Chapter 9 - Mechanics of Options Markets

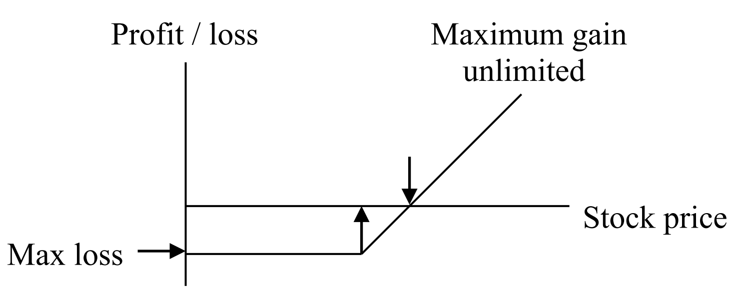
- Types of options
- Option positions and profit/loss diagrams
- Underlying assets
- Specifications
- Trading options
- Margins
- Taxation
- Warrants, employee stock options, and convertibles

- Types of options
  Two types of options: call options vs. put options
  Four positions: buy a call, sell (write) a call, buy a put, sell (write) a put

- Option positions and profit/loss diagrams
  Notations
  S_0: the current price of the underlying asset
  K: the exercised (strike) price
  T: the time to expiration of option
  S_T: the price of the underlying asset at time T
  C: the call price (premium) of an American option
  c: the call price (premium) of a European option
  P: the put price (premium) of an American option
  p: the put price (premium) of a European option
  r: the risk-free interest rate
  σ : the volatility (standard deviation) of the underlying asset price

  (1) Buy a European call option: buy a June 90 call option at $2.50

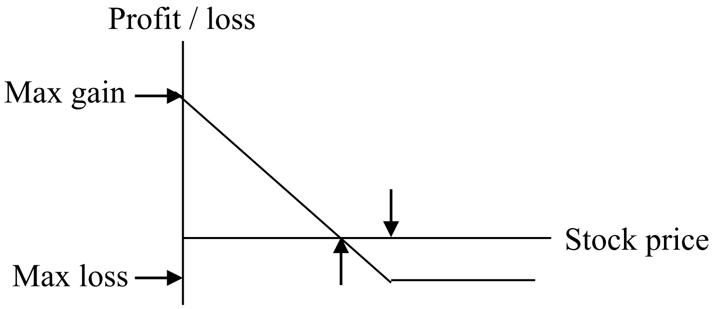
		Stock price at expiration			
		0	70	90	110
Buy June 90 call @ $2.50		-2.50	-2.50	-2.50	17.50
Net cost	$2.50	-2.50	-2.50	-2.50	17.50



Write a European call option: write a June 90 call at \$2.50 (exercise for students, reverse the above example)

Buy a European put option: buy a July 85 put at \$2.00

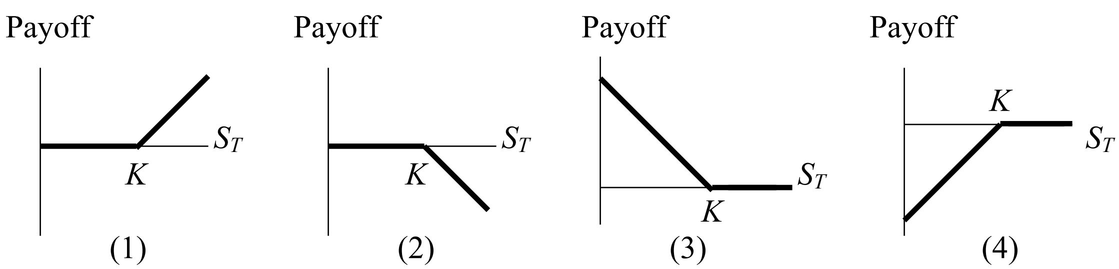
		Stock price at expiration			
		0	65	85	105
Buy June 85 put @ \$2.00		83.00	18.00	-2.00	-2.00
Net cost	\$2.00	83.00	18.00	-2.00	-2.00



Write a European put option: write a July 85 put at \$2.00 (exercise for students, reverse the above example)

In general, the payoff at time T:

(1) For a long European call option is = max (S_T - K, 0)
(2) For a short European call option is = min (K - S_T, 0) = -max (S_T - K, 0)
(3) For a long European put option is = max (K - S_T, 0)
(4) For a short European put option is = min (S_T - K, 0) = -max (K - S_T, 0)



In-the-money options: $S > K$ for calls and $S < K$ for puts

Out-of-the-money options: $S < K$ for calls and $S > K$ for puts

At-the-money options: $S = K$ for both calls and puts

Intrinsic value = max (S - K, 0) for a call option

Intrinsic value = max (K - S, 0) for a put option

C (or P) = intrinsic value + time value

Suppose a June 85 call option sells for \$2.50 and the market price of the stock is \$86, then the intrinsic value = 86 – 85 = \$1; time value = 2.50 - 1 = \$1.50

