Chapter 1 - Introduction

- Derivative securities
- Futures contracts
- Forward contracts
- Futures and forward markets
- Comparison of futures and forward contracts
- Options contracts
- Options markets
- Comparison of futures and options
- Types of traders
- Applications

- Derivative securities

    Securities whose values are derived from the values of other underlying assets
    Futures and forward contracts
    Options
    Swaps
    Others

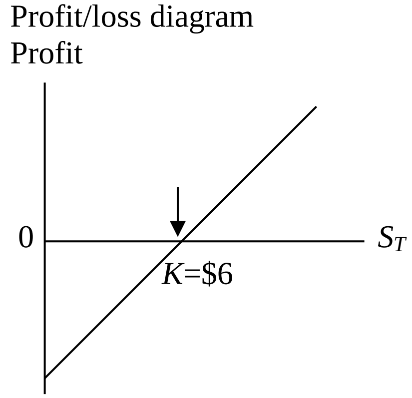
- Futures contracts

    An agreement between two parties to either buy or sell an asset at a certain time in the future for a certain price

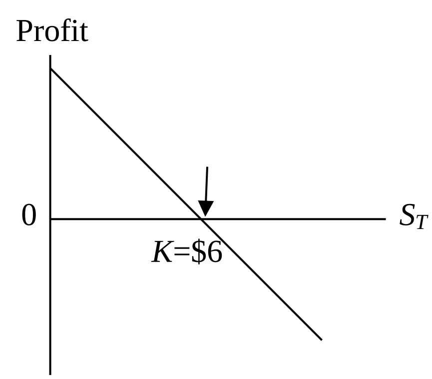
    For example, in March a trader buys a June futures contract on corn at 600 cents (or $6) per bushel

March: A trader buys a June futures contract on corn at 600 cents/bushel →	June: The trader must buy 5,000 bushels of corn for $30,000 →	If S_T > $6 (gain) If S_T = $6 (no gain/loss) If S_T < $6 (lose)

Profit/loss diagram



Long position (buy a futures contract)



Short position (sell a futures contract)

K = delivery price = \$6/bushel and S_T = the spot price at maturity (can be greater than, equal to, or less than \$6/bushel)

If S_T is greater than K, the person with a long position gains (S_T - K) and the person with a short position loses (K - S_T) - zero sum game (someone's gain is someone else's loss)

If S_T is less than K, the person with a long position loses (S_T - K) and the person with a short position gains (K - S_T) - zero sum game

If S_T is equal to K, there is no gain or loss on both sides - zero sum game

Corn: underlying asset - commodity (commodity futures contract)
Buy a futures contract: long position - promise to buy
600 cents/bushel: futures price/delivery price
5,000 bushels: contract size - standardized
June: delivery month
Spot price: actual price in the market for immediate delivery

More examples
(1) Long futures positions: agree to buy or call for delivery
On February 1, you buy a June gold futures contract at 1,300: you agree to buy (or call for delivery) 100 ounces of gold in June at 1,300 dollars per troy ounce

Contract details
1,300 dollars per ounce - futures price of gold on February 1 for June delivery, also called delivery price (Note: the futures price of gold on February 2 for June delivery may be different)
100 ounces - contract size
June - delivery month
Underlying asset: gold - commodity (commodity futures contract)
Position: long position
Actual market price of gold - spot price which can be different from the futures price

(2) Short futures positions: agree to sell or promise to deliver
On February 1, you sell a June Yen futures contract at 1.1250: you agree to sell (or promise to deliver) 12,500,000 Yen in June at \$1.1250 per 100 Yen (\$1 for 88.8889 Yen)

Contract details
\$1.1250 per 100 Yen - futures price (exchange rate), also called delivery price
12,500,000 Yen - contract size
June - delivery month
Underlying asset: foreign currency - financial asset (financial futures contract)
Position: short position
Actual market exchange rate - spot exchange rate which can be different from the futures exchange rate

Futures contracts can be written on different assets (underling assets):
Commodities - commodity futures, for example, grains, livestock, meat, metals, and oil
Financial assets - financial futures, for example, stock indices, bonds, currencies
Others

Technical details (covered in later chapters)
Margin requirements
Daily settlement procedures - marking to market
Bid-offer spreads
Clearinghouses
Delivery

- Forward contracts

  A forward contract is similar to a futures contract in that it is an agreement between two parties to either buy or sell an asset at a certain time in the future for a certain price. But forward contracts are less formal, traded only in OTC markets, and contract sizes are not standardized.

