

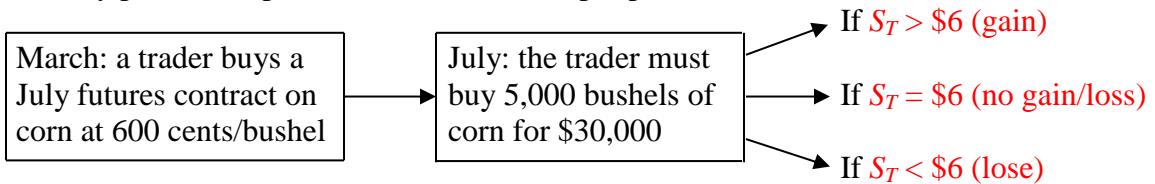
## Chapter 1 - Introduction

- Derivative securities
- Futures contracts
- Forward contracts
- Futures and forward markets
- Comparison of futures and forward contracts
- Options contracts
- Options markets
- Comparison of futures and options
- Types of traders
- Applications

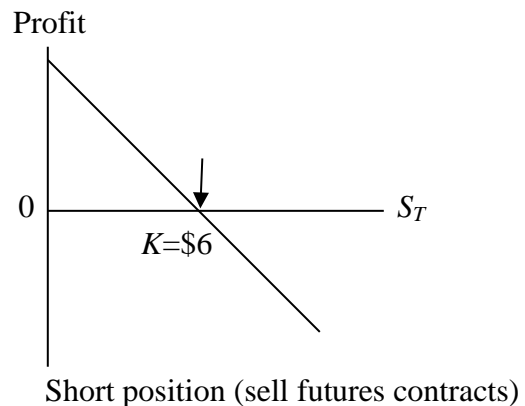
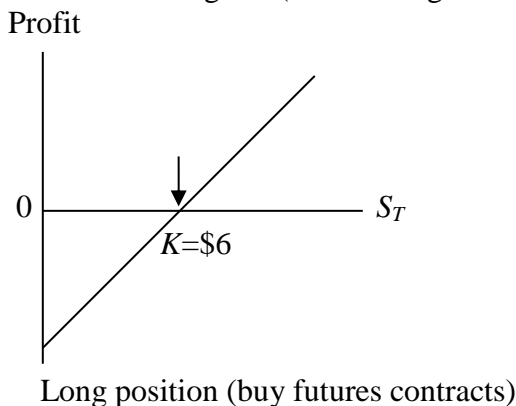
- Derivative securities
  - Securities whose values are derived from the values of other underlying assets
  - Futures and forward contracts
  - Options
  - Swaps
  - Others

- Futures contracts
  - An agreement between two parties to either buy or sell an asset at a certain time in the future for a certain price

For example, in March a trader buys a July futures contract on corn at 600 cents (the delivery price is \$6/per bushel and  $S_T$  is the spot price when the contract matures)



Profit/Loss Diagram (Refer to Figure 2.3)



$K$  = delivery price = \$6/bushel and  $S_T$  = the spot price at maturity, which can be greater than, equal to, or less than \$6/per bushel)

If  $S_T$  is greater than  $K$ , the person with a long position gains ( $S_T - K$ ) and the person with a short position loses ( $K - S_T$ ): a zero sum game (someone's gain is someone else's loss)

If  $S_T$  is less than  $K$ , the person with a long position loses ( $S_T - K$ ) and the person with a short position gains ( $K - S_T$ ): a zero sum game again

If  $S_T$  is equal to  $K$ , there is no gain or loss on both sides: a zero sum game

Corn: underlying asset - commodity (commodity futures contract)

Buy a futures contract: long position - promise to buy

600 cents/per bushel: futures price (delivery price)

5,000 bushels: contract size - standardized

July: delivery month

Spot price: the actual price in the market for immediate delivery

More examples

(1) **Long** (buy) futures positions: agree to buy or call for delivery

On February 1, you buy a June gold futures contract at 1,300: you agree to buy (or call for delivery) 100 ounces of gold in June at \$1,300 per troy ounce

Contract details

1,300 dollars per ounce - futures price of gold on February 1 for June delivery, also called delivery price (Note: the futures price of gold on February 2 for June delivery may be different)

100 ounces - contract size (standardized)