Suppose a June 85 put option sells for \$1.00 and the market price of the stock is \$86, then the intrinsic value = 0; time value = 1 - 0 = \$1

Naked call option writing: the process of writing a call option on a stock that the option writer does not own

Naked options vs. covered options

- Underlying assets
  If underlying assets are stocks - stock options
  If underlying assets are foreign currencies - currency options
  If underlying assets are stock indexes - stock index options
  If underlying assets are commodity futures contracts - futures options
  If the underlying assets are futures on fixed income securities (T-bonds, T-notes) - interest-rate options

- Specifications
  Dividends and stock splits: exchange-traded options are not adjusted for cash dividends but are adjusted for stock splits

  Position limits: the CBOE specifies a position limit for each stock on which options are traded. There is an exercise limit as well (equal to position limit)

  Expiration date: the third Friday of the month

- Trading options
  Market maker system (specialist) and floor broker

  Offsetting orders: by issuing an offsetting order

  Bid-offer spread

  Commissions

- Margins

    Writing naked options are subject to margin requirements

    The initial margin for writing a naked call option is the greater of
    (1) A total of 100% of proceeds plus 20% of the underlying share price less the amount, if any, by which the option is out of the money
    (2) A total of 100% of proceeds plus 10% of the underlying share price

    The initial margin for writing a naked put option is the greater of
    (1) A total of 100% of proceeds plus 20% of the underlying share price less the amount, if any, by which the option is out of the money
    (2) A total of 100% of proceeds plus 10% of the exercise price

    For example, an investor writes four naked call options on a stock. The option price is $5, the exercise price is $40, and the stock price is $38. Because the option is $2 out of the money, the first calculation gives 400*(5+0.2*38-2) = $4,240 while the second calculation gives 400*(5+0.1*38) = $3,520. So the initial margin is $4,240.

    If the options were puts, it would be $2 in the money. The initial margin from the first calculation would be 400*(5+0.2*38) = $5,040 while it would be 400*(5+0.1*40) = $3,600 from the second calculation. So the initial margin would be $5,040.

    Buying options requires cash payments and there are no margin requirements

    Writing covered options are not subject to margin requirements (stocks as collateral)

- Taxation

    In general, gains or losses are taxed as capital gains or losses. If the option is exercised, the gain or loss from the option is rolled over to the position taken in the stock.

    Wash sale rule: when the repurchase is within 30 days of the sale, the loss on the sale is not tax deductible

- Warrants, employee stock options, and convertibles

    Warrants are options issued by a financial institution or a non-financial corporation. Employee stock options are call options issued to executives by their company to motivate them to act in the best interest of the company's shareholders. Convertible bonds are bonds issued by a company that can be converted into common stocks.

- Assignments

    Quiz (required)
    Practice Questions: 9.9, 9.10 and 9.12

Chapter 10 - Properties of Stock Options

- Factors affecting option prices
- Upper and lower bounds for option prices
- Put-call parity
- Early exercise
- Effect of dividends

- Factors affecting option prices
  Six factors:
  Current stock price, S_0
  Strike (exercise) price, K
  Time to expiration, T
  Volatility of the stock price, σ
  Risk-free interest rate, r
  Dividends expected during the life of the option

  Refer to Table 10.1

Variables	European call	European put	American call	American put
Stock price	+	-	+	-
Strike price	-	+	-	+
Time to expiration	n/a	n/a	+	+
Volatility	+	+	+	+
Risk-free rate	+	-	+	-
Dividends	-	+	-	+

  Refer to Figures 10.1 and 10.2
  + indicates that two variables have a positive relationship (partial derivative is positive)
  - indicates that two variables have a negative relationship (partial derivative is negative)

- Upper and lower bounds for options prices
  Upper bounds for calls: $c \leq S_0$ and $C \leq S_0$
  If the condition is violated, arbitrage exists by buying the stock and writing the call

  Upper bounds for puts: $p \leq K$ and $P \leq K$
  For European put options, it must be: $p \leq Ke^{-rT}$
  If the condition is violated, arbitrage exists by writing the put and investing the proceeds at the risk-free rate

  Lower bound for European calls on nondividend-paying stocks: $c \geq S_0 - Ke^{-rT}$
  Lower bound for American calls on nondividend-paying stocks: $C \geq S_0 - Ke^{-rT}$
  If the condition is violated, arbitrage exists by buying the call, shorting the stock, and investing the proceeds

Lower bound for European puts on nondividend-paying stocks: $p \geq Ke^{-rT} - S_0$
Lower bound for American puts on nondividend-paying stocks: $P \geq K - S_0$
If violated, arbitrage exists by borrowing money and buying the put and the stock

