

## Which statistical Tests Should be used

One Independent Variable (Two-Levels)	
Related (Correlated)	Unrelated (Uncorrelated)
Paired difference t-test (parametric) Wilcoxon Rank Sum Test (non-parametric)	Independent Samples t-test (Parametric) Mann-Whitney U-Test (non-parametric)

One Independent Variable (More Than Two-Levels)	
Related (Correlated)	Unrelated (Uncorrelated)
Randomized Block Design ANOVA Repeated Measures ANOVA Friedman Test	Completely Randomized Design ANOVA Kruskal-Wallis H-Test

Two Independent Variable (Two-Levels) Factorial Design		
Related (Correlated)	Unrelated (Uncorrelated)	Mixed
Within Subjects ANOVA Ss used in ALL Treatment Conditions (Both IVs Active)	Between Subjects ANOVA Each S gets only one treatment combination. IVs can be active or Attribute	Split-Plot ANOVA Each S gets one within Subjects Treatment and One Between Subjects Treatment.

Examples for each are given below.

### Steps in Hypothesis Testing.

1. Statement of the null hypothesis:  $H_0$ .
2. Statement of the alternative hypothesis:  $H_1$ .
3. Test statistic computed from empirical data
4. Creation of the decision rule and the decision made from applying the rule.
5. Conclusion.

#### One Sample t-test

1. Compute the mean and standard deviation of the sample,  $M$ ,  $S$
2. Establish the value of  $\mu_0$ , the hypothesized true value of the mean to be tested. This is from the research problem itself.
3. Calculate the standard error of the mean,  $s_M$ , where  $s_M = \frac{S}{\sqrt{n}}$
4. Compute  $t = \frac{M - \mu_0}{s_M}$
5. With  $n - 1$  degrees of freedom, where  $n$  is the sample size, determine the critical value from the table of the t distribution
6. Decision Rule: Reject  $H_0$  if  $|t| >$  critical value otherwise do not reject  $H_0$ .
7. Relate the decision back to the original research question/statement.

Example: A manufacturer of TV sets claimed that a particular model has an average defect-free life of 3 years. Five households that purchased that set have observed a failure before 3 years. The failure times were 1.9, 2.5, 2.9, 2.7 and 2.4 years, respectively. Conduct the appropriate hypothesis test to determine if the data contradicts the manufacturer's claim. Use  $\alpha = .05$ .

$$M = 2.480 \quad S = 0.377 \quad s_M = \frac{S}{\sqrt{n}} = \frac{0.377}{\sqrt{5}} = \frac{0.377}{2.236} = .169$$

$H_0: \mu = 3 \text{ years}$      $H_1: \mu < 3 \text{ years}$

$$t = \frac{2.480 - 3.00}{.169} = \frac{-0.520}{.169} = -3.077$$

DECISION RULE: Reject  $H_0$  if  $|t| >$  critical value, otherwise do not reject  $H_0$ .

To find Critical value we need  $H_1$ ,  $\alpha$ , and df (degrees of freedom),  $df = n - 1 = 5 - 1 = 4$ .  $\alpha = .05$  and  $H_1$  tells us it is a one-tailed test. Now go to the table for the t distribution to find the critical value.

**Table 9.A. t-distribution values**

df	Probability					two-tailed one-tailed
	0.50	0.10	0.05	0.02	0.01	
	0.25	<b>0.05</b>	0.025	0.01	0.005	
1	1.00	6.34	12.71	31.82	63.66	
2	.816	2.92	4.30	6.96	9.92	
3	.765	2.35	3.18	4.54	5.84	
<b>4</b>	.741	<b>2.13</b>	2.78	3.75	4.60	
5	.727	2.02	2.57	3.36	4.03	
6	.718	1.94	2.45	3.14	3.71	
7	.711	1.90	2.36	3.00	3.50	
8	.706	1.86	2.31	2.90	3.36	
9	.703	1.83	2.26	2.82	3.25	

The critical value is 2.13. Since  $|t| = 3.077$  is greater than 2.13, reject  $H_0$ .

CONCLUSION: There is evidence that the failure rate of the TV set is less than 3 years.

For the two samples t-test, there are two variations. One is used when the samples are correlated. The other is for uncorrelated samples.

To determine whether two samples are correlated or uncorrelated, the researcher needs to ask how the data were collected. This can be done using two questions:

1. Are the participants measured twice (more than once) on the same variable or measure? (Repeated Measures). If the answer is "yes", then the samples are correlated. Each person has a data point in each group or sample. If the answer is "no", proceed to question number 2. A "no" answer tells us that each person was measured only once.
2. Are the participants in the two samples or groups matched or paired in any meaningful way? If the answer is "yes" then the samples (groups) are correlated (paired). If the answer is "no." then the samples (groups) are uncorrelated.