- Futures and forward markets

  (1) Exchange-traded markets
  Chicago Board of Trade (CBOT): futures contracts
  Chicago Mercantile Exchange (CME): futures contracts

  Open-outcry system: traders physically meet on the floor of the exchange and use a complicated set of hand signals to trade

  Electronic trading: increasingly replacing the open-outcry system to match buyers and sellers

  (2) Over-the-counter (OTC) markets
  Telephone- and computer-linked network of dealers

  Flexibility - tailor your needs

  Credit risk - the risk that your contract will not be honored

- Comparison of futures and forward contracts

	Exchange trading	Standardized contract size	Marking to market	Delivery	Delivery time	Default risk
Forward	No	No	No	Yes or cash settlement	One date	Some credit risk
Futures	Yes	Yes	Yes	Usually closed out	Range of dates	Virtually no risk

- Options contracts

  Rights to buy or sell an asset by a certain date for a certain price

  American options vs. European options
  American options can be exercised at any time before the expiration date
  European options can only be exercised on the maturity date

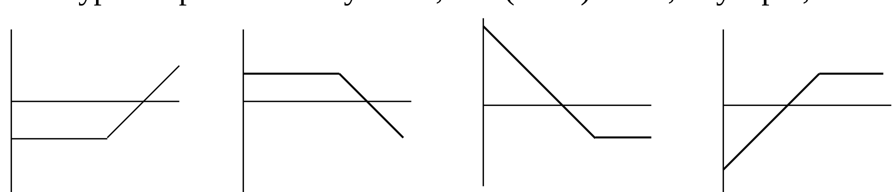
  Keeping other things the same, which type of options should be worth more and why?
  Answer: American options because they can be exercised at any time before expiration

  Two types of options: call options vs. put options

  A call option gives the right to buy an asset for a certain price by a certain date
  For example, you buy an IBM June 190 call option for $3.00
  Option type: call option - the right to buy
  The underlying asset - IBM stock
  Exercise (strike) price - $190 per share
  Expiration date - the third Friday in June
  Contract size: 100 shares
  Option premium (price of the option): $300.00

  A put option gives the right to sell an asset for a certain price by a certain date
  For example, you buy a GE June 24 put option for $2.00
  Option type: put option - the right to sell
  The underlying asset - GE stock
  Exercise (strike) price - $24 per share
  Expiration date - the third Friday in June
  Contract size: 100 shares
  Option premium: $200.00

  Four types of positions: buy a call, sell (write) a call, buy a put, and sell (write) a put



(1) (2) (3) (4)

(1) Buy a call (2) Sell a call (3) Buy a put (4) Sell a put

  Options can be written on different assets:
  Stocks - stock options
  Stock indices - index options
  Currencies - currency options
  Futures - futures options
  Others

- Options markets
  Exchange traded markets
  (1) Chicago Board Options Exchange (CBOE): options

  (2) Over-the-counter (OTC) markets

- Comparison of futures and options
  Rights (options) vs. obligations (futures)
  Initial outlay (buying options requires an initial outlay while buying futures doesn't)

  Both need a margin account (futures are subject to marking to market daily)
  Both use leverage

- Types of traders
  (1) Hedgers: use options and futures markets to reduce price uncertainty (risk) in the future

  For example, a farmer can sell corn futures contracts to lock in a price and an investor can buy a put option to protect a potential downward movement of a particular stock

  More details: a company can use forward contracts for hedging currency risk
  Import Co. purchased goods from a British supplier in June and needs to pay 10 million British pounds in September. A local financial institution offers forward contracts for British pounds. The quotes are shown below:

	Bid	Offer
Spot	1.6382	1.6386
1-month forward	1.6380	1.6385
3-month forward	1.6378	1.6384
6-month forward	1.6376	1.6383

  How should Import Co. hedge the exchange rate risk?

