the price of interest rate futures is going to rise

A speculator with a short position is betting that the interest rate is going to rise so the price of interest rate futures is going to fall

Speculation with spreads

An intra-commodity T-bill spread: speculating the term structure of interest rates, for example, T-bill futures with different maturities (nearby vs. distant)

An inter-commodity spread: speculating on shifting risk levels between different instruments, for example, T-bill futures vs. T-bond futures

Hedging with interest rate futures

A long hedge: take a long position in futures market to reduce interest rate risk

A short hedge: take a short position in futures market to reduce interest rate risk

- Assignments

Quiz (required)

Practice Questions: 6.8, 6.9, 6.10 and 6.11

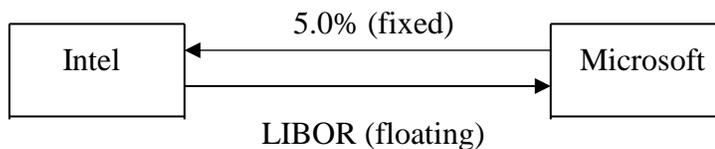
## Chapter 7 - Swaps

- Swaps
- Interest-rate swaps
- Role of financial intermediary
- Comparative advantage
- Valuation of interest-rate swaps
- Currency swaps
- Valuation of currency swaps
- Other types of swaps

- Swaps

A swap is a private agreement between two companies to exchange cash flows in the future according to a prearranged formula: an extension of a forward contract.

For example, Intel and Microsoft agreed a swap in interest payments on a notional principal of \$100 million. Microsoft agreed to pay Intel a fixed rate of 5% per year while Intel agreed to pay Microsoft the 6-month LIBOR.



Cash flows to Microsoft in a \$100 million 3-year interest rate swap when a fixed rate of 5% is paid and LIBOR is received:

Date	6-month LIBOR	Floating cash flow	Fixed cash flow	Net cash flow
3-5-2013	4.20%			
9-5-2013	4.80%	+2.10 million	-2.50 million	-0.40 million
3-5-2014	5.30%	+2.40 million	-2.50 million	-0.10million
9-5-2014	5.50%	+2.65 million	-2.50 million	+0.15 million
3-5-2015	5.60%	+2.75 million	-2.50 million	+0.25 million
9-5-2015	5.90%	+2.80 million	-2.50 million	+0.30 million
3-5-2016		+2.95 million	-2.50 million	+0.45 million

Cash flows to Intel will be the same in amounts but opposite in signs.

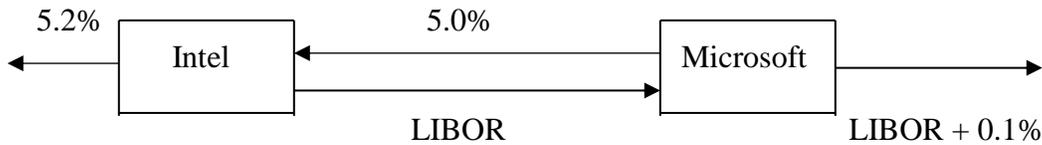
The floating rate in most interest rate swaps is the London Interbank Offered Rate (LIBOR). It is the rate of interest for deposits between large international banks.

We ignored the day count convention (LIBOR is quoted on an actual/360 basis while a fixed rate uses 365 days per year). Since there are 184 days between March 5, 2013 and September 5, 2013 the actual payment should be  $100 * 0.042 * 184 / 360 = \$2.1467$  million

- Interest-rate swaps

Using swaps to transform a liability

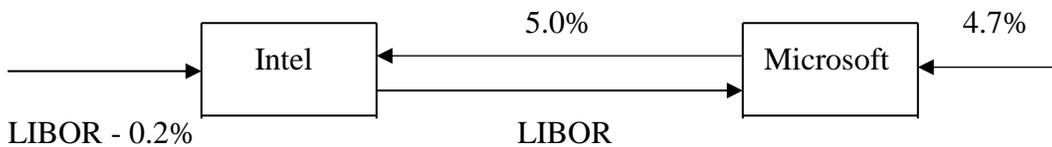
For example, Microsoft could transform a floating-rate loan to a fixed rate loan while Intel could transform a fixed rate loan to a floating-rate loan