Note that SPSS calls correlated samples "paired." Some textbooks call the samples "dependent." Also SPSS calls uncorrelated samples as "independent" samples.

The null and alternative hypotheses cannot tell you whether the samples are correlated or uncorrelated. You can only determine this by asking how the data for the two samples were collected.

#### Two sample (correlated samples) t-test

1. Form the difference d, for each pair.
2. Compute the mean of the difference,  $M_d$ .

3. Calculate the standard error,  $s_{M_d} = \frac{S_d}{\sqrt{n_{pairs}}}$

4. Compute  $t = \frac{M_d}{s_{M_d}}$

- With  $n_{\text{pairs}} - 1$  degrees of freedom, determine the critical value from the table of the  $t$  distribution
- Decision Rule: Reject  $H_0$  if  $|t| >$  critical value otherwise do not reject  $H_0$ .
- Relate the decision back to the original research question/statement.

Example: A company wanted to determine whether to buy for all employees a MAC or PC computer. Since work production was very important as well as cost, only 6 employees were chosen for this test. Three employees were randomly chosen to test the MAC first and then test the PC. The other 3 employees were chosen to receive the PC first followed by the MAC. (This is called counterbalancing and is used to eliminate order effects). After testing both computers, each employee responded to a 7 point rating scale where "1" = worst to "7" = best. Using the data below, conduct the appropriate hypothesis test to determine which computer is preferred. Use  $\alpha = .05$ .

Person	1	2	3	4	5	6	$M_d = 2.000$
MAC	7	6	2	5	6	5	$S_d = 2.828$
PC	2	2	5	4	3	3	$H_0: \mu_{\text{Mac}} = \mu_{\text{pc}}$
$d = \text{MAC} - \text{PC}$	5	4	-3	1	3	2	$H_1: \mu_{\text{Mac}} \neq \mu_{\text{pc}}$

$$s_{Md} = \frac{S_d}{\sqrt{n_{\text{pairs}}}} = \frac{2.828}{\sqrt{6}} = \frac{2.828}{2.449} = 1.171 \quad t = \frac{M_d}{s_{Md}} = \frac{2.000}{1.171} = 1.708$$

Decision Rule: Reject  $H_0$  if  $|t| >$  critical value otherwise do not reject  $H_0$ .

Critical value:  $H_1$  tells us it is a 2-tailed test,  $\alpha = .05$ , and  $df = n_{\text{pairs}} - 1 = 6 - 1 = 5$ . From the table the critical value is 2.57. Since  $|t| = 1.708$  is less than 2.57, do not reject  $H_0$ .

CONCLUSION: There is insufficient evidence that the preference ratings for the two computers: MAC and PC are different.

#### Two sample (uncorrelated samples) t-test

- Find the mean and standard deviation for sample 1,  $M_1$  and  $S_1$ .
- Find the mean and standard deviation for sample 2,  $M_2$  and  $S_2$ .

$$3. \text{ Computer the standard error: } S_{DM} = \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

$$4. \text{ Compute } t = \frac{M_1 - M_2}{S_{DM}}$$

- With  $n_1 + n_2 - 2$  degrees of freedom, determine the critical value of from the table of the  $t$  distribution
- Decision Rule: Reject  $H_0$  if  $|t| >$  critical value otherwise do not reject  $H_0$ .
- Relate the decision back to the original research question/statement.

Example: An educational psychologist wanted to determine if 7-year old girls do better than 7-year old boys on a visual perception task. Five girls and 5 boys were randomly selected from the entire set of 7-year olds in a school. Each were then administered a visual perception test. Using the data below, develop the appropriate hypothesis test to determine if the psychologist's hypothesis was correct. Higher scores indicate better visual perception. Use  $\alpha = 0.01$

Girls	17	16	21	15	16	$M_G = 17.000$	$S_G = 2.345$	$H_0: \mu_G = \mu_B$
Boys	12	12	15	14	13	$M_B = 13.200$	$S_B = 1.304$	$H_1: \mu_G > \mu_B$

$$S_{DM} = \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} = \sqrt{\frac{2.345^2}{5} + \frac{1.304^2}{5}} = \sqrt{\frac{5.50}{5} + \frac{1.70}{5}} = \sqrt{1.440} = 1.20$$

$$t = \frac{M_1 - M_2}{S_{DM}} = \frac{17.00 - 13.20}{1.20} = \frac{3.80}{1.20} = 3.167$$

Decision Rule: Reject  $H_0$  if  $|t| >$  critical value otherwise do not reject  $H_0$ .