June - delivery month

Underlying asset: gold - commodity (it is a commodity futures contract)

Position: long position

Actual market price of gold - spot price which can be different from the futures price

(2) **Short** (sell) futures positions: agree to sell or promise to deliver

On February 1, you sell a June Yen futures contract at 0.9850: you agree to sell (or promise to deliver) 12,500,000 Yen in June at \$0.9850 per 100 Yen (\$1 for 101.52 Yen)

Contract details

\$0.9850 per 100 Yen - futures price (exchange rate), also called delivery price

12,500,000 Yen - contract size

June - delivery month

Underlying asset: foreign currency - financial asset (it is a financial futures contract)

Position: short position

Actual market exchange rate - spot exchange rate which can be different from the futures exchange rate

Futures contracts can be written on different assets (underling assets):  
 Commodities - commodity futures, for example, grains, livestock, meat, metals, and oil  
 Financial assets - financial futures, for example, stock indices, bonds, currencies  
 Others

Technical details (covered in later chapters)  
 Margin requirements; Daily settlement procedures - marking to market  
 Bid-offer spreads; Clearinghouses; Delivery

- Forward contracts  
 A forward contract is similar to a futures contract in that it is an agreement between two parties to either buy or sell an asset at a certain time in the future for a certain price. But forward contracts are less formal, traded only in OTC markets, and contract sizes are not standardized.

- Futures and forward markets  
 (1) Exchange-traded markets  
 Chicago Board of Trade (CBOT): futures contracts  
 Chicago Mercantile Exchange (CME): futures contracts

Open-outcry system: traders physically meet on the floor of the exchange and use a complicated set of hand signals to trade

Electronic trading: increasingly replacing the open-outcry system to match buyers and sellers

- (2) Over-the-counter (OTC) markets  
 Telephone- and computer-linked network of dealers

Flexibility - tailor your needs

Credit risk - the risk that your contract will not be honored

- Comparison of futures and forward contracts (Refer to Table 2.3)

	Exchange trading	Standardized contract size	Marking to market	Delivery	Delivery time	Default risk
Forward	No	No	No	Yes or cash settlement	One date	Some credit risk
Futures	Yes	Yes	Yes	Usually closed out	Range of dates	Virtually no risk

- Options contracts

Rights to buy or sell an asset by a certain date for a certain price

American options vs. European options

American options can be exercised at any time before the expiration date

European options can only be exercised on the maturity date

Keeping other things the same, which type of options should be worth more and why?

Answer: American options because they can be exercised at any time before expiration

Two types of options: call options vs. put options

A call option gives the right to buy an asset for a certain price by a certain date

For example, you buy an IBM June 190 call option for \$3.00

Option type: call option - the right to buy

The underlying asset - IBM stock

Exercise (strike) price - \$190 per share

Expiration date - the third Friday in June

Contract size: 100 shares

Option premium (price of the option): \$300.00

A put option gives the right to sell an asset for a certain price by a certain date

For example, you buy a GE June 26 put option for \$2.00

Option type: put option - the right to sell

The underlying asset - GE stock

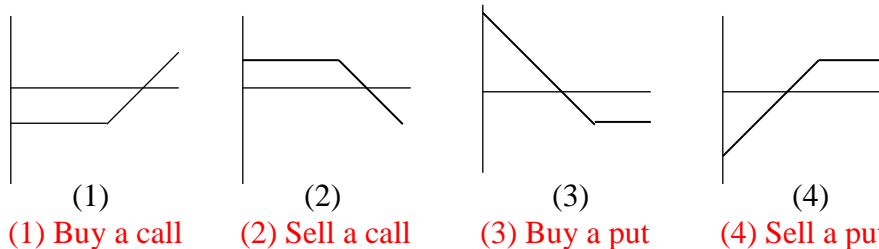
Exercise (strike) price - \$26 per share

Expiration date - the third Friday in June

Contract size: 100 shares

Option premium: \$200.00

Four types of positions: buy a call, sell (write) a call, buy a put, and sell (write) a put



Options can be written on different assets:

Stocks - stock options

Stock indices - index options

Currencies - currency options

Futures - futures options

Others

- Options markets
  - Exchange traded markets
    - (1) Chicago Board Options Exchange (CBOE): options
    - (2) Over-the-counter (OTC) markets
- Comparison of futures and options
  - Rights (options) vs. obligations (futures)
  - Initial outlay (buying options requires an initial outlay while buying futures doesn't)
  - Both need a margin account (futures are subject to marking to market daily)
  - Both use leverage

- Types of traders
  - (1) Hedgers: use options and futures markets to reduce price uncertainty (risk) in the future

For example, a farmer can sell corn futures contracts to lock in a price and an investor can buy a put option to protect a potential downward movement of a particular stock

More details: a company can use forward contracts for hedging currency risk  
 Import Co. purchased goods from a British supplier in June and needs to pay 10 million British pounds in September. A local financial institution offers forward contracts for British pounds. The quotes are shown below:

	Bid	Offer
Spot	1.6382	1.6386
1-month forward	1.6380	1.6385
3-month forward	1.6378	1.6384
6-month forward	1.6376	1.6383

How should Import Co. hedge the exchange rate risk?

Answer: Import Co. should buy 10 million British pounds in the three-month forward market to lock in the exchange rate of 1.6384 (or 16.384 million dollars for 10 million pounds for September delivery)

When you sell pounds to the financial institution you get the bid price  
 When you buy pounds from the financial institution you pay the offer price  
 The difference between bid and offer is called bid-offer spread which is the profit for the institution

(2) Speculators: bet on price movement

For example, you buy gold futures contracts because you bet that the price of gold will go up in the future, or you buy a put option on Intel if you bet that Intel stock price will drop

More details: a trader uses options for speculation (leverage effect)

A trader with \$2,000 to invest bets that the price of ORCL will increase in the near future and has the following quotes:

Current stock price: \$20.00

ORCL June call option with exercise price of \$22.50 sells for \$1.00

Alternative 1: buy 100 shares of ORCL

Alternative 2: buy 20 ORCL June call options with exercise price of \$22.50

Possible outcomes:

If ORCL stock price rises to \$27

Alternative 1: profit of \$700 =  $100 \times (27 - 20)$

Alternative 2: profit of \$7,000 =  $20 \times 100(27 - 22.5 - 1)$

If ORCL stock price falls to \$17

Alternative 1: loss of \$300 =  $100 \times (17 - 20)$

Alternative 2: loss of \$2,000 (the options are worthless)

If ORCL stock price rises to \$23.5

Alternative 1: profit of \$350 =  $100 \times (23.5 - 20)$

Alternative 2: break-even ( $23.5 - 22.5 - 1$ )

(3) Arbitrageurs: look for risk-free profits by taking the advantage of mispricing in two or more markets simultaneously

Conditions for arbitrage:

Zero net cost

No risk

Positive profit

Example: use stocks and exchange rates for arbitrage

A stock is traded on both NYSE and London Stock Exchange. The following quotes are obtained at a particular time:

Stock price at NYSE: \$20/share

Stock price at London Stock Exchange: 13 pounds/share

Spot exchange rate: 1 pound = \$1.60

Detailed arbitrage process:

Borrow \$2,000 to buy 100 shares at NYSE

Sell the shares at London Stock Exchange for 1,300 pounds

Convert the sale proceeds from pounds to dollars at the spot exchange rate to receive  
 $\$2,080 = 1,300 * 1.60$

Repay the loan of \$2,000

Arbitrage profit:  $2,080 - 2,000 = \$80$  (ignoring transaction costs)

(4) Dangers: start from hedgers or arbitrageurs and consciously or unconsciously become speculators

- Applications

Market completeness - any and all identifiable payoffs can be obtained by trading the derivative securities available in the market (Financial Engineering)

Risk management - reduce risk, hedging

Trading efficiency - increase market efficiency

Price discovery - futures price is the best expected spot price in the future

Speculation - bet on prices

- Assignments

Quiz (required)

Practice Questions: 1.11, 1.13, 1.14, 1.20, and 1.21

## Chapter 2 - Mechanics of Futures Markets

- Specification of a futures contract
  - Convergence of futures price to spot price
  - Operation of margins
  - Quotes and prices
  - Delivery
  - Types of orders
  - Regulation
  - Accounting and taxes
- 
- Specification of futures contracts
    - Opening a futures position vs. closing a futures position