After swap, Intel pays LIBOR + 0.2% and Microsoft pays 5.1%

Using swaps to transform an asset

For example, Microsoft could transform an asset earning fixed-rate to an asset earning floating-rate while Intel could transform an asset that earns floating-rate to an asset that earns fixed-rate



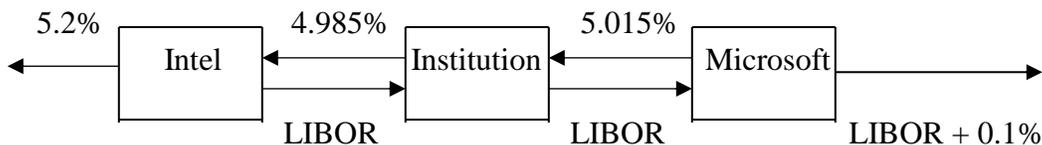
After swap, Intel earns 4.8% and Microsoft earns LIBOR - 0.3%

We will ignore the day count convention (LIBOR uses 360 days per year while a fixed rate uses 365 days per year)

- Role of financial intermediary

Arranging the swaps to earn about 3-4 basis points (0.03-0.04%)

Refer to the swap between Microsoft and Intel again when a financial institution is involved to earn 0.03% (shared evenly by both companies)



After swap, Intel pays LIBOR + 0.215%, Microsoft pays 5.115%, and the financial institution earns 0.03%

- Comparative advantage

Some companies have a comparative advantage when borrowing in fixed rate (U.S dollars) markets, whereas others have a comparative advantage in floating-rate (foreign currency) markets.

Let us consider two corporations, AAACorp and BBBCorp. Both companies are going to borrow \$10 million and are facing the following rates. Assume that AAACorp wants floating rate and BBBCorp wants fixed rate:

	Fixed	Floating
AAACorp	4.00%	LIBOR - 0.1%
BBBCorp	5.20%	LIBOR + 0.6%

How can a swap benefit both companies?

Answer: AAACorp borrows at the fixed rate and BBBCorp borrows at the floating rate and then two companies engage in a swap

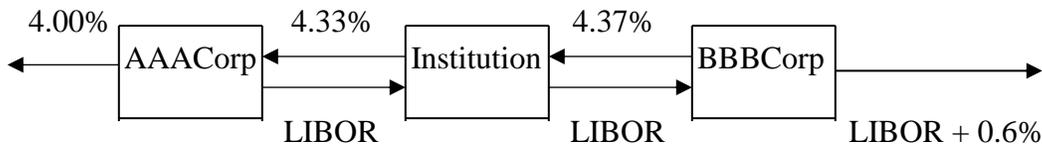
Without a financial institution involved



After swap, AAACorp pays LIBOR - 0.35% and BBBCorp pays 4.95% (both benefit by 0.25%)

The total benefit is equal to a - b (0.5%), where a is the difference between the interest rates in fixed rate markets (1.20%) and b is the difference between the interest rates in floating rate markets (0.7%). The total gain doesn't have to be shared evenly.

When a financial institution is involved and it earns 4 basis points (0.04%):



After swap, AAACorp pays LIBOR - 0.33% and BBBCorp pays 4.97% (both benefit by 0.23%) while the financial institution earns 0.04%. The total gain remains at 0.5%.

- Valuation of interest-rate swaps

$V_{\text{swap}} = B_{\text{fl}} - B_{\text{fix}}$  (or  $V_{\text{swap}} = B_{\text{fix}} - B_{\text{fl}}$ ), where  $B_{\text{fix}}$  is the present value of fixed-rate bond underlying the swap and  $B_{\text{fl}}$  is the present value of floating-rate bond underlying the swap

Suppose that a financial institution has agreed to pay 6-month LIBOR and receive 8% fixed rate (semiannual compounding) on a notional principal of \$100 million. The swap has a remaining life of 1.25 years. The LIBOR rates with continuous compounding for 3-month, 9-month, and 15-month maturities are 10%, 10.5%, and 11%, respectively. The 6-month LIBOR rate at the last payment date was 10.2%.