Critical value:  $H_1$  tells us it is a 1-tailed test,  $\alpha = .01$ , and  $df = n_1 + n_2 - 2 = 5 + 5 - 2 = 8$ . From the table the critical value is 2.90. Since  $|t| = 3.167$  is greater than 2.90, reject  $H_0$ .

CONCLUSION: There is evidence that 7-year old girls have better visual perception than 7-year old boys.

### Important Notes:

Hypothesis testing can be very complex to some students. However, if each student follows the steps involved, she or he can easily do a hypothesis test without a full understanding of hypothesis testing and inferential statistics. It is the hope of this instructor that a functional student will eventually learn and understand hypothesis testing at a later time. It could be as early as the next hour or as late as several decades.

Apply the two questions to the correlated and uncorrelated sample problems. See if you come up with the conclusion that one is correlated samples and the other is uncorrelated samples.

Two-tailed tests are non-directional. Words such as "different" or "difference" with no other specification indicate a two-tailed test. One-tailed tests usually have words such as "greater than," "less than." "Better," "worse," "improve," and such words to indicate a direction.

Use "\*" in the alternative,  $H_1$  hypothesis for two-tailed tests. Use either ">" or "<" in the alternative hypothesis for a one-tailed test. The null hypothesis,  $H_0$ , always (in this class) contains the equal "=" sign.

Also, in this class avoid using words such as "prove" or "proof." Instead substitute the words "show" or "demonstrate." "Prove" is much too strong of a word to use in hypotheses tests in psychology and the behavioral sciences.

The decision rule for this class on each parametric hypothesis test is written exactly the same way. The only difference is what symbol is used for the test statistic.

In writing the conclusion, if  $H_0$  was rejected start the statement with "There is evidence...." If  $H_0$  was not rejected then start the conclusion statement with "There is insufficient evidence....."

## Psychology 320: Practice Problems for ANOVA

1. Four groups of fifth graders are randomly selected and assigned to a different educational program in computer literacy. At the end of the educational program, each student is examined on a standardized examination. Twenty-eight students participated, but only 21 finished the course. The data are given below.

Educ. Program	Score on exam					
A	85	80	62	45	81	70
B	73	72	55	76	69	71
C	86	83	40	90	62	
D	66	79	65	80		

Do an analysis of variance of these data using  $\alpha = 0.05$  to determine if there are any differences between the different educational programs.

2. A consumer research company is hired to see if three additives used in automobile gasoline improves mileage. Twelve cars of the same make, year, and model are used in the study. The data after three tanks of gasoline are given below.

Additive	Average miles per gallon			
S	23	25	26	22
V	20	19	21	17
T	21	24	18	19

Do an analysis of variance at  $\alpha = 0.05$  to determine if there is any difference in average miles per gallon between the different additives.

3. Referring to question 2, do all pairwise comparisons of means using the Scheffé test ( $\alpha = .05$ ).

4. Do all pairwise comparisons using the Tukey HSD test for the data in question 2.

5. In a study to determine whether highly active individuals are more optimistic than less active individuals, 20 college students were administered an optimism-pessimism scale in addition to being asked how often they exercise. The optimism scores for five highly active, eight moderately active and seven inactive participants are shown. Using the data below, conduct the appropriate hypothesis test to determine if a significant difference exists between the three groups.

Inactive		Moderate		Highly Active
58	60	59	49	62
48	46	54	59	48
53		50	62	63
56		46		47
50		61		55

6. A consumer psychologist wants to determine the effect of label information of the perceived quality of wine. The study has 3 conditions. In each condition the participant tastes the wine and then rates the wine on a 1 to 20 scale where 20 has the highest perceived quality. In all 3 conditions the wine is identical. The only difference is the label on the bottle.. One label indicates it is a French wine, another says it is an Italian wine and the third says it is an American wine. Six participants are used and each participant sample the wine in all 3 conditions. The only difference from one participant to the other is that the order of the wine is randomly presented. The data are given below. Conduct the appropriate hypothesis test to determine if label differences influence the perceived quality of the wine.

Participant	French	Italian	American
1	14	10	9
2	16	12	12
3	17	13	14
4	16	14	16
5	15	12	19
6	12	11	8

7. A researcher in eating disorders studies the amount of effort expended by obese and average-weight people to obtain food for a snack. Each participant in the study is told to fill out an attitude questionnaire. At the table where the participant sits is a bowl of peanuts. Half of the participants, the bowl contained shelled peanuts. The other half got peanuts that are still in the shell. The researcher randomly selects 20 obese and 20 average-weight participants from a pool of volunteers and records the number of peanuts each person eats while filling out the questionnaire. The data are given below. Conduct the appropriate hypothesis test to determine (1) if a significant difference exists between obese and non-obese people, (2) if a significant difference exists in preference for the type of peanuts and (3) if an interaction exists between people and peanut-type.