- Put -call parity

  Considers the relationship between p and c written on the same stock with same exercise price and same maturity date

  Portfolio A: buy a European call option at c_t and invest Ke^{-rT}

	Stock price at expiration	
Portfolio A	$S_T > K$	$S_T \leq K$
Buy call @ c_t	$S_T - K$	0
Invest Ke^{-rT}	K	K
Net	S_T	K

Portfolio B: buy a European put option at p_t and buy one share of stock S_t

	Stock price at expiration	
Portfolio B	$S_T > K$	$S_T \leq K$
Buy put @ p	0	$K - S_T$
Buy stock at S_t	S_T	S_T
Net	S_T	K

Since two portfolios are worth the same at expiration, they should have the same value (cost) today. Therefore, we have the put-call parity for European options

$c_t + Ke^{-r(T-t)} = p_t + S_t$ or $c + Ke^{-rT} = p + S_0$ if $t = 0$ for today

Arbitrage exists if the parity does not hold

Example

You are interested in XYZ stock options. You noticed that a 6-month \$50 call sells for \$4.00, while a 6-month \$50 put sells for \$3.00. The 6-month interest rate is 6%, and the current stock price is \$48. There is an arbitrage opportunity present. Show how you can take the advantage of it.

Answer: $c + Ke^{-rT} = 4 + 50\, e^{-0.06\,(0.5)} = 52.52$
$p + S_0 = 3 + 48.00 = 51.00$
Arbitrage opportunity exists with a risk-free profit of \$1.52

Rationale: the stock and put are undervalued relative to the call

		Stock Price at expiration	
		If $S_T > 50$	If $S_T \leq 50$
Write a 50 call @	4.00	$(50 - S_T)$	0
Borrow \$48.52 (present value of 50)	48.52	- 50	- 50
Buy a share @ \$48.00	- 48.00	S_T	S_T
Buy a 50 put @ \$3.00	- 3.00	0	$(50 - S_T)$
Net	\$1.52	0	0

Put-call parity for American options: $S_0 - K \leq C - P \leq S_0 - Ke^{-rT}$

- Early exercise
  For American call options
  Nondividend-paying stocks: never early exercise
  (1) You can always sell the call at a higher price (intrinsic value + time value)
  (2) Insurance reason (what if the stock price drops after you exercise the option?)

  Dividend-paying stocks: early exercise may be optimal if dividends are large enough

  For American put options
  Nondividend-paying stocks: early exercise can be optimal if the option is deep in-the-money

- Effect of dividends
  Adjust for dividends (D is the present value of cash dividends)

  Lower bonds for calls with adjustments of dividends: $c \geq (S_0 - D) - Ke^{-rT}$
  Lower bonds for puts with adjustments of dividends: $p \geq Ke^{-rT} - (S_0 - D)$

  Since dividends lower the stock price, we use the adjusted stock price, $(S_0 - D)$ in the put-call parity. For stocks that pay dividends the put-call parity for European and American options can be written respectively as

$$c + K^{-rT} = p + (S_0 - D) \qquad \text{and} \qquad (S_0 - D) - K \leq C - P \leq S_0 - Ke^{-rT}$$

- Assignments
  Quiz (required)
  Practice Questions: 10.9, 10.10, 10.11 and 10.12

Chapter 11 - Trading Strategies Involving Options

- Strategies with a single option and a stock
- Spreads
- Combinations

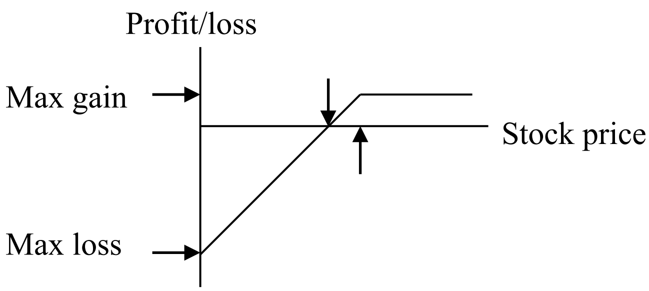
- Strategies with a single option and a stock

  A strategy involves an option and the underlying stock

  Strategy (1) - Long a stock and write a call (writing a covered call)

  Example: buy a stock at $86 and write a Dec. 90 call on the stock at $2.00

		Stock price at expiration			
		0	45	90	135
Buy stock @	86	-86	-41	4	49
Write Dec. 90 call @	2	2	2	2	-43
Net	-84	-84	-39	6	6



