Problem Set 12
Quantum Field Theory and Many Body Physics (SS2016)

Due: July 14, 2016 at the beginning of the lecture

Anf finally some simple considerations of Feynman diagrams for the thermodynamic potential.

Problem 1 Second order Feynman diagrams (15 + 10 points)

Consider the Hamiltonian of interacting fermions

$$H = \sum_{\alpha} (\epsilon_\alpha - \mu) \psi^\dagger_\alpha \psi_\alpha + \frac{1}{2} \sum_{\alpha\beta\gamma\delta} v_{\alpha\beta\gamma\delta} \psi^\dagger_\alpha \psi^\dagger_\beta \psi_\delta \psi_\gamma \tag{1}$$

where α labels all the orbital and spin states.

(a) Draw all the unlabeled Feynman diagrams obtained to second order in the perturbative expansion of the thermodynamic potential. Give the corresponding symmetry factor for each diagram.

(b) For each diagram write the corresponding analytical expression in terms of exact eigenstates of the non interacting Hamiltonian (including the symmetry factor).

Problem 2: **Spinless fermions with short-range interactions** (15 + 10 points)

Consider a system of spinless fermions with two-particle interaction $V(\mathbf{x};\mathbf{y}) = v_0 \delta(\mathbf{x} - \mathbf{y})$.

(a) Draw and calculate the Feynman diagrams for the thermodynamic potential obtained to first order in the interaction.

(b) Sum the results in (a) to obtain the total first order correction to the thermodynamic potential. Can you give a physical explanation for your result?