Time Years	$B_{\text{fix}}$ cash flow	$B_{\text{fl}}$ cash flow	Discount factor	Present value $B_{\text{fix}}$ cash flow	Present value $B_{\text{fl}}$ cash flow
0.25	4.0	105.10	0.9753	3.901	102.505
0.75	4.0		0.9243	3.697	
1.25	104.0		0.8715	90.640	

$$V_{\text{swap}} = B_{\text{fix}} - B_{\text{fl}} = (3.901 + 3.697 + 90.640) - 102.505 = -4.267 \text{ million}$$

Note:  $B_{\text{fl}} = L$  (notional principal immediately after an interest payment) and therefore  $B_{\text{fl}} = 100 + 100 * 0.051 = 105.10$  million after 3 months

If the financial institution pays fixed and receives floating, the value of the swap would be +4.267 million (again a zero-sum game)

- Currency swaps

A currency swap is an agreement to exchange interest payments and principal in one currency for principal and interest payments in another currency. It can transform a loan in one currency into a loan in another currency.

Let us consider two corporations, GE and Qantas Airway. Both companies are going to borrow money and are facing the following rates. Assume GE wants to borrow AUD and Qantas Airway wants to borrow USD.

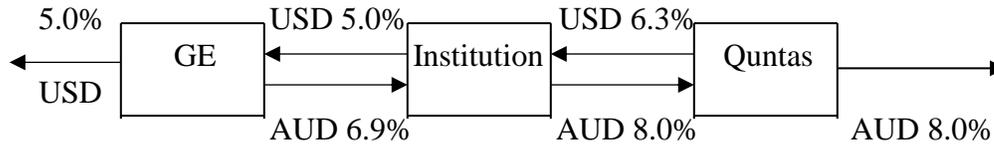
	USD	AUD
GE	5.00%	7.60%
Qantas	7.00%	8.00%

How can a swap benefit both companies?

Answer: GE borrows USD and Qantas borrows AUD and then two companies engage in a swap (fixed-for-fixed currency swap)

Since  $a = 2.00\%$  and  $b = 0.4\%$ , therefore the net gain =  $a - b = 1.6\%$

Assuming a financial institution arranges the swap and earns 0.2% (by taking the exchange rate risk)



Net outcome:

GE borrows AUD at 6.9% (0.7% better than it would be if it went directly to AUD markets)

Qantas borrows USD at 6.3% (0.7% better than it would be if it went directly to USD markets)

Financial institution receives 1.3% USD and pays 1.1% AUD and has a net gain of 0.2%

- Valuation of currency swaps

$V_{\text{swap}} = B_D - S_0 B_F$  (or  $V_{\text{swap}} = S_0 B_F - B_D$ ), where  $S_0$  is the spot exchange rate,  $B_F$  and  $B_D$  are the values of the foreign-denominated bond underlying the swap and the U.S. dollar bond underlying the swap

- Other types of swaps

Fixed-for-floating currency swap: a floating interest rate in one currency is exchanged for a fixed interest rate in another currency

Floating-for-floating currency swap: a floating interest rate in one currency is exchanged for a floating interest rate in another currency

Credit default swap (CDS): allows companies to hedge credit risks in the same way that they have hedged market risks - buys insurance to hedge default risk

Equity swap: an agreement to exchange the total returns (dividends and capital gains) realized on an equity index for either a fixed or floating rate of interest

Commodity swaps

Volatility swaps

- Assignments

Quiz (required)

Practice Questions: 7.9, 7.10 and 7.11

## Swap examples

1. Companies X and Y have been offered the following rates per year on a \$5 million ten-year loan:

	Fixed-rate	Floating-rate
Company X	7.0%	LIBOR + 0.5%
Company Y	8.8%	LIBOR + 1.5%

Company X requires a floating-rate loan and Company Y requires a fixed-rate loan. Design a swap that will net a financial institution acting as an intermediary 0.1% per year and it will be equally attractive to X and Y.