Opening a futures position can be either a long (buy) position or a short (sell) position  
Closing a futures position involves entering an opposite trade to the original one

The underlying asset or commodity: must be clearly specified

The contract size: standardized  
Corn and wheat: 5,000 bushels per contract  
Live cattle: 40,000 pounds per contract  
Cotton: 50,000 pounds per contract  
Gold: 100 troy ounces per contract  
DJIA: \$10\*index  
Mini DJIA: \$5\*index  
S&P 500: \$250\*index  
Mini S&P 500: \$50\*index

Delivery month: set by the exchange

Delivery place: specified by the exchange

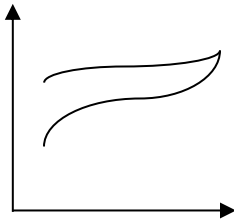
Daily price limits: usually specified by the exchange, it is the restriction on the day-to-day price change of an underlying commodity. If in a day the price moves down from the previous day's close by an amount equal to the daily price limit, we say the contract is limit down. If in a day the price moves up from the previous day's close by an amount equal to the daily price limit, we say the contract is limit up.

Position limits: the maximum number of contracts that a speculator may hold (Why?)  
Ask students to check for price limits, position limits, and how Hunt brothers cornered the silver market in 1974

Tick: the minimum price fluctuation, for example, ¼ cent per bushel for wheat



- Convergence of futures price to spot price  
As the delivery period approaches, the futures price converges to the spot price of the underlying asset



Why? Show that the arbitrage opportunity exists if it doesn't happen

If the futures price is above the spot price as the delivery period is reached

- (1) Short a futures contract
- (2) Buy the asset at the spot price
- (3) Make the delivery

If the futures price is below the spot price as the delivery period is reached

- (1) Buy a futures contract
- (2) Short sell the asset and deposit the proceeds
- (3) Take the delivery and return the asset

- Operation of margins  
Margin account: an account maintained by an investor with a brokerage firm in which borrowing is allowed

Initial margin: minimum initial deposit

Maintenance margin: the minimum actual margin that a brokerage firm will permit investors to keep their margin accounts

Margin call: a demand on an investor by a brokerage firm to increase the equity in the margin account

Variation margin: the extra fund needs to be deposited by an investor

Marking to market: the procedure that the margin account is adjusted to reflect the gain or loss at the end of each trading day

Example: suppose an investor buys two gold futures contracts. The initial margin is \$6,000 per contract (or \$12,000 for two contracts) and the maintenance margin is \$4,500 per contract (or \$9,000 for two contracts). The contract is entered into on Day 1 at \$1,650 and closed out on Day 16 at \$1,626.90. (Gold is trading around \$1,300 per ounce now)  
 Ask students to check the gold futures prices now

Day	Trade/Futures Price (\$)	Settlement Price (\$)	Daily Gain (\$)	Cumulative Gain (\$)	Margin Balance (\$)	Margin Call (\$)
1	1,650.00				12,000	
1		1,641.00	-1,800	-1,800	10,200	
2		1,638.30	-540	-2,340	9,660	
.....		.....	.....	.....	.....	
6		1,636.20	-780	-2,760	9,240	
7		1,629.90	-1,260	-4,020	7,980	4,020
8		1,630.80	180	-3,840	12,180	
.....		.....	.....	.....	.....	
16	1,626.90		780	-4,620	15,180	

Note: the investor earns interest on the balance in the margin account. The investor can also use other assets, such as T-bills to serve as collateral (at a discount)

Clearing house: an intermediary to guarantee in futures transactions

An individual investor is required to maintain a margin account with a broker (or a brokerage firm) while a broker is also required to maintain a margin account with the clearing house, known as a clearing margin

- Quotes and prices

Opening price, highest and lowest prices, and settlement price

Settlement price: the average of the prices immediately before the closing bell and marking to market is based on the settlement price

**Quotes: Crude Oil Trading on July 13, 2012 (from Table 2.2, page 35)**

	Open	High	Low	Prior settle	Last trade	Change	Volume
Aug 2012	85.86	87.61	85.58	86.08	87.28	+1.20	223,698
Sept 2012	86.33	88.00	85.95	86.46	87.68	+1.22	87,931
Dec 2012	87.45	89.21	87.39	87.73	88.94	+1.21	31,701
Dec 2013	88.85	90.15	88.78	88.92	89.95	+1.03	11,128
Dec 2014	87.20	87.74	87.20	86.98	87.74	+0.76	2,388

Ask students to check for the crude oil futures prices (it is trading around \$100 per barrel)

Open interest: the total number of contracts outstanding at a particular time

Open interest: an example

Given the following trading activities, what is the open interest at the end of each day?

