

Week 4: Lecture

One of the key components to research designs in the behavioral sciences is randomization. Randomization occurs in three parts.

- (1) Randomly select participants or units from the population
- (2) Randomly assign selected participants to groups
- (3) Randomly assign the groups to treatment.

Using the participant pool does not satisfy the requirements of randomization. Participants chosen this way do not satisfy the first condition. They are not randomly selected from the population. However, the remaining two are usually doable.

The Completing Randomized Design abbreviated CR-n by Kirk involves only one independent variable. Remember an independent variable must have a minimum of two levels. CR designs are generally analyzed using the One-way analysis of variance. There are several variations of the one-way analysis of variance (ANOVA). The completely randomized design is one of those. This simple design forms the basis for the more complex designs we will consider later.

The simple completely randomized design is one where each treatment is independently administered to a different group of participants. All groups were originally drawn at random from the same parent population. After treatments have been administered, these groups may be regarded as random samples from a single population only if the treatments all have identical effects on the dependent variable. Otherwise the group receiving treatment A may be regarded as a random sample from an imaginary or hypothetical population which is like the original parent population except all participants have received treatment A. Groups receiving other treatment conditions would be considered in a similar fashion. These hypothetical populations are generally referred to as "treatment populations." The hypothesis that we wish to test is the mean of the dependent variable for all treatment populations are identical. This hypothesis is called "the overall null hypothesis" because it is concerned with all of the treatments instead of any specific hypothesis.

Assumptions

In the CR design, there are two (2) assumptions made about the dependent variable.

- (1) the dependent variable is normally distributed in each treatment population
- (2) the variance of the dependent variable in the treatment populations are equal.

More specifically,

Given the model: $y_{ij} = \mu + \alpha_i + e_{ij}$ for the completely randomized ANOVA, we assume

- (1) e_{ij} is normally distributed for each i and j
- (2) $E(e_{ij}) = 0$ for each i and j .
- (3) $V(e_{ij}) = \sigma^2$ for each i and j
- (4) The treatment effect, α_i is a fixed effect.
- (5) The error terms e_{ij} are independent random variables

The first three assumptions concerning the random variable, e_{ij} will assure us that y_{ij} is normally distributed and that homoscedasticity (equal variances) assumption is satisfied. To treat α_i as a fixed effect implies that we are only concerned with the treatment effects in the experiment. Inferences drawn from the analysis of variance procedure will be made about these treatment effects only. Alternatively, we can consider the treatments as representing a sample taken from a larger population of treatments, in which case, the α_i 's are referred to as random effects. If this is the case, then an additional assumption need to be

made. Each treatment effect would have to be normally distributed and be in itself a independently distributed random variable.

Assumption (5) can usually be satisfied if participants are randomly assigned to treatment groups. If there is no treatment effect, then the model reduces to:

$$y_{ij} = \mu_i + e_{ij}$$

To determine if assumption (3) is tenable, we can conduct the Fmax test. The formula for the Fmax test is

$$F_{\max} = \frac{\hat{\sigma}_{largest}^2}{\hat{\sigma}_{smallest}^2} \text{ with } k \text{ and } n-1 \text{ degrees of freedom.}$$

Here k = the number of groups or levels of the IV (also the number of treatment groups) and n is the sample size of each group (assumes they are all equal). If F_{\max} is larger than the critical value found in a F-value table, we have evidence that homoscedasticity is not tenable. $\hat{\sigma}_{largest}^2$ would be the treatment group with the largest variance when compared to the others and $\hat{\sigma}_{smallest}^2$ is the treatment group with the smallest group variance.

The null hypothesis for the CR-ANOVA is

$$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \dots = \mu_k$$

the alternative or research hypothesis, H_1 , would be that at least one treatment condition is different from the others.