  Answer: Import Co. should buy 10 million British pounds in the three-month forward market to lock in the exchange rate of 1.6384 (or 16.384 million dollars for 10 million pounds for September delivery)

  When you sell pounds to the financial institution you get the bid price
  When you buy pounds from the financial institution you pay the offer price
  The difference between bid and offer is called bid-offer spread which is the profit for the institution

  Hedging using options – Example 1.2 on page 13

(2) Speculators: bet on price movement

For example, you buy gold futures contracts because you bet that the price of gold will go up in the future, or you buy a put option on Intel if you bet that Intel stock price will drop

More details: a trader uses options for speculation (leverage effect)
A trader with $2,000 to invest bets that the price of ORCL will increase in the near future and has the following quotes:
Current stock price: $20.00
ORCL June call option with exercise price of $22.50 sells for $1.00

Alternative 1: buy 100 shares of ORCL
Alternative 2: buy 20 ORCL June call options with exercise price of $22.50

Possible outcomes:
If ORCL stock price rises to $27
Alternative 1: profit of $700 = 100*(27 - 20)
Alternative 2: profit of $7,000 = 20*100(27 - 22.5 - 1)

If ORCL stock price falls to $17
Alternative 1: loss of $300 = 100*(17 - 20)
Alternative 2: loss of $2,000 (the options are worthless)

If ORCL stock price rises to $23.5
Alternative 1: profit of $350 = 100*(23.5 - 20)
Alternative 2: break-even (23.5 - 22.5 - 1)

(3) Arbitrageurs: look for risk-free profits by taking the advantage of mispricing in two or more markets simultaneously

Conditions for arbitrage:
Zero net cost
No risk
Positive profit

Example: use stocks and exchange rates for arbitrage
A stock is traded on both NYSE and London Stock Exchange. The following quotes are obtained at a particular time:

Stock price at NYSE: $20/share
Stock price at London Stock Exchange: 13 pounds/share
Spot exchange rate: 1 pound = $1.60

Detailed arbitrage process:
Borrow $2,000 to buy 100 shares at NYSE
Sell the shares at London Stock Exchange for 1,300 pounds
Covert the sale proceeds from pounds to dollars at the spot exchange rate to receive $2,080 = 1,300*1.60
Repay the loan of $2,000
Arbitrage profit: 2,080 – 2,000 = $80 (ignoring transaction costs)

Example 1.3 on page 17

(4) Dangers: start from hedgers or arbitrageurs and consciously or unconsciously become speculators

- Applications

  Market completeness - any and all identifiable payoffs can be obtained by trading the derivative securities available in the market (Financial Engineering)

  Risk management - reduce risk or hedging

  Trading efficiency - increase market efficiency

  Price discovery - futures price is the best expected spot price in the future

  Speculation - bet on prices

- Assignments

  Quiz (required)
  Practice Questions: 1.11, 1.12, 1.13, 1.14, 1.20 and 1.21

Chapter 2 - Mechanics of Futures Markets

- Specification of a futures contract
- Convergence of futures price to spot price
- Operation of margins
- Quotes and prices
- Delivery
- Types of orders
- Regulation
- Accounting and taxes

- Specification of futures contracts

  Opening a futures position vs. closing a futures position

  Opening a futures position can be either a long position or a short position
  (i.e., the opening position can be either to buy or to sell a futures contract)
  Closing a futures position involves entering an opposite trade to the original one

  The underlying asset or commodity: must be clearly specified

  The contract size: standardized
  Corn and wheat: 5,000 bushels per contract
  Live cattle: 40,000 pounds per contract
  Cotton: 50,000 pounds per contract
  Gold: 100 troy ounces per contract
  DJIA: $10*index
  Mini DJIA: $5*index
  S&P 500: $250*index
  Mini S&P 500: $50*index

  Delivery month: set by the exchange

  Delivery place: specified by the exchange

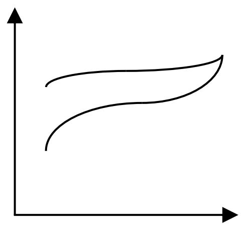
  Daily price limits: usually specified by the exchange, it is the restriction on the day-to-day price change of an underlying commodity. For example, the price of corn can change by no more than 10 cents per bushel from one day to the next and the price of wheat can change by no more than 20 cents per bushel from the preceding closing price

  Position limits: the maximum number of contracts that a speculator may hold
  For example, less than 1,000 contracts total with no more than 300 contracts in any one delivery month for random-length lumber contract in CME

  Tick: the minimum price fluctuation, for example, ¼ cent per bushel for wheat

- Convergence of futures price to spot price

  As the delivery period approaches, the futures price converges to the spot price of the underlying asset (no matter the futures price is higher or lower the spot price now)