Time	Actions	Open Interest
t = 0	Trading opens for gold contract	0
t = 1	Trader A buys 2 and trader B sells 2 gold contracts	2
t = 2	Trader A sells 1 and trader C buys 1 gold contract	2
t = 3	Trader D sells 2 and trader C buys 2 gold contracts	4

Ask students to check open interest

- Delivery

Very few futures contracts lead to actual delivery and most of futures contracts are closed out prior to the delivery month

If delivery takes place, the decision on when to deliver is made by the party with the short position

Cash settlement: some financial futures, for example, stock index futures are cash settled (No actual delivery)

- Types of orders

Market order: the best price currently available in the market

Limit order: specifies a price (limit price) and your order will be executed only at that price or a more favorable price to you

More specifically, you will buy at or below a specified price (limit price) or you will sell at or above a specified price (limit price)

For example, current futures price of gold = \$1,300, you can specify to buy a futures contract on gold if the futures price  $\leq$  \$1,280 or you can specify to sell a futures contract on gold if the futures price  $\geq$  \$1,350

Stop (stop-loss) order: specifies a price (stop price) and your order will become a market order if the stop price is reached

For example, current futures price of gold = \$1,300, you can specify to sell a futures contract if the futures price  $\leq$  \$1,260 or you can specify to buy a futures contract if the futures price  $\geq$  \$1,350

The main difference between a limit order and a stop order:

With a limit order you buy when the price drops and you sell when the price rises

With a stop order you buy when the price rises and you sell when the price drops

Stop-limit order: a combination of a stop order and a limit order

For example, current futures price of gold = \$1,300

A stop-limit order to sell at a stop price \$1,250 with a limit price \$1,220

A stop-limit order to buy at a stop price \$1,320 with a limit price \$1,350

Market-if-touched order: executed at the best available price after a specified price is reached

For example, current futures price of gold = \$1,300

A market-if-touched order to sell if price  $\geq$  \$1,320

Day order: valid for the day

Open order (good-till-canceled): in effect until the end of trading in a particular contract

- Regulation

Futures markets are mainly regulated by the Commodity Futures Trading Commission (CFTC). Other agencies, for example, National Futures Association (NFA), Securities and Exchange Committee (SEC), the Federal Reserve Board, and U.S. Treasury Department also step in from time to time.

The Commodity Futures Trading Commission (CFTC): to approve new contracts, set up daily maximum price fluctuation, minimum price movements, and certain features of delivery process

The National Futures Association (NFA): to prevent fraudulent and manipulative acts and practices

Trading irregularities: corner the market - take a huge long futures position and also try to exercise some control of the underlying commodity

Example: Hunt brothers' price manipulation in the silver market in 1979-1980

- Accounting and taxes

Changes in the market value of a futures contract are recognized when they occur unless the contract is qualified as a hedge

For example, consider a company with a December year end. In September 2013 it buys a March 2014 corn futures contract at 750 (750 cents for one bushel). At the end of 2013, the price for March 2014 corn futures contract is 770 and it is 780 when the contract is closed out in February 2014.

If the contract doesn't qualify as a hedge, the gains are

$(7.70 - 7.50) * (5,000) = \$1,000$  in 2013

$(7.80 - 7.70) * (5,000) = \$500$  in 2014

For speculators, all paper gains or losses on futures contracts are treated as though they were realized at the end of the tax year - marking to market at the end of the year

40% of any gains or losses are to be treated as short-term and

60% of any gains or losses are to be treated as long-term

If the contract qualifies as a hedge, all the gains (\$1,500) are realized in 2014

For hedgers, all paper gains or losses are realized when the contracts are closed out

The 40% short-term and 60% long-term rule doesn't apply for hedgers

- Assignments

Quiz (required)