(1) Long a stock + write a call = write a put

Strategy (2) - Short a stock and buy a call

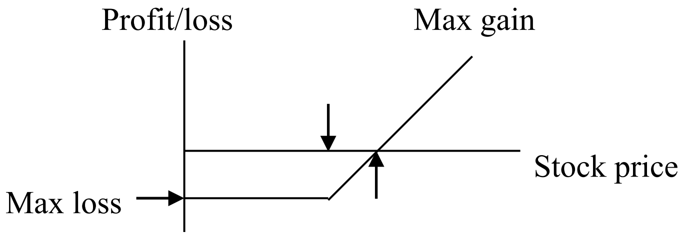
Example: short a stock at $86 and buy a Dec. 90 call on the stock at $2.00

(2) Short a stock + buy a call = buy a put (exercise for students, reverse strategy 1)

Strategy (3) - Long a stock and buy a put (protective put)

Example: buy a stock at $86 and buy a Dec. 85 put on the stock at $2.00

		Stock price at expiration			
		0	45	85	125
Buy stock @	86	-86	-41	-1	39
Buy Dec. 85 put @	2	83	38	-2	-2
Net	-88	-3	-3	-3	37



(3) Long a stock + buy a put = buy a call

Strategy (4) - Short a stock and write a put

Example: short a stock at $86 and write a Dec. 85 put on the stock at $2.00

(4) Short a stock + write a put = write a call (exercise for students, reverse strategy 3)

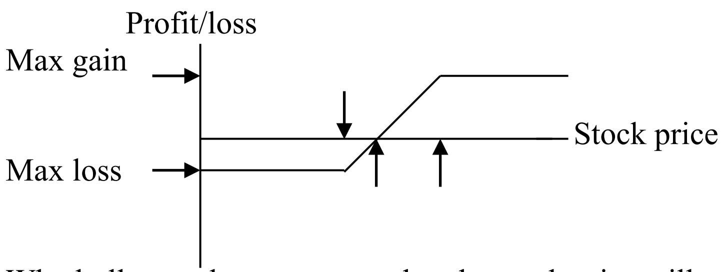
- Spreads

  A spread involves a position in two or more options of the same type

  Bull spreads: buy a call on a stock with a certain strike price and sell a call on the same stock with a higher strike price

  Example: buy a Dec. 85 call at $3 and write a Dec. 90 call at $1.00

		Stock price at expiration				
		0	45	85	90	125
Buy Dec. 85 call @	3	-3	-3	-3	2	37
Write Dec. 90 call @	1	1	1	1	1	-34
Net	-2	-2	-2	-2	3	3



Why bull spreads: you expect that the stock price will go up

Bear spreads: buy a call on a stock with a certain strike price and sell a call on the same stock with a lower strike price

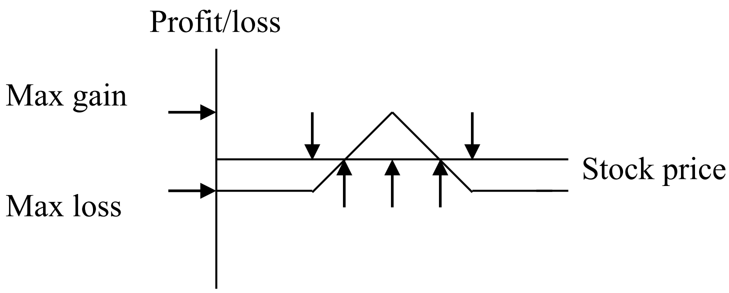
Example: write a Dec. 85 call at $3 and buy a Dec. 90 call at $1 (reverse the bull spread)

Why bear spreads: you expect that the stock price will go down

Butterfly spreads: involve four options (same type) with three different strike prices

Example: buy a Dec. 80 call at $7.00, write 2 Dec. 85 calls at $3.00, and buy a Dec. 90 call at $1.00

		Stock price at expiration					
		0	45	80	85	90	125
Buy a Dec. 80 call @	7	-7	-7	-7	-2	3	38
Write 2 Dec. 85 calls @	3	6	6	6	6	-4	-74
Buy a Dec. 90 call @	1	-1	-1	-1	-1	-1	34
Net	-2	-2	-2	-2	3	-2	-2



Why butterfly spreads

Other spreads: calendar spreads, diagonal spreads, etc

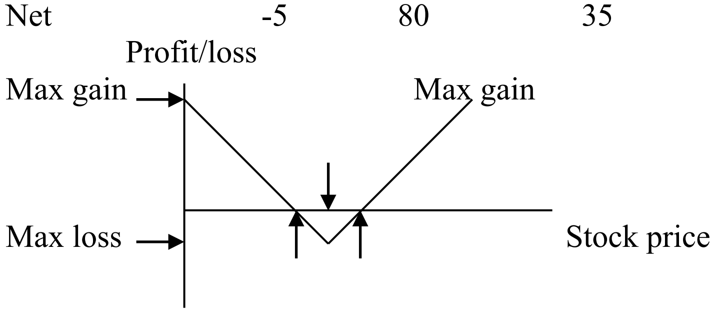
- Combinations

A combination involves a position in both calls and puts on the same stock

Straddle: involves buying a call and a put with the same strike price and expiration date