Obese		Average	
Shelled	Unshelled	Shelled	Unshelled
16, 17, 12, 14, 13, 10, 9, 17, 16, 16	4, 2, 2, 0, 1, 3, 2, 1, 1, 4	10, 5, 9, 10, 9, 7, 6, 8, 6, 11	7, 9, 6, 4, 3, 8, 7, 10, 4, 2

8. A consumer psychologist wanted to determine what type of advertising works best in people's perception of quality. One variable was the mode of presentation: visual or audio and the second variable was media style: common versus unusual. Ten participants were recruited and all 10 people were exposed to all four-treatment combinations. The only difference between individuals was the order the treatment combinations were presented. The data are given below. Conduct the appropriate hypothesis test to determine of presentation mode, media style and their interaction were statistically significant.

Ss	A <sub>1</sub> : Visual		A <sub>2</sub> : Audio	
	B <sub>1</sub> : Common	B <sub>2</sub> : Unusual	B <sub>1</sub> : Common	B <sub>2</sub> : Unusual
1	6	4	3	6
2	7	3	2	3
3	5	2	5	8
4	8	5	4	5
5	3	6	2	5
6	6	4	7	7
7	8	5	3	6
8	9	2	4	6
9	3	4	3	4
10	6	2	3	5

9. A researcher wanted to know if females and males differ on their preference for two different webpage layouts. To do this, the researcher randomly selected 5 females and 5 males for the study. Each participant was shown both layouts and asked to indicate a preference using a rating scale from 1 to 10, where 10 = the most desirable. Each participant was shown the two layouts in random order. The data are given below. Conduct the appropriate hypothesis tests to determine if there is a gender difference, layout difference and an interaction between gender and layouts.

Participant	Gender	Layout 1	Layout 2
1	F	6	4
2	F	5	2
3	F	9	6
4	F	10	4
5	F	8	4
6	M	9	3
7	M	7	2
8	M	8	1
9	M	10	4
10	M	7	2

**Solution:**

1.  $H_0: \mu_A = \mu_B = \mu_C = \mu_D$        $H_1: \mu_A \neq \mu_B \quad \mu_A \neq \mu_C \quad \mu_A \neq \mu_D \quad \mu_B \neq \mu_C \quad \mu_B \neq \mu_D \quad \mu_C \neq \mu_D$   
At least one is true.

$$M = 70.952 \quad M^2 = 5034.186 \quad M^2(N) = 105717.906 \quad S = 13.044 \quad S^2 = 170.146$$

$$SS_{TOT} = 170.146(21-1) = 3402.920 \quad T_A = 423 \quad T_B = 416 \quad T_C = 361 \quad T_D = 290$$

$$n_A = n_B = 6 \quad n_C = 5 \quad n_D = 4$$

$$SSB = \left[ \frac{T_A^2}{n_A} + \frac{T_B^2}{n_B} + \frac{T_C^2}{n_C} + \frac{T_D^2}{n_D} \right] - M^2(N) = \left[ \frac{423^2}{6} + \frac{416^2}{6} + \frac{361^2}{5} + \frac{290^2}{4} \right] - 105717.906$$

$$= [29821.500 + 28842.667 + 26064.200 + 21025.000] - 105717.906$$

$$= 105753.367 - 105717.906 = 35.461$$

$$SSW = 3402.920 - 35.461 = 3438.381$$

$$Df_B = 4 - 1 = 3 \quad df_W = 21 - 4 = 17 \quad df_{TOT} = 21 - 1 = 20$$

$$MSB = \frac{35.461}{3} = 11.820 \quad MSW = \frac{3438.381}{17} = 202.258 \quad F = \frac{11.820}{202.258} = .058$$

Source of variation	Sum of squares	df	Mean square (variance estimate)	F
Between-groups	35.461	3	11.820	0.058
Within-groups	3438.381	17	202.258	
Total	3402.920	20		

$$F_{.05} = 3.20 \quad F_{.01} = 5.18$$

Reject  $H_0$  if  $F >$  critical value, otherwise do not reject  $H_0$ . Since  $F = 0.058 < 3.20$ , do not reject  $H_0$ .