Practice Questions: 2.11, 2.15, 2.16, and 2.23

## Chapter 3 - Hedging Strategies Using Futures

- Hedging principles
- Arguments for and against hedging
- Basis risk
- Cross hedging
- Stock index futures
- Rolling hedging

- Hedging principles
  - Hedging: to reduce risk
  - Complete hedging: to eliminate risk

Short hedges: use short positions in futures contracts to reduce or eliminate risk

For example, an oil producer can sell oil futures contracts to reduce oil price uncertainty in the future

Long hedge: use long positions in futures contracts to reduce or eliminate risk

For example, a brewer buys wheat futures contracts to reduce future price uncertainty in wheat

- Arguments for and against hedging
  - Hedging can reduce risk but it has a cost

(1) Firms don't need to hedge because shareholders can hedge by themselves

(2) Hedging may cause profit margin to fluctuate

(3) Hedging may offset potential gains

- Basis risk
  - Hedging usually cannot be perfect for the following reasons:

The asset whose price is to be hedged may not be exactly the same as the asset underlying the futures contract (e.g., stock index futures and your stock portfolio)

The hedger may be uncertain as to the exact date when the asset will be bought or sold

The hedge may require the futures contract to be closed out well before its expiration date

Basis

Basis ( $b$ ) = spot price ( $S$ ) - futures price ( $F$ )

Basis risk

Let  $S_1, F_1$ , and  $b_1$  be the spot price, futures price, and basis at time  $t_1$  and  $S_2, F_2$ , and  $b_2$  be the spot price, futures price, and basis at time  $t_2$ , then  $b_1 = S_1 - F_1$  at time  $t_1$  and  $b_2 = S_2 - F_2$  at time  $t_2$ .

Consider a hedger who knows that the asset will be sold at time  $t_2$  and takes a short position at time  $t_1$ . The spot price at time  $t_2$  is  $S_2$  and the payoff on the futures position is  $(F_1 - F_2)$  at time  $t_2$ . The effective price is

$S_2 + F_1 - F_2 = F_1 + b_2$ , where  $b_2$  refers to the basis risk

Basis risk in a short hedge

Suppose it is March 1 ( $t_1$ ). You expect to receive 50 million yen at the end of July ( $t_2$ ). The September futures price (exchange rate) is currently 0.8900 dollar for 100 yen.

Hedging strategy:

- (1) Sell four September yen futures contracts on March 1 (Since the contract size is 12.5 million yen 4 contracts will cover 50 million yen)
- (2) Close out the contracts when yen arrives at the end of July

Basis risk: arises from the uncertainty as to the difference between the spot price and September futures price of yen at the end of July ( $S_2 - F_2 = b_2$ )

The outcome: at the end of July, suppose the spot price was 0.9150 and the September futures price was 0.9180, then the basis  $b_2 = 0.9150 - 0.9180 = -0.0030$

Gain on futures contract  $F_1 - F_2 = 0.8900 - 0.9180 = -0.0280$

Effective price  $F_1 + b_2 = 0.8900 - 0.0030 = 0.8870$  or

Effective price  $S_2 + F_1 - F_2 = 0.9150 - 0.0280 = 0.8870$

Detailed illustration:

Sell 4 Sept. yen futures contracts	Receive 50 million yen, exchange yen to \$ at $S_2$ , close Sept. contracts	Delivery month
↓	↓	↓
March 1 ( $t_1$ ) (Know $S_1, F_1$ , and $b_1$ ) (Don't know $S_2, F_2$ , and $b_2$ )	End of July ( $t_2$ ) (Receive 50 million yen) (Know $S_2, F_2$ , and $b_2$ )	September  ( $F_1 = 0.8900$ , known in $t_1$ ) ( $F_2 = 0.9180$ , known in $t_2$ ) (Effective price = $S_2 + F_1 - F_2$ )

Choice of contract for hedging:

- (1) Same underlying asset (or closely related)
- (2) Delivery month (usually a later delivery month)

- Cross hedging

If the asset underlying the futures contract is the same as the asset whose price is being hedged, the hedge ratio usually is 1.0. For example, if a farmer expects to harvest 10,000 bushels of corn, the farmer should sell 2 corn futures contracts.