Why? Show that the arbitrage opportunity exists if it doesn't happen

If the futures price is above the spot price as the delivery period is reached

(1) Short a futures contract
(2) Buy the asset at the spot price
(3) Make the delivery

If the futures price is below the spot price as the delivery period is reached

(1) Buy a futures contract
(2) Short sell the asset and deposit the proceeds
(3) Take the delivery and return the asset

- Operation of margins

  Margin account: an account maintained by an investor with a brokerage firm in which borrowing is allowed

  Initial margin: minimum initial deposit

  Maintenance margin: the minimum actual margin that a brokerage firm will permit investors to keep their margin accounts

  Margin call: a demand on an investor by a brokerage firm to increase the equity in the margin account

  Variation margin: the extra fund needs to be deposited by an investor

  Marking to market (daily settlement): the procedure that the margin account is adjusted to reflect the gain or loss at the end of each trading day

Example: suppose an investor buys two gold futures contracts. The initial margin is \$2,000 per contract (or \$4,000 for two contracts) and the maintenance margin is \$1,500 per contract (or \$3,000 for two contracts). The contract is entered into on June 5 at \$850 and closed out on June 18 at \$840.50. (Gold is trading around \$1,300 per ounce now.)

Day	Futures price (settlement)	Daily gain (loss)	Cumulative gain (loss)	Margin balance	Margin call (variation margin)
	850.00			4,000	
June 5	848.00	(400)	(400)	3,600	
June 6	847.50	(100)	(500)	3,500	
June 9	848.50	200	(300)	3,700	
June 10	846.00	(500)	(800)	3,200	
June 11	844.50	(300)	(1,100)	2,900	Yes (1,100)
June 12	845.00	100	(1,000)	4,100	
June 13	846.50	300	(700)	4,400	
June 16	842.50	(800)	(1,500)	3,600	
June 17	838.00	(900)	(2,400)	2,700	Yes (1,300)
June 18	840.50	500	(1,900)	4,500	

Note: the investor earns interest on the balance in the margin account. The investor can also use other assets, such as T-bills to serve as collateral (at a discount)

Clearinghouse: an intermediary to guarantee in futures transactions

- Quotes and prices

  Opening price, highest and lowest prices, and settlement price – Table 2.2

  Bid price vs. offer price

  Bid price: the price an institution is prepared to buy
  Offer price: the price an institution is prepared to sell

  Settlement price: the average of the prices immediately before the closing bell and marking to market is based on the settlement price

  Open interest: the total number of contracts outstanding at a particular time

  Open interest: an example
  Given the following trading activities, what is the open interest at the end of each day?

Time	Actions	Open Interest
t = 0	Trading opens for gold contract	0
t = 1	Trader A buys 2 and trader B sells 2 gold contracts	2
t = 2	Trader A sells 1 and trader C buys 1 gold contract	2
t = 3	Trader D sells 2 and trader C buys 2 gold contracts	4

- Delivery

  Very few futures contracts lead to actual delivery and most of futures contracts are closed out prior to the delivery month

  If delivery takes place, the decision on when to deliver is made by the party with the short position

  Cash settlement: some financial futures, for example, stock index futures are cash settled (No actual delivery)

- Types of orders

  Market order: the best price currently available in the market

  Limit order: specifies a price (limit price) and your order will be executed only at that price or a more favorable price to you

  More specifically, you will buy at or below a specified price (limit price) or you will sell at or above a specified price (limit price)

  For example, current price of gold = $1,300, you can specify to buy a futures contract on gold if the futures price $\leq$ $1,250 or you can specify to sell a futures contract on gold if the futures price $\geq$ $1,350

  Stop (stop-loss) order: specifies a price (stop price) and your order will become a market order if the stop price is reached

  For example, current price of gold = $1,300, you can specify to sell a futures contract if the futures price $\leq$ $1,250 or you can specify to buy a futures contract if the futures price $\geq$ $1,350 (short term momentum)

  The main difference between a limit order and a stop order:
  With a limit order you buy when the price drops and you sell when the price rises
  With a stop order you buy when the price rises and you sell when the price drops

  Stop-limit order: a combination of a stop order and a limit order
  For example, current price of gold = $1,300
  A stop-limit order to sell at a stop price $1,250 with a limit price $1,200
  A stop-limit order to buy at a stop price $1,350 with a limit price $1,400

  Market-if-touched order: executed at the best available price after a specified price is reached

  For example, current price of gold = $1,300
  A market-if-touched order to sell if price $\geq$ $1,350

Day order: valid for the day

Open order (good-till-canceled): in effect until the end of trading in a particular contract

- Regulation

  Futures markets are mainly regulated by the Commodity Futures Trading Commission (CFTC). Other agencies, for example, National Futures Association (NFA), Securities and Exchange Committee (SEC), the Federal Reserve Board, and U.S. Treasury Department also step in from time to time.