Example: long a Dec. 85 straddle by buying a Dec. call at $3.00 and a Dec. put at $2.00

		Stock price at expiration			
		0	45	85	125
Buy Dec. 85 call @	3	-3	-3	-3	37
Buy Dec. 85 put @	2	83	38	-2	-2
Net	-5	80	35	-5	35

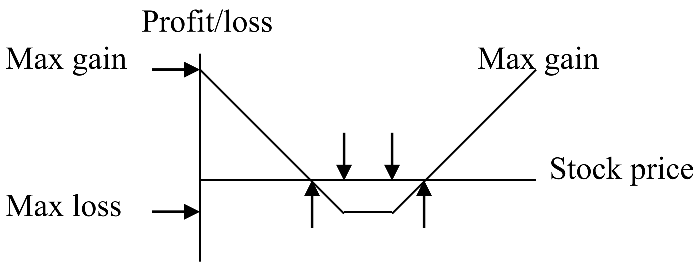


Why straddle

Strangle: involves buying a put and a call with same expiration date but different strike prices

Example: long a Dec. Strangle by buying a Dec. 90 call at $2.00 and a Dec. 85 put at $3.00

		Stock price at expiration				
		0	45	85	90	130
Buy Dec. 85 put @	3	82	37	-3	-3	-3
Buy Dec. 90 call @	2	-2	-2	-2	-2	38
Net	-5	80	35	-5	-5	35



Why strangle

Strips and straps: different numbers of calls and puts

- Assignments
  Quiz (required)
  Practice Questions: 11.10 and 11.12

Chapter 12 - Binomial Option Pricing Model

- A one-step binomial model
- Risk-neutral valuation
- Two-step binomial model
- Matching volatility with *u* and *d*
- Options on other assets

- One-step binomial model

  A numerical example: consider a European call option with 3 months to mature. The underlying stock price is \$20 and it is known that it will be either \$22 or \$18 in 3 months. The exercise price of the call is \$21. The risk-free rate is 12% per year. What should be the price of the option?

Stock price = \$22 → will be worth 22 Δ
Option price = \$1 → will be worth 21-22 = -1

Stock price = \$20
Option price = ?

Stock price = \$18 → will be worth 18 Δ
Option price = 0 → will be worth 0

Consider a portfolio: long (buy) Δ shares of the stock and short a call. We calculate the value of Δ to make the portfolio risk-free

$22\Delta - 1 = 18\Delta$

Solving for $\Delta = 0.25$ = hedge ratio (It means that you need to long 0.25 shares of the stock for one short call to construct the risk-free portfolio. Δ is positive for calls and negative for puts.)

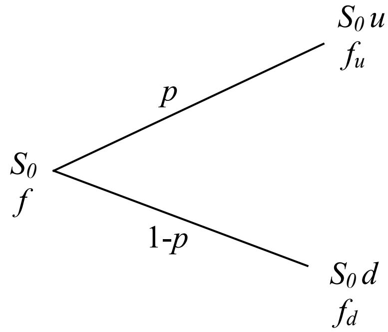
The value of the portfolio is worth \$4.5 in 3 months (22*0.25 - 1 = 18*0.25 = 4.5)

The present value of the portfolio is $4.5e^{-0.12*3/12} = 4.367$

Let *f* be the option price today. Since the stock price today is known at \$20, we have

$20*0.25 - f = 4.367$, so $f = 0.633$, the option is worth 0.633

Generalization



For a risk-free portfolio: $S_0 u \Delta - f_u = S_0 d \Delta - f_d$

$\Delta = \dfrac{f_u - f_d}{S_0 u - S_0 d} = \dfrac{1-0}{22-18} = 0.25$, hedge ratio called delta (long 0.25 shares of the stock for one short call), here $S_0 = 20$, $u = 1.1$, $d = 0.9$, $f_u = 1$, and $f_d = 0$

$f = S_0\Delta - (S_0 u\Delta - f_u)e^{-rT} = 0.633$ or

$p = \dfrac{e^{rT} - d}{u - d} = \dfrac{e^{0.12*3/12} - 0.9}{1.1 - 0.9} = 0.6523$ and $1 - p = 0.3477$

$f = e^{-rT}[p * f_u + (1\text{-}p) * f_d] = e^{-0.12*3/12}[0.6523*1+0.3477*0) = 0.633$

where f is the value of the option, S_0 is the current price of the stock, T is the time until the option expires, $S_0 u$ is a new price level if the price rises and $S_0 d$ is a new price level if the price drops (u>1 and d<1), f_u is the option payoff if the stock price rises, f_d is the option payoff if the stock price drops, and e^{-rT} is the continuous discounting factor

f: the present value of expected future payoffs

- Risk-neutral valuation

  Risk neutral: all individuals are indifferent to risk

  Risk neutral valuation: stocks' expected returns are irrelevant and investors don't require additional compensation for taking risk