There is insufficient evidence that the educational programs are different.

$$2. H_0: \mu_S = \mu_V = \mu_T$$

$$H_1: \mu_S \neq \mu_V \quad \mu_S \neq \mu_T \quad \mu_V \neq \mu_T$$

At least one is true.

$$M = 21.250 \quad M^2 = 451.563 \quad M^2(N) = 5418.756 \quad S = 2.832 \quad S^2 = 8.020$$

$$SS_{TOT} = 8.020(12-1) = 88.222 \quad T_S = 96 \quad T_V = 77 \quad T_T = 82$$

$$n_S = n_V = n_T = 4$$

$$SSB = \left[ \frac{T_S^2}{n_S} + \frac{T_V^2}{n_V} + \frac{T_T^2}{n_T} \right] - M^2(N) = \left[ \frac{96^2}{4} + \frac{77^2}{4} + \frac{82^2}{4} \right] - 5418.756$$

$$= [2304 + 1482.250 + 1681] - 5418.756$$

$$= 5467.250 - 5418.756 = 48.494$$

$$SSW = 88.222 - 48.494 = 39.728$$

$$Df_B = 3 - 1 = 2 \quad df_W = 12 - 3 = 9 \quad df_{TOT} = 12 - 1 = 11$$

$$MSB = \frac{48.494}{2} = 24.247 \quad MSW = \frac{39.728}{9} = 4.414 \quad F = \frac{24.247}{4.414} = 5.493$$

Source of variation	Sum of squares	df	Mean square (variance estimate)	F
Between-groups	48.494	2	24.247	5.493
Within-groups	39.728	9	4.414	
Total	88.222	11		

$$F_{.05} = 4.26 \quad F_{.01} = 8.02$$

Reject  $H_0$  if  $F >$  critical value, otherwise do not reject  $H_0$ . Since  $F = 5.493 > 4.26$ , reject  $H_0$ .

There is evidence that the gas additives have a different effect on mileage..

$$3. M_S = 24.000 \quad M_V = 18.250 \quad M_T = 20.50$$

$$\text{Scheffé critical value} = \sqrt{(k-1)F_{.05}} = \sqrt{(3-1) \times 4.26} = \sqrt{8.520} = 2.919$$

$$t_{SV} = \frac{M_S - M_V}{\sqrt{MSW \left( \frac{1}{n_S} + \frac{1}{n_V} \right)}} = \frac{24.000 - 18.250}{\sqrt{4.414 \left( \frac{1}{4} + \frac{1}{4} \right)}} = \frac{5.750}{\sqrt{2.207}} = \frac{5.750}{1.486} = |3.869| > 2.919(\text{sig})$$

$$t_{ST} = \frac{M_S - M_T}{\sqrt{MSW \left( \frac{1}{n_S} + \frac{1}{n_T} \right)}} = \frac{24.000 - 20.500}{\sqrt{4.414 \left( \frac{1}{4} + \frac{1}{4} \right)}} = \frac{3.500}{\sqrt{2.207}} = \frac{3.500}{1.486} = |2.355| < 2.919(\text{n.s.})$$

$$t_{VT} = \frac{M_V - M_T}{\sqrt{MSW \left( \frac{1}{n_V} + \frac{1}{n_T} \right)}} = \frac{18.250 - 20.500}{\sqrt{4.414 \left( \frac{1}{4} + \frac{1}{4} \right)}} = \frac{-2.250}{\sqrt{2.207}} = \frac{-2.250}{1.486} = |-1.514| < 2.919(\text{n.s.})$$

$$4. M_S = 24.000 \quad M_V = 18.250 \quad M_T = 20.50$$

$$\text{Tukey Critical Value} = q_{k,v} \sqrt{\frac{MSW}{n}} = q_{k,v} \sqrt{\frac{4.414}{4}} = 3.95 \sqrt{1.104} = 3.95(1.051) = 4.151$$

$$D_1 = |M_S - M_V| = |24.000 - 18.250| = 5.75 > 4.151 (\text{sig})$$

$$D_2 = |M_S - M_T| = |24.000 - 20.500| = 3.500 < 4.151 (\text{n.s.})$$

$$D_3 = |M_V - M_T| = |18.250 - 20.500| = 2.250 < 4.151 (\text{n.s.})$$

$$5. H_0: \mu_I = \mu_M = \mu_H$$

$$H_1: \mu_I \neq \mu_M \quad \mu_I \neq \mu_H \quad \mu_M \neq \mu_H$$

At least one is true.