In our previous discussion about the Central Limit Theorem, we established that the sampling distribution of the sample means is approximately normal. In addition, the mean of the sample means is equal to the population mean. Also for very large populations and large sample sizes, the standard deviation of the sampling distribution (also called the standard error of the mean) is equal to:

$$\frac{\sigma_X^2}{n} = \sigma_{\bar{X}}^2$$

If the null hypothesis is true then the population variance, σ_X^2 , should be equal to n times the variance of the sample means, $\sigma_{\bar{X}}^2$. If the null hypothesis is not true, then $n\sigma_{\bar{X}}^2$ will be greater than σ_X^2 .

The null and alternative hypotheses can be restated as:

$$H_0: \sigma_X^2 = n\sigma_{\bar{X}}^2 \text{ and } H_1: \sigma_X^2 < n\sigma_{\bar{X}}^2.$$

The analysis of variance procedure does analyze variances in comparing means.

The two variances can be compared using the following formula:

$$f = \frac{n\hat{\sigma}_{\bar{X}}^2}{\hat{\sigma}_X^2} = \frac{\text{Mean Square for Treatments}}{\text{Mean Square for Error}}$$

where $df_1 = k - 1$ and $df_2 = k(n - 1)$.

The Analysis of Variance Table

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F-Ratio
Between Treatments	$k - 1$	SS (Treatment)	$MST = SS \text{ Treatment} / (k - 1)$	$F = MST/MSE$
Within Treatments	$k(n - 1)$	SS (Error)	$MSE = SS \text{ Error} / (k(n - 1))$	
Total	$kn - 1$	SS Total		

$$\sum_{i=1}^k \sum_{j=1}^n (x_{ij} - \bar{x}_{..})^2 = n \sum_{i=1}^k (\bar{x}_i - \bar{x}_{..})^2 + \sum_{i=1}^k \sum_{j=1}^n (x_{ij} - \bar{x}_i)^2$$

Total Sum of Squares

Sum of Squares Due to Treatments

Error Sum of Squares

$$\sum_{i=1}^k \sum_{j=1}^n (x_{ij} - M)^2 = n \sum_{i=1}^k (M_i - M)^2 + \sum_{i=1}^k \sum_{j=1}^n (x_{ij} - M_i)^2$$

Computationally, these formulas would not be used. An easier set of formulas would be:

$$\text{Total Sums of Squares} = \sum_{i=1}^k \sum_{j=1}^n x_{ij}^2 - \frac{\left(\sum_{i=1}^k \sum_{j=1}^n x_{ij} \right)^2}{kn} = [BS] - [X]$$

$$\text{Correction for the Mean} = \frac{\left(\sum_{i=1}^k \sum_{j=1}^n x_{ij} \right)^2}{kn} = \frac{\left(\sum_{i=1}^k \sum_{j=1}^n x_{ij} \right)^2}{N} = [X]$$

$$\text{Between Treatment Groups Sum of Squares} = \sum_{i=1}^k \frac{\left(\sum_{j=1}^n x_{ij} \right)^2}{n} - \frac{\left(\sum_{i=1}^k \sum_{j=1}^n x_{ij} \right)^2}{kn} = [B] - [X]$$

$$\text{Within Groups Sum of Squares} = \sum_{i=1}^k \sum_{j=1}^n x_{ij}^2 - \sum_{i=1}^k \frac{\left(\sum_{j=1}^n x_{ij} \right)^2}{n} = [BS] - [B]$$

Some simplified computational formulas would be:

$$\text{Total SS} = SD^2(N) \quad \text{Between SS} = \sum \frac{T_i^2}{n_i} - M^2(N)$$

$$\text{Within SS} = \text{Total SS} - \text{Between SS} = N (SD^2 + M^2) - \sum \frac{T_i^2}{n_i}$$

The hypothesis test can be influenced by sample size. The larger the sample, the greater the chance that small differences become statistically significant (i.e. reject H_0). To determine whether or not a real effect has taken place, we can compute one of many indices. Each one of these indices is estimated without directly considering sample size. Also these indices can be helpful in those cases where the null hypothesis was not rejected. When H_0 is not rejected and there is a real effect for the IV, this is due to a sample that is too small. The computation of these indices that measure strength of association between the IV and DV can tell a researcher that the non significant hypothesis test contained a real effect. The remedy is to collect more data for the study.