  Expected payoff of the option at $T = p\, f_u + (1\text{-}p)\, f_d$
  where p is the probability that the stock price will move higher and (1-p) is the probability that the stock price will be lower in a risk-neutral world

Expected stock price at $T = E(S_T) = p(S_0 u) + (1\text{-}p)(S_0 d) = S_0 e^{rT}$
Stock price grows on average at the risk-free rate. The expected return on all securities is the risk-free rate.

Real world vs. risk-neutral world
In the about numerical example, we assume that the risk-free rate is 12% per year. We have $p = 0.6523$ and $1\text{-}p = 0.3477$. The option price is 0.633. What would happen if the expected rate of return on the stock is 16% (r^*) in the real world?

Let p^* be the probability of an up movement in stock price, the expected stock price at T must satisfy the following condition:

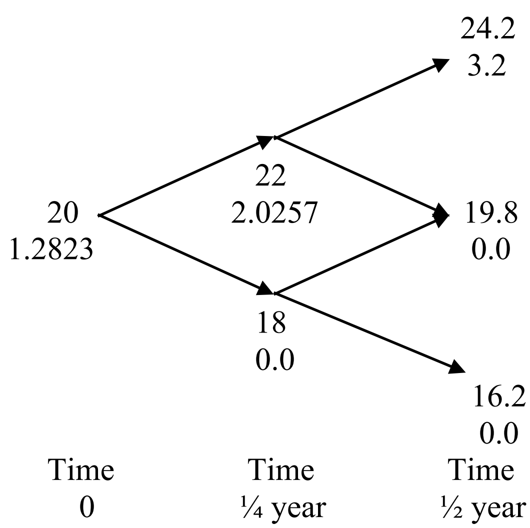
$22p^* + 18(1\text{-}p^*) = 20\ e^{0.16*3/12}$, solving for $p^* = 0.7041$ and $1\text{-}p^* = 0.2959$

$f = e^{-r^*T}\ [(p^*)*f_u + (1\text{-}p^*)*f_d] = e^{-0.16*3/12}[0.7041*1+0.2959*0] = 0.676$

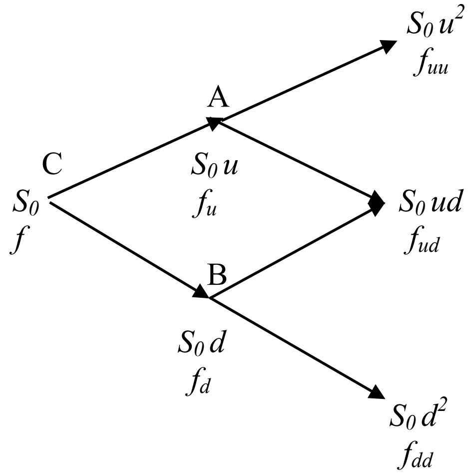
Note: in the real world, it is difficult to determine the appropriate discount rate to price options since options are riskier than stocks

- Two-step binomial model
  A numerical example: consider a European call option with 6 months to mature (two-steps with 3 months in each step). The underlying stock price is \$20 and it is known that it will be either rise or drop by 10% in each step. The exercise price of the call is \$21. The risk-free rate is 12% per year. What should be the price of the option?

  Since $u = 1.1$ and $d = 0.9$, $K = 21$, each step is ¼ year (3 months), and $r = 12\%$, we work backwards to figure out what should be the price of the option price in 3 months. We then calculate how much the option should be worth today.



Generalization



Each step is Δt years

At node A: $f_u = e^{-r\Delta t}[pf_{uu} + (1-p)f_{ud}]$

At node B: $f_d = e^{-r\Delta t}[pf_{ud} + (1-p)f_{dd}]$

At node C: $f = e^{-r\Delta t}[pf_u + (1-p)f_d]$

Since $p = \dfrac{e^{r\Delta t} - d}{u - d}$ for each step, we have

$$f = e^{-2r\Delta t}[p^2 f_{uu} + 2p(1-p)f_{ud} + (1-p)^2 f_{dd}]$$

Notes: in the two-step binomial model, Δ (delta) changes in each step

- Matching volatility with *u* and *d*

  In practice, we choose *u* and *d* to match the volatility of the underlying stock price. The expected stock price in the real world at Δt (from 0 to Δt) must satisfy

  $p^* S_0 u + (1-p^*)S_0 d = S_0 e^{\mu\Delta t}$, where μ is the expected rate of return for the stock