$$M = 54.300 \quad M^2 = 2948.490 \quad M^2(N) = 58969.800 \quad S = 5.939 \quad S^2 = 35.629$$

$$SS_{TOT} = 35.629(20-1) = 676.951 \quad T_I = 371 \quad T_M = 440 \quad T_H = 275$$

$$n_I = 7 \quad n_M = 8 \quad n_H = 5$$

$$SSB = \left[ \frac{T_I^2}{n_I} + \frac{T_M^2}{n_M} + \frac{T_H^2}{n_H} \right] - M^2(N) = \left[ \frac{371^2}{7} + \frac{440^2}{8} + \frac{275^2}{5} \right] - 58969.800$$

$$= [19663 + 24200 + 15125] - 58969.800$$

$$= 58988 - 58969.800 = 18.200$$

$$SSW = 676.951 - 18.200 = 658.751$$

$$Df_B = 3 - 1 = 2$$

$$df_W = 20 - 3 = 17$$

$$df_{TOT} = 20 - 1 = 19$$

$$MSB = \frac{18.200}{2} = 9.100$$

$$MSW = \frac{658.751}{17} = 38.750$$

$$F = \frac{9.100}{38.750} = 0.235$$

Source of variation	Sum of squares	df	Mean square (variance estimate)	F
Between-groups	18.200	2	9.100	0.235
Within-groups	658.751	17	38.750	

Total	676.951	19		
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$$F_{.05} = 3.59 \quad F_{.01} = 6.11$$

Reject  $H_0$  if  $F >$  critical value, otherwise do not reject  $H_0$ . Since  $F = 0.235 < 3.59$ , do not reject  $H_0$ .

There is insufficient evidence that people with different activity levels differ on optimism...

6.  $H_0: \mu_F = \mu_I = \mu_A$                        $H_1: \mu_F \neq \mu_I \quad \mu_F \neq \mu_A \quad \mu_I \neq \mu_A$   
At least one is true.

$$\begin{aligned} M &= 12.833 & M^2 &= 164.686 & M^2(N) &= 2964.348 & S &= 2.618 & S^2 &= 6.854 \\ SS_{TOT} &= 116.517 & T_F &= 90 & T_I &= 72 & T_A &= 69 & T_{S1} &= 33 \\ T_{S2} &= 40 & T_{S3} &= 44 & T_{S4} &= 46 & T_{S5} &= 37 & T_{S6} &= 31 \\ n_F &= n_I = n_A = 6 & n_{S1} &= n_{S2} = n_{S3} = n_{S4} = n_{S5} = n_{S6} &= 3 \end{aligned}$$

$$SSB = \left[ \frac{T_F^2}{n_F} + \frac{T_I^2}{n_I} + \frac{T_A^2}{n_A} \right] - M^2(N) = \left[ \frac{90^2}{6} + \frac{72^2}{6} + \frac{69^2}{6} \right] - 2964.346 = 3007.500 - 2964.348 = 43.152$$

$$\begin{aligned} SSP &= \left[ \frac{T_{S1}^2}{n_{S1}} + \frac{T_{S2}^2}{n_{S2}} + \frac{T_{S3}^2}{n_{S3}} + \frac{T_{S4}^2}{n_{S4}} + \frac{T_{S5}^2}{n_{S5}} + \frac{T_{S6}^2}{n_{S6}} \right] - M^2(N) \\ &= \left[ \frac{33^2}{3} + \frac{40^2}{3} + \frac{44^2}{3} + \frac{46^2}{3} + \frac{37^2}{3} + \frac{31^2}{3} \right] - 2964.348 = 3023.667 - 2964.348 = 59.319 \end{aligned}$$

$$SSW = 116.517 - 43.152 - 59.319 = 14.046$$

Source of variation	Sum of squares	df	Mean square (variance estimate)	F
People	59.319	5		
Between-groups	43.152	2	21.576	15.357
Within-groups	14.046	10	1.405	
Total	116.517	17		

$$F_{.05} = 4.10 \quad F_{.01} = 7.56$$

Reject  $H_0$  if  $F >$  critical value, otherwise do not reject  $H_0$ . Since  $F = 15.357 > 7.56$ , reject  $H_0$ .

There is evidence that the different wine labels affect the perceived quality of wine....