Cross hedging: two different assets (but closely related)

Minimum variance hedge ratio: the ratio of the size of the position taken in futures contract to the size of the exposure to minimize the variance of the hedged position

Define

$\Delta S$  : change in spot price,  $S$

$\Delta F$  : change in futures price,  $F$

$\sigma_S$  : standard deviation of  $\Delta S$

$\sigma_F$  : standard deviation of  $\Delta F$

$\rho$  : correlation coefficient between  $\Delta S$  and  $\Delta F$

When the hedger is long the asset and short futures, the change in value of the hedged position is  $\Delta S - h\Delta F$

When the hedger is short the asset and long futures, the change in value of the hedged position is  $h\Delta F - \Delta S$

Taking variance of the hedged position, both positions yield

$$v = \sigma_S^2 + h^2 \sigma_F^2 - 2h\rho\sigma_S\sigma_F$$

Taking the first order derivative of  $v$  with respect to  $h$  in order to minimize the risk

Minimum variance hedge ratio,  $h^* = \rho \frac{\sigma_S}{\sigma_F}$ , where  $\rho$  is the correlation coefficient

Note:  $h^*$  is the estimated slope coefficient in a linear regression of  $\Delta F$  on  $\Delta S$  and  $\rho^2$  is the  $R^2$  from the regression and it is called hedge effectiveness

Optional number of contracts is given by:

$N^* = h^* Q_A / Q_F$ , where  $Q_A$  is the size of position being hedged, and  $Q_F$  is the size of futures contract

For example, if an airline wants to purchase 2 million gallons of jet fuel and decides to use heating oil futures to hedge, and if  $h^*$  is 0.78 and the contract size for heating oil is 42,000 gallons, then the airline should buy 37 contracts to hedge (futures contracts on jet fuel are not available in the market).



- Stock index futures  
Stock indices: price weighted vs. value weighted

Price weighted indices: for example, DJIA  
Value weighted indices: for example, S&P 500

For stock index futures, the optional number of contracts is given by:

$N^* = \beta V_A/V_F$ , where  $V_A$  is the current value of the portfolio,  $V_F$  is the current value of the stocks underlying one futures contract, and  $\beta$  is the beta of the portfolio

For example, if you want to hedge a stock portfolio using the S&P 500 futures contract, if  $V_A = \$5,000,000$ ,  $\beta = 1.5$ , S&P 500 index = 1,000,  $V_F = 250 * 1,000 = 250,000$ , then

$$N^* = 1.5 * 5,000,000 / 250,000 = 30 \text{ contracts (short)}$$

Changing beta of a portfolio from  $\beta$  to  $\beta^*$  ( $\beta > \beta^*$ )

$$N^* = (\beta - \beta^*) V_A / V_F$$

For example, if you want to reduce the portfolio beta to 0.75 from 1.5

$$N^* = 15 \text{ contracts (short)}$$

Changing beta of a portfolio from  $\beta$  to  $\beta^*$  ( $\beta < \beta^*$ )

$$N^* = (\beta^* - \beta) V_A / V_F$$

For example, if you want to increase the portfolio beta to 2 from 1.5

$$N^* = 10 \text{ contracts (long)}$$

- Rolling hedging  
Rolling the hedge forward multiple times (n times)

Short futures contract 1	Close out future contract 1 and short futures contract 2	Close out futures contract 2 and short futures contract 3	Close out futures contract n
Time 1 ( $t_1$ )	Time 2 ( $t_2$ )	Time 3 ( $t_3$ )	Time T ( $t_n$ )

- Assignments  
Quiz (required)  
Practice Questions: 3.12, 3.16, 3.18, and 3.24

## Chapter 4 - Interest Rates

- Types of interest rates
- Measuring interest rates
- Zero rates
- Bond pricing
- Forward rates
- Term structure theories

- Interest rates

Treasury rates - risk-free rates

T-bill rates vs. T-bond rates

LIBOR (London Interbank Offer Rate): used between large international banks

LIBID (London Interbank Bid Rate): used between large international banks

LIBOR > LIBID

Repo rate: an investment dealer sells its securities to another company and agrees to buy them back later at a slightly higher price - the percentage change in prices is the repo rate

- Measuring interest rates

Nominal rate vs. effective rate

**Effective rate** =  $(1 + R/m)^m - 1$ , where  $R$  is the annual nominal rate and  $m$  is the number of compounding within a year