  Taking variance on both sides (by eliminating S_0 first), we have

  $p^* u^2 + (1-p^*)d^2 - [p^* u + (1-p^*)d]^2 = \sigma^2 \Delta t$, derived from $Var(X) = E(X^2) - [E(X)]^2$

Substituting $p^* = \dfrac{e^{\mu\Delta T} - d}{u - d}$ into the above equation gives

$$e^{\mu\Delta t}(u + d) - ud - e^{2\mu\Delta t} = \sigma^2 \Delta t$$

Ignoring Δt^2 and higher powers of Δt, one solution is

$u = e^{\sigma\sqrt{\Delta t}}$ and $d = e^{-\sigma\sqrt{\Delta t}}$ (volatility matching *u* and *d*)

For example, consider an American put option. The current stock price is \$50 and the exercise price is \$52. The risk-free rate is 5% per year and the life of the option is 2 years. There are two steps (Δt = 1 year in this case). Suppose the volatility is 20% per year. Then

$u = e^{\sigma\sqrt{\Delta t}}$ = 1.2214 and $d = e^{-\sigma\sqrt{\Delta t}}$ = 0.8187

- Options on other assets
  Binomial models can be used to price options on stocks paying a continuous dividend yield, on stock indices, on currencies, and on futures. To increase the number of steps, we use the software included in the textbook.

- Assignments
  Quiz (required)
  Practice Questions: 12.9 12.10, 12.11 and 12.12

Chapters 13 - Black-Scholes Option Pricing Model

- Lognormal property of stock prices
- Distribution of the rate of return
- Volatility
- Black-Scholes option pricing model
- Risk-neutral valuation
- Implied volatility
- Dividends
- Greek Letters
- Extensions

- Lognormal property of stock prices

  If percentage changes in a stock price in a short period of time, Δt, follow a normal distribution:

  $\frac{\Delta S}{S} \sim \phi(\mu \Delta t, \sigma^2 \Delta t)$, then between times 0 and *T*, it follows

  $\ln \frac{S_T}{S_0} \sim \phi[(\mu - \frac{\sigma^2}{2})T, \sigma^2 T]$ and

  $\ln S_T \sim \phi[(\ln S_0 + (\mu - \frac{\sigma^2}{2})T, \sigma^2 T]$

  Stock price follows a lognormal distribution

  For example, consider a stock with an initial price of $40. The expected return is 16% per year and a volatility of 20%. The probability distribution of the stock price in 6 months ($T = 0.5$) is

  $$\ln S_T \sim \phi[\ln 40 + (0.16 - \frac{0.2^2}{2})0.5, 0.2^2 0.5] = \phi(3.759, 0.02)$$

  The 95% confidence interval (2 σ rule) is (3.759 - 1.96*0.141, 3.759 + 1.96*0.141), where 0.141 is the standard deviation ($\sqrt{0.02} = 0.141$).

  Thus, there is a 95% probability that the stock price in 6 months will be (32.55, 56.56)

  $32.55 = e^{3.759-1.96*0.141} < S_T < e^{3.759+1.96*0.141} = 56.56$

  The mean of S_T = 43.33 and the variance of S_T = 37.93 (using formula 13.3)

- Distribution of the rate of return

  If a stock price follows a lognormal distribution, then the stock return follows a normal distribution.

  Let R be the continuous compounded rate of return per year realized between times 0 and T, then

  $S_T = S_0 e^{RT}$ or $R = \frac{1}{T}\ln\frac{S_T}{S_0}$. Therefore, $R = \frac{1}{T}\ln\frac{S_T}{S_0} \sim \phi(\mu - \frac{\sigma^2}{2}, \frac{\sigma^2}{T})$

  For example, consider a stock with an expected return of 17% per year and a volatility of 20% per year. The probability distribution for the average rate of return (continuously compounding) over 3 years is normally distributed

  $R \sim \phi(0.17 - \frac{0.2^2}{2}, \frac{0.2^2}{3})$ or $R \sim \phi(0.15, 0.0133)$

  i.e., the mean is 15% per year over 3 years and the standard deviation is 11.55% ($\sqrt{0.0133} = 0.1155$)

- Volatility

  Stocks typically have volatilities (standard deviation) between 15% and 50% per year. In a small interval, Δt, $\sigma^2 \Delta t$ is approximately equal to the variance of the percentage change in the stock price. Therefore, $\sigma\sqrt{\Delta t}$ is the standard deviation of the percentage change in the stock price.