Answer:  $a = 1.8\%$  ( $8.8\% - 7.0\%$ ) and  $b = 1.0\%$  [ $(\text{LIBOR} + 1.5\%) - (\text{LIBOR} + 0.5\%)$ ], swap provides a net gain of 0.8% ( $1.8\% - 1.0\%$ ), less 0.1% for the bank, leaving 0.7% net gain to be shared by X and Y (0.35% each)

After the swap, X pays floating at  $\text{LIBOR} + 0.15\%$ , Y pays fixed at 8.45%, and the institution earns 0.1%

2. Companies A and B are facing the following annual interest rates in US and UK:

	Sterling	U.S. Dollar
Company A	8.0%	7.0%
Company B	7.6%	6.2%

A wants to borrow dollar and B wants to borrow sterling. Design a swap that will net a financial institution acting as an intermediary 0.10% per year and it will be equally attractive to both companies.

Answer:  $a = 0.4\%$  ( $8.0\% - 7.6\%$ ) and  $b = 0.8\%$  ( $7.0 - 6.2\%$ ), a swap provides a total gain of 0.4% ( $0.8\% - 0.4\%$ ), less 0.1% for the bank, leaving 0.3% net gain to be shared by A and B (0.15% each)

After the swap, A borrows dollar at 6.85%, B borrows sterling at 7.45%, and the financial institution earns 0.1%.

## Chapter 8 - Securitization and the Credit Crisis of 2007

- Securitization
- The U.S. housing market
- What went wrong?
- Aftermath

- Securitization

Banks could not keep pace with the increasing demand for residential mortgages, especially during real estate booming periods. This led to the development of the mortgage-backed security (MBS) market. Portfolios of mortgages were created and the cash flows (interests and principal payments) generated by the portfolios were packaged as securities and sold to investors.

Asset-backed security (ABS): a portfolio of income-producing assets, such as loans, is sold by the originating banks to a special purpose vehicle (SPV) and the cash flows from the assets are then allocated to tranches.

A tranche is one of a number of related securities offered as part of the same transaction.

Figure 8.1

Cash flows are allocated to different tranches by specifying what is known as a waterfall

Figure 8.2

The extent to which the tranches get their principal back depends on losses on the underlying assets. The first 5% of losses are borne by the equity tranche. If losses exceed 5% the equity tranche loses its entire principal and some losses are borne by the principal of the mezzanine tranche. If losses exceed 20%, the mezzanine tranche loses its entire principal and some losses are borne by the principal of the senior tranche.

Usually, the senior tranche is rated AAA. The mezzanine is rated BBB. The equity tranche is not rated.

Finding investors to buy the senior tranche (AAA-rated) was not difficult since it offered attractive return with relatively low risk.

Equity tranche usually was sold to hedge funds (high risk high returns)

It is difficult to find investors to buy mezzanine tranche.

Figure 8.3

- The U.S. housing market  
The relaxation of lending standards

Subprime mortgage securitization

The S&P/Case-Shiller Composite-10 index of U.S. real estate prices

Figure 8.4

The bubble burst in 2007: many mortgage holders cannot afford their payments, housing price drops, leading to many foreclosures.

The borrowers hold a free American-style put option since when there is a default, the lender is able to take the possession of the house.

The losses: far greater than the losses in house prices

- What went wrong?  
Irrational exuberance

Mortgage lending standards

Mortgage-backed products

Rating agencies

Default risk

Incentives (agency problems)

- Aftermath  
More regulations

Higher standards for mortgage lending

- Assignments  
Quiz (required)  
Practice Questions: 8.9, 8.10, 8.11 and 8.12