7. For weight:  $H_0: \mu_O = \mu_A$                        $H_1: \mu_O \neq \mu_A$   
For Nut Type:  $H_0: \mu_S = \mu_{US}$                        $H_1: \mu_S \neq \mu_{US}$   
For interaction:  $H_0: \mu_{W \times N} = 0$                        $H_1: \mu_{W \times N} \neq 0$

$$\begin{aligned} M &= 7.500 & M^2 &= 56.250 & M^2(N) &= 56.250(40) = 2250 & S &= 4.930 & S^2 &= 24.308 \\ SS_{TOT} &= 948.000 \end{aligned}$$

	Shelled	Unshelled	
Obese	140 (10)	20 (10)	160 (20)
Average	80 (10)	60 (10)	140 (20)
	220 (20)	80 (20)	

$$SS_{we} = \left[ \frac{T_O^2}{n_O} + \frac{T_A^2}{n_A} \right] - M^2(N) = \left[ \frac{160^2}{20} + \frac{140^2}{20} \right] - 2250 = 2260 - 2250 = 10.000$$

$$SS_{NT} = \left[ \frac{T_S^2}{n_S} + \frac{T_{UN}^2}{n_{UN}} \right] - M^2(N) = \left[ \frac{220^2}{20} + \frac{80^2}{20} \right] - 2250 = 2740 - 2250 = 490$$

$$SS_{INT} = \left[ \frac{T_{OS}^2}{n_{OS}} + \frac{T_{OUN}^2}{n_{OUN}} + \frac{T_{AS}^2}{n_{AS}} + \frac{T_{AUN}^2}{n_{AUN}} \right] - SS_W - SS_{NT} - M^2(N)$$

$$= \left[ \frac{140^2}{10} + \frac{20^2}{10} + \frac{80^2}{10} + \frac{60^2}{10} \right] - 10 - 490 - 2250 = 3000 - 10 - 490 - 2250 = 250$$

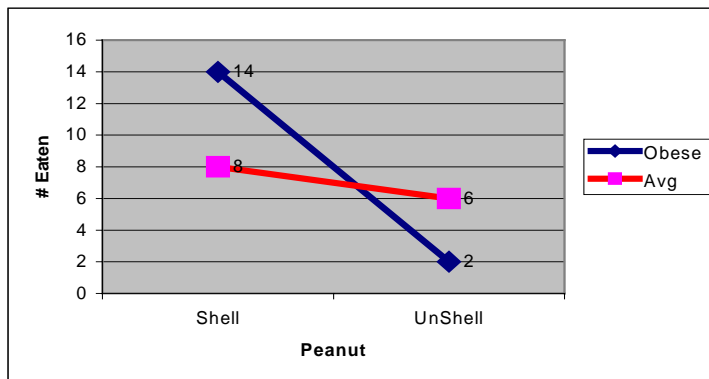
Source of variation	Sum of squares	df	Mean square (variance estimate)	F
Weight	10.000	1	10.000	1.818
Nut Type	490.000	1	490.000	89.091
Interaction	250.000	1	250.000	45.455
Error	198.000	36	5.500	
Total	948.000	39		

Critical Value for Weight = 4.12 or 7.40  
 Critical Value for Nut Type = 4.12 or 7.40  
 Critical Value for Interaction = 4.12 or 7.40

For Weight: Reject  $H_0$  if  $F >$  critical value, otherwise do not reject  $H_0$ . Since  $F = 1.818 < 4.12$ , do not reject  $H_0$ . There is insufficient evidence that people with different weight types differ on number of nuts eaten.

For Nut Type: Reject  $H_0$  if  $F >$  critical value, otherwise do not reject  $H_0$ . Since  $F = 89.091 > 4.12$ , reject  $H_0$ . There is evidence that different nut types differ on the number of nuts eaten.

For Interaction: Reject  $H_0$  if  $F >$  critical value, otherwise do not reject  $H_0$ . Since  $F = 45.455 > 4.12$ , reject  $H_0$ . There is evidence that an interaction between weight type and t nut types differ on the number of nuts eaten.



8. For mode:  $H_0: \mu_V = \mu_A$                        $H_1: \mu_V \neq \mu_A$   
 For media:  $H_0: \mu_C = \mu_{UN}$                        $H_1: \mu_S \neq \mu_{UN}$   
 For interaction:  $H_0: \mu_{MoxMe} = 0$                        $H_1: \mu_{MoxMe} \neq 0$

$M = 4.725$        $M^2 = 22.326$        $M^2(N) = 893.040$        $S = 1.908$        $S^2 = 3.640$   
 $SS_{TOT} = 141.960$

	A1	A2	
B1	61 (10)	36(10)	97 (20)
B2	37 (10)	55 (10)	92 (20)
	98(20)	91 (20)	