  The Commodities Futures Trading Commission (CFTC): to approve new contracts, set up daily maximum price fluctuation, minimum price movements, and certain features of delivery process

  The National Futures Association (NFA): to prevent fraudulent and manipulative acts and practices

  Trading irregularities: corner the market - take a huge long futures position and also try to exercise some control of the underlying commodity
  Example: Hunt brothers' price manipulation in the silver market in 1979-1980

  Trading irregularities: front running - take the advantage of inside information

- Accounting and taxes

  Changes in the market value of a futures contract are recognized when they occur unless the contract is qualified as a hedge

  For speculators, all paper gains or losses on futures contracts are treated as though they were realized at the end of the tax year - marking to market at the end of the year

  40% of any gains or losses are to be treated as short-term and
  60% of any gains or losses are to be treated as long-term

  For hedgers, all paper gains or losses are realized when the contracts are closed out

  The 40% short-term and 60% long-term rule doesn't apply for hedgers

- Assignments

  Quiz (required)
  Practice Questions: 2.11, 2.15, 2.16 and 2.23

Chapter 3 - Hedging Strategies Using Futures

- Hedging principles
- Arguments for and against hedging
- Basis risk
- Cross hedging
- Stock index futures
- Rolling hedging

- Hedging principles

  Hedging: to reduce risk
  Complete hedging: to eliminate all risk

  Short hedges: use short positions in futures contracts to reduce or eliminate risk

  For example, an oil producer can sell oil futures contracts to reduce oil price uncertainty in the future

  Long hedge: use long positions in futures contracts to reduce or eliminate risk

  For example, a brewer buys wheat futures contracts to reduce future price uncertainty in wheat

- Arguments for and against hedging

  Hedging can reduce risk but it has a cost

  (1) Firms don't need to hedge because shareholders can hedge by themselves

  (2) Hedging may cause profit margin to fluctuate

  (3) Hedging may offset potential gains (or lead to a worse outcome)

- Basis risk

  Hedging usually cannot be perfect for the following reasons:

  The asset whose price is to be hedged may not be exactly the same as the asset underlying the futures contract (e.g., stock index futures and your stock portfolio)

  The hedger may be uncertain as to the exact date when the asset will be bought or sold

  The hedge may require the futures contract to be closed out well before its expiration date

Basis

Basis (b) = spot price (S) - futures price (F)

Basis risk

Let S_1, F_1, and b_1 be the spot price, futures price, and basis at time t_1 and S_2, F_2, and b_2 be the spot price, futures price, and basis at time t_2, then $b_1 = S_1 - F_1$ at time t_1 and $b_2 = S_2 - F_2$ at time t_2.

Consider a hedger who knows that the asset will be sold at time t_2 and takes a short position at time t_1. The spot price at time t_2 is S_2 and the payoff on the futures position is $(F_1 - F_2)$ at time t_2. The effective price is

$S_2 + F_1 - F_2 = F_1 + b_2$, where b_2 refers to the basis risk

Basis risk in a short hedge

Suppose it is March 1 (t_1). You expect to receive 50 million yen at the end of July (t_2). The September futures price (exchange rate) is currently 0.8900 dollar for 100 yen.

Hedging strategy:

(1) Sell four September yen futures contracts on March 1 (Since the contract size is 12.5 million yen 4 contracts will cover 50 million yen)

(2) Close out the contracts when yen arrives at the end of July

Basis risk: arises from the uncertainty as to the difference between the spot price and September futures price of yen at the end of July ($S_2 - F_2 = b_2$)

The outcome: at the end of July, suppose the spot price was 0.9150 and the September futures price was 0.9180, then the basis $b_2 = 0.9150 - 0.9180 = -0.0030$