The three we will consider are (1) eta-squared η^2 , $\hat{\rho}I$, and ω^2 .

$$\eta^2 = \frac{SS \text{ between}}{SS \text{ Total}}$$

$$\hat{\rho}I = \frac{MS \text{ Between} - MS \text{ Within}}{MS \text{ Between} + (n - 1) MS \text{ Within}}$$

$$\omega^2 = \frac{SS \text{ Between} - (k - 1) MS \text{ Within}}{SS \text{ Total} + MS \text{ Within}}$$

$\hat{\rho}I$ is used if we are using a random-effects model and ω^2 is used if we have a fixed effects model. The simplest and easiest to use is η^2 . All of these measure the proportion of variance in the dependent variable, Y accounted for by variations in the independent variable, X.

Week 4 Lab

Continue with SPSS exercise. Last week, we saw how to read in the data stored in Excel format on our floppy disk. Assignment 1 calls for the creation of 6 groups. Note that Grade = 1 for letter grade, Grade = 2 for Mastery Grade. For Feedback: Feedback = 1 for no feedback, Feedback = 2 for minimum feedback and Feedback = 3 for maximum feedback. A combination of these levels will give us our 6 groups.

Letter Grade with No feedback is Group 1
 Letter Grade with Minimum Feedback is Group 2
 Letter Grade with Maximum Feedback is Group 3
 Mastery Grade with No Feedback is Group 4
 Mastery Grade with Minimum Feedback is Group 5
 Mastery Grade with Maximum Feedback is Group 6

In logical statements this would look like:

If Grade = 1 and Feedback = 1 then Grade = 1
 If Grade = 1 and Feedback = 2 then Grade = 2
 If Grade = 1 and Feedback = 3 then Grade = 3
 If Grade = 2 and Feedback = 1 then Grade = 4
 If Grade = 2 and Feedback = 2 then Grade = 5
 If Grade = 2 and Feedback = 3 then Grade = 6

To do this SPSS, we need to use the “Transform” function. When we click on “Transform” on the top bar, we get a menu. Select from the menu “Compute”.

For the first logical statement, follow these steps:

1. In the first space “Target Variable” enter in “group”
2. In the space under “Numeric Expression” enter in “1”.
3. Click on the “IF..” button. In the subsequent screen
4. Choose “Include if case satisfies condition”
5. In the space type in “grade = 1 and feedback = 1
6. click on “continue” button
7. click on “OK” button.

This will create a new variable named “group” and place the number 1 in the space where the condition is satisfied.

Repeat these steps for each logical statement. For the second one, the target variable is still “group” but the numeric expression is now “2”. In the window following clicking on the “IF..” button, the condition will be “grade = 1 and feedback =2”.

If the program asks you if you want to “Change existing variable?” choose the “Yes” button.

Under the “group” variable, we should see a block of “1” followed by blocks of 4, 2, 5, 3 and 6. This will repeat until all 72 participants have been assigned to one of 6 groups.

We are now ready to do the one way anova. The steps are as follows:

1. Select “Statistics”
2. In the menu choose “Compare Means”
3. This gives another menu, from it pick “One-Way ANOVA”
4. Move the variable “Score2” from the left side list into the “dependent list”
5. Move the variable “group” from the left side list to the “factor” box.
6. Click on “Options” and choose “Descriptive Statistics” and “Homogeneity of Variance.”
7. Click on “continue”
8. Click on “Post Hoc” and from the following window pick Scheffe, S-N-K and Tukey tests, click on “continue”
9. Clicking on the “OK” button within the “One-Way ANOVA” window will start the analyses.

The output and the detail will be discussed in subsequent lectures and labs.