If  $m$  goes to infinity, we have effective rate under continuous compounding,  $e^R - 1$

In general, the FV of \$A compounded continuously for  $n$  years at a nominal annual rate of  $R$  is **FV** =  $Ae^{Rn}$

In the same way, the PV of \$A discounted continuously at a nominal rate of  $R$  for  $n$  years is **PV** =  $Ae^{-Rn}$

Relationship between continuous compounding and compounding  $m$  times per year:

$e^{R_c} = (1 + R_m/m)^m$ , where  $R_c$  is a rate of interest with continuous compounding and  $R_m$  is

the equivalent rate with compounding  $m$  times per year, or  $R_c = m * \ln(1 + R_m/m)$

For example, for a 10% annual rate with semiannual compounding, the equivalent rate with continuous compounding is  $R_c = 2 * \ln(1 + 0.1/2) = 9.758\%$

- Zero rates  
Zero rates: n-year zero-coupon interest rate is the rate of interest earned on an investment that starts today and lasts for n years. There are no intermediate coupon payments.
- Bond pricing  
When pricing a bond, you should use different zero rates to discount all expected future cash flows (coupon payments and face value) to the present.

Determining Treasury zero rates using T-bond price quotes

Bond principal (\$)	Time to maturity (years)	Annual coupon (\$)	Bond price (\$)	Zero rate (%) (cont. comp.)
100	0.25 (3 months)	0	97.5	10.127
100	0.50 (6 months)	0	94.9	10.469
100	1.00 (12 months)	0	90.0	10.536
100	1.50 (18 months)	8	96.0	10.681
100	2.00 (24 months)	12	101.6	10.808

For a 3 month T-bond, the price is 97.5 for \$100 face value. With quarterly compounding, the 3 month T-bond has a zero rate of  $4*(2.5)/97.5 = 10.256\%$ . Convert that rate to continuous compounding

$$R_c = m*\ln(1 + R_m/m) = 0.10127 = 10.127\% \text{ (with } m = 4 \text{ and } R_m \text{ is } 10.256\%)$$

Similarly, we can obtain the 6 month and 1 year T-bond zero rates of 10.469% and 10.536% for continuous compounding.

To determine the 1.5 year zero-rate, we solve

$$4e^{-0.10469*0.5} + 4e^{-0.10536*1} + 104e^{-R*1.5} = 96 \text{ (semiannual coupon payments)}$$

$$R = 10.681\%$$

In a similar way, we can obtain the 2 year zero-rate of 10.808%

- Forward rates  
Spot rate vs. forward rate  
An n-year spot rate is the interest rate on an investment that starts today and lasts for n years

A forward rate is an interest rate that is implied by the current spot rates for periods of time in the future. In general,  $R_F = (R_2T_2 - R_1T_1) / (T_2 - T_1)$

For example, if  $T_1 = 3$ ,  $T_2 = 4$ ,  $R_1 = 0.046 = 4.6\%$ , and  $R_2 = 0.05 = 5\%$ , then  $R_F = 6.2\%$

- Term structure theories

Term structure of interest rates and yield curves

Term structure of interest rates: relationship between interest rates (yields) and time to maturity

Yield curve: a graph showing the term structure of interest rates

Expectation theory: long-term interest rates should reflect expected futures short-term interest rates

For example, if a 1-year bond yields a zero rate of 5% and a 2-year bond yields a zero rate of 5.5%, then the 1-year forward rate between year 1 and year 2 is expected to be 6%.

$$R_F = (R_2T_2 - R_1T_1) / (T_2 - T_1) = (5.5\%*2 - 5\%*1) / (2 - 1) = 6\%$$

If a yield curve is upward sloping, it indicates that the short term interest rates in the future will rise.

Market segmentation theory: interest rates are determined by the demand and supply in each of different markets (short-, medium-, and long-term markets, for example)

If the demand for short-term funds is higher than the demand for medium- and long-term funds we should observe a downward sloping yield curve

If the demand for long-term funds is higher than the demand for medium- and short-term funds we should observe an upward sloping yield curve

Liquidity preference theory: investors prefer to invest in short-term funds while firms prefer to borrow for long periods - yield curves tend to be upward sloping

- Assignments

Quiz (required)

Practice Questions: 4.10, 4.12, and 4.14