Gain on futures contract $F_1 - F_2 = 0.8900 - 0.9180 = -0.0280$

Effective price $F_1 + b_2 = 0.8900 - 0.0030 = 0.8870$ or

Effective price $S_2 + F_1 - F_2 = 0.9150 - 0.0280 = 0.8870$

Detailed illustration:

Sell 4 Sept. Yen futures contracts	Receive 50 million Yen, exchange Yen to \$ at S_2, close Sept. contracts	Delivery month
↓	↓	↓
March 1 (t_1)	End of July (t_2)	September
(Know S_1, F_1, and b_1)	(Receive 50 million yen)	(F_1 = 0.8900, known in t_1)
(Don't know S_2, F_2, and b_2)	(Know S_2, F_2, and b_2)	(F_2 = 0.9180, known in t_2)
	(Effective price = $S_2 + F_1 - F_2$)	

Choice of contract for hedging:

(1) Same underlying asset (or closely related)

(2) Delivery month (usually a later delivery month)

- Cross hedging

If the asset underlying the futures contract is the same as the asset whose price is being hedged, the hedge ratio usually is 1.0. For example, if a farmer expects to harvest 10,000 bushels of corn, the farmer should sell 2 corn futures contracts.

Cross hedging: two different assets (but closely related)

Minimum variance hedge ratio: the ratio of the size of the position taken in futures contract to the size of the exposure to minimize the variance of the hedged position

Define
ΔS : Change in spot price, S
ΔF : Change in futures price, F
σ_S : Standard deviation of ΔS
σ_F : Standard deviation of ΔF
ρ : Correlation coefficient between ΔS and ΔF

When the hedger is long the asset and short futures, the change in value of the hedged position is $\Delta S - h\Delta F$

When the hedger is short the asset and long futures, the change in value of the hedged position is $h\Delta F - \Delta S$

Taking variance of the hedged position, both positions yield

$$v = \sigma_S^2 + h^2\sigma_F^2 - 2h\rho\sigma_S\sigma_F$$

Taking the first order derivative of v with respect to h in order to minimize the risk

Minimum variance hedge ratio, $h^* = \rho \dfrac{\sigma_S}{\sigma_F}$, where ρ is the correlation coefficient

Note: h^* is the estimated slope coefficient in a linear regression of ΔF on ΔS and ρ^2 is the R^2 from the regression and it is called hedge effectiveness

Optional number of contracts is given by:

$N^* = h^* Q_A/Q_F$, where Q_A is the size of position being hedged, and Q_F is the size of futures contract

For example, if an airline wants to purchase 2 million gallons of jet fuel and decides to use heating oil futures to hedge, and if h^* is 0.78 and the contract size for heating oil is 42,000 gallons, then the airline should buy 37 contracts to hedge (futures contracts on jet fuel are not available in the market).

- Stock index futures

  Stock indices: price weighted vs. value weighted – Table 3.3

  Price weighted indices: for example, DJIA

  Value weighted indices: for example, S&P 500

  For stock index futures, the optional number of contracts is given by:

  $N^* = \beta V_A/V_F$, where V_A is the current value of the portfolio, V_F is the current value of the stocks underlying one futures contract, and β is the beta of the portfolio

  For example, if you want to hedge a stock portfolio using the S&P 500 futures contract, if P = \$5,000,000, β = 1.5, S&P 500 index = 1,000, A = 250*1,000 = 250,000, then

  N^* = 1.5*5,000,000/250,000 = 30 contracts (short)

  Changing beta of a portfolio from β to β^* ($\beta > \beta^*$)

  $N^* = (\beta - \beta^*)V_A/V_F$

  For example, if you want to reduce the portfolio beta from 1.5 to 0.75

  N^* = 15 contracts (short)

- Rolling hedging

  Rolling the hedge forward multiple times (n times)

Short futures contract 1	Close out future contract 1 and short futures contract 2	Close out futures contract 2 and short futures contract 3		Close out futures contract *n*
Time 1 (t_1)	Time 2 (t_2)	Time 3 (t_3)	- - - - - -	Time T (t_n)

- Assignments

  Quiz (required)

  Practice Questions: 3.12, 3.16 and 3.18

- Appendix: review of standard deviation, correlation, linear regression, and CAPM

Chapter 4 - Interest Rates

- Types of interest rates
- Measuring interest rates
- Zero rates
- Bond pricing
- Forward rates
- Forward rate agreements
- Term structure theories