  For example, if $\sigma = 30\% = 0.3$ then the standard deviation of the percentage change in the stock price in 1 week is a approximately $30 * \sqrt{1/52} = 4.16\%$

  Estimating volatility from historical data

  (1) Collect price data, S_i (daily, weekly, monthly, etc.) over time period τ (in years)

  (2) Obtain returns $\mu_i = \ln(\frac{S_i}{S_{i-1}})$

  (3) Estimate standard deviation of μ_i, which is s

  (4) The estimated standard deviation in τ years is $\hat{\sigma} = \frac{s}{\sqrt{\tau}}$

- Black-Scholes option pricing model

  Assumptions:

  (1) Stock price follows a lognormal distribution with u and σ constant
  (2) Short selling with full use of proceeds is allowed
  (3) No transaction costs or taxes
  (4) All securities are divisible
  (5) No dividends
  (6) No arbitrage opportunities
  (7) Continuous trading
  (8) Constant risk-free rate, r

  The price of a European call option on a non-dividend paying stock at time 0 and with maturity T is

  $$c = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

  and the price of a European put option on a non-dividend paying stock at time 0 and with maturity T is

  $$p = Ke^{-rT} N(-d_2) - S_0 N(-d_1)$$

  where $d_1 = \dfrac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$ and $d_2 = \dfrac{\ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$

  $N(x)$ is the cumulative distribution function for a standardized normal distribution

  Cumulative normal distribution function: a polynomial approximation gives six-decimal-place accuracy (refer to Tables on pages 590-591)

  For example, if $S_0 = 42$, $K = 40$, $r = 0.1 = 10\%$ per year, $T = 0.5$ (6 months), and $\sigma = 0.2 = 20\%$ per year, then

  $d_1 = 0.7693$; $d_2 = 0.6278$; $Ke^{-rT} = 40e^{-0.1*0.5} = 38.049$

  $N(d_1) = N(0.7693) = 0.7791$, $N(d_2) = N(0.6278) = 0.7349$

  $N(-d_1) = N(-0.7693) = 0.2209$, $N(-d_2) = N(-0.6278) = 0.2651$

  $c = 42*N(0.7693) - 38.049*N(0.6278) = 42*(0.7791) - 38.049*(0.7349) = 4.76$

  $p = 38.049*N(-0.6278) - 42*N(-0.7693) = 38.049*(0.2651) - 42*(0.2209) = 0.81$

- Risk-neutral valuation

  The Black-Scholes option pricing model doesn't contain the expected return of the stock, μ, which should be higher for investors with higher risk aversion. It seems to work in a risk-neutral world. Actually, the model works in all worlds. When we move from a risk-neutral world to a risk-averse world, two things happen simultaneously: the expected growth rate in the stock price changes and the discount rate changes. It happens that these two changes always offset each other exactly

- Implied volatility

  In the Black-Scholes option pricing model, only σ is not directly observable. One way is to estimate it using the historical data. In practice, traders usually work with what are called implied volatilities. These are the volatilities implied by option prices observed in the market.

- Dividends

  How to adjust for dividends?

  Since dividends lower the stock price we first calculate the present value of dividends during the life of the option, D, and then subtract it from the current stock price, S_0, to obtain the adjusted price, $S_0^* = S_0 - D$. We use the adjusted price, S_0^*, in the Black-Scholes option pricing model.

- Greek Letters (refer to Chapter 17, optional)

  Delta: the first order partial derivative of an option price with respect to the current underlying stock price

  Delta ($\frac{\partial f}{\partial S} = N(d_1) > 0$ for a call and $\frac{\partial f}{\partial S} = -N(-d_1) = N(d_1) - 1 < 0$ for a put)

  (1) Option sensitivity: how sensitive the option price is with respect to the underlying stock price

  (2) Hedge ratio: how many long shares of stock needed for short a call

  (3) Likelihood of becoming in-the-money: the probability that the option will be in-the-money at expiration

  Gamma ($\frac{\partial^2 f}{\partial S^2}$): the second order partial derivative of an option price with respect to the current underlying stock price (how often the portfolio needs to be rebalanced)

Theta ($\frac{\partial f}{\partial t} < 0$ for American options): the first order partial derivative of an option price with respect to the passage of time (time left to maturity is getting shorter, time decay)

Vega ($\frac{\partial f}{\partial \sigma} > 0$): the first order partial derivative of an option price with respect to the volatility of the underlying stock

Rho ($\frac{\partial f}{\partial r} > 0$ for a call and $\frac{\partial f}{\partial r} < 0$ for a put): the first order partial derivative of an option premium with respect to the risk-free interest rate

Variables	European call	European put	American call	American put
Stock price	+	-	+	-
Strike price	-	+	-	+
Time to expiration	n/a	n/a	+	+
Volatility	+	+	+	+
Risk-free rate	+	-	+	-
Dividends	-	+	-	+

- Extensions
    Options on stock indexes and currencies - Chapter 15
    Options on futures - Chapter 16
    Interest rate options - Chapter 21

- Assignments
    Quiz (required)
    Practice Questions: 13.9 and 13.14