- Interest rates

  Treasury rates - risk-free rates
  T-bill rates vs. T-bond rates

  LIBOR (London Interbank Offer Rate): used between large international banks
  LIBID (London Interbank Bid Rate): used between large international banks
  LIBOR > LIBID

  Repo rate: an investment dealer sells its securities to another company and agrees to buy them back later at a slightly higher price - the percentage change in prices is the repo rate

- Measuring interest rates

  Nominal rate vs. effective rate

  Effective rate $= (1 + R/m)^m - 1$, where R is the annual nominal rate and m is the number of compounding within a year

  If m goes to infinity, we have effective rate under continues compounding, $e^R - 1$

  In general, the FV of \$A compounded continuously for n years at a nominal annual rate of R is FV = Ae^{Rn}

  In the same way, the PV of \$A discounted continuously at a nominal rate of R for n years is PV = Ae^{-Rn}

  Relationship between continues compounding and compounding m times per year:

  $e^{R_C} = (1 + R_m/m)^m$, where R_c is a rate of interest with continues compounding and R_m is the equivalent rate with compounding m times per year, or $R_C = m * \ln(1 + R_m/m)$
  For example, for a 10% annual rate with semiannual compounding, the equivalent rate with continuous compounding is R_c = 2*ln (1 + 0.1/2) = 9.758%

- Zero rates

  Zero rates: n-year zero-coupon interest rate is the rate of interest earned on an investment that starts today and lasts for n years

- Bond pricing

  When pricing a bond, you should use different zero rates to discount all expected future cash flows (coupon payments and face value) to the present.

  Determining Treasury zero rates using T-bond price quotes

Bond principal ($)	Time to maturity (years)	Annual coupon ($)	Bond price (S)
100	0.25	0	97.5
100	0.50	0	94.9
100	1.00	0	90.0
100	1.50	8	96.0
100	2.00	12	101.6

  For a 3 month T-bond, the price is 97.5 for $100 face value. With quarterly compounding, the 3 month T-bond has a zero rate of 4*(2.5)/97.5 = 10.256%. Convert that rate to continuous compounding

  $R_c = m*\ln(1 + R_m/m) = 0.10127 = 10.127\%$ (with $m = 4$ and R_m is 10.256%)

  Similarly, we can obtain the 6 month and 1 year T-bond zero rates of 10.469% and 10.536% for continuous compounding.

  To determine the 1.5 year zero-rate, we solve

  $4e^{-0.10469*0.5} + 4e^{-0.10536*1} + 104e^{-R*1.5} = 96$ (semiannual coupon payments)

  $R = 10.681\%$

  In a similar way, we can obtain the 2 year zero-rate of 10.808%

- Forward rates

  Spot rate vs. forward rate

  An n-year spot rate is the interest rate on an investment that starts today and lasts for n years

  A forward rate is an interest rate that is implied by the current spot rates for periods of time in the future

  In general, $R_F = (R_2T_2 - R_1T_1) / (T_2 - T_1)$

  For example, if $T_1 = 3$, $T_2 = 4$, $R_1 = 0.046 = 4.6\%$, and $R_2 = 0.05 = 5\%$, then $R_F = 6.2\%$

- Forward rate agreements

  A forward rate agreement (FRA) is an over-the-counter agreement designed to ensure that a certain interest rate will apply to either borrowing or lending a certain principal during a specified future period of time.

  Example 4.2

- Term structure theories

  Term structure of interest rates and yield curves

  Term structure of interest rates: relationship between interest rates (yields) and time to maturity

  Yield curve: a graph showing the term structure of interest rates

  Expectation theory: long-term interest rates should reflect expected futures short-term interest rates

  For example, if a 1-year T-bond yields 5% and a 2-year T-bond yields 5.5%, then the 1-year forward rate between year 1 and year 2 is expected to be 6%.

  $R_F = (R_2T_2 - R_1T_1) / (T_2 - T_1) = (5.5\%*2 - 5\%*1) / (2 - 1) = 6\%$

  If a yield curve is upward sloping, it indicates that the short term interest rates in the future will rise.

  Market segmentation theory: interest rates are determined by the demand and supply in each of different markets (short-, medium-, and long-term markets, for example)

  Liquidity preference theory: investors prefer to invest in short-term funds while firms prefer to borrow for long periods - yield curves tend to be upward sloping

- Assignments

  Quiz (required)

  Practice Questions: 4.10, 4.11 and 4.14