Ss	A1	A2	B1	B2	T <sub>s</sub>
1	10 (2)	9 (2)	9 (2)	10 (2)	19
2	9 (2)	5 (2)	9 (2)	5 (2)	14
3	8 (2)	13 (2)	10 (2)	11 (2)	21
4	13 (2)	9 (2)	12 (2)	10 (2)	22
5	9 (2)	7 (2)	5 (2)	11 (2)	16
6	10 (2)	14 (2)	13 (2)	11 (2)	24
7	13 (2)	9 (2)	11 (2)	11 (2)	22
8	11 (2)	10 (2)	13 (2)	8 (2)	21
9	7 (2)	7 (2)	6 (2)	8 (2)	14
10	8 (2)	8 (2)	9 (2)	7 (2)	16

$$SS_A = \left[ \frac{T_{A1}^2}{n_{A1}} + \frac{T_{A2}^2}{n_{A2}} \right] - M^2(N) = \left[ \frac{98^2}{20} + \frac{91^2}{20} \right] - 893.040 = 894.250 - 893.040 = 1.210$$

$$SS_B = \left[ \frac{T_{B1}^2}{n_{B1}} + \frac{T_{B2}^2}{n_{B2}} \right] - M^2(N) = \left[ \frac{97^2}{20} + \frac{92^2}{20} \right] - 893.040 = 893.650 - 893.040 = 0.610$$

$$\begin{aligned} SS_{AB} &= \left[ \frac{T_{A1B1}^2}{n_{A1B1}} + \frac{T_{A1B2}^2}{n_{A1B2}} + \frac{T_{A2B1}^2}{n_{A2B1}} + \frac{T_{A2B2}^2}{n_{A2B2}} \right] - SS_A - SS_B - M^2(N) \\ &= \left[ \frac{61^2}{10} + \frac{37^2}{10} + \frac{36^2}{10} + \frac{55^2}{10} \right] - 1.210 - 0.610 - 893.040 = 941.100 - 1.210 - 0.610 - 893.040 = 46.240 \end{aligned}$$

$$\begin{aligned} SS_S &= \left[ \frac{T_{S1}^2}{n_{S1}} + \frac{T_{S2}^2}{n_{S2}} + \frac{T_{S3}^2}{n_{S3}} + \frac{T_{S4}^2}{n_{S4}} + \frac{T_{S5}^2}{n_{S5}} + \frac{T_{S6}^2}{n_{S6}} + \frac{T_{S7}^2}{n_{S7}} + \frac{T_{S8}^2}{n_{S8}} + \frac{T_{S9}^2}{n_{S9}} + \frac{T_{S10}^2}{n_{S10}} \right] - M^2(N) \\ &= \left[ \frac{19^2}{4} + \frac{14^2}{4} + \frac{21^2}{4} + \frac{22^2}{4} + \frac{16^2}{4} + \frac{24^2}{4} + \frac{22^2}{4} + \frac{21^2}{4} + \frac{14^2}{4} + \frac{16^2}{4} \right] - 893.040 \\ &= 922.750 - 893.040 = 29.710 \end{aligned}$$

$$\begin{aligned} SS_{AS} &= \left[ \frac{T_{A1S1}^2}{n_{A1S1}} + \frac{T_{A1S2}^2}{n_{A1S2}} + \frac{T_{A1S3}^2}{n_{A1S3}} + \dots + \frac{T_{A2S8}^2}{n_{A2S8}} + \frac{T_{A2S9}^2}{n_{A2S9}} + \frac{T_{A2S10}^2}{n_{A2S10}} \right] - SS_A - SS_S - M^2(N) \\ &= \left[ \frac{10^2}{2} + \frac{9^2}{2} + \frac{8^2}{2} + \dots + \frac{10^2}{2} + \frac{7^2}{2} + \frac{8^2}{2} \right] - 1.210 - 29.710 - 893.040 \\ &= 946.500 - 1.210 - 29.710 - 893.040 = 22.540 \end{aligned}$$

$$\begin{aligned} SS_{BS} &= \left[ \frac{T_{B1S1}^2}{n_{B1S1}} + \frac{T_{B1S2}^2}{n_{B1S2}} + \frac{T_{B1S3}^2}{n_{B1S3}} + \dots + \frac{T_{B2S8}^2}{n_{B2S8}} + \frac{T_{B2S9}^2}{n_{B2S9}} + \frac{T_{B2S10}^2}{n_{B2S10}} \right] - SS_B - SS_S - M^2(N) \\ &= \left[ \frac{9^2}{2} + \frac{9^2}{2} + \frac{10^2}{2} + \dots + \frac{8^2}{2} + \frac{8^2}{2} + \frac{7^2}{2} \right] - 0.610 - 29.710 - 893.040 \\ &= 946.500 - 1.210 - 29.710 - 893.040 = 23.140 \end{aligned}$$

$$SS_{ABS} = 141.960 - 0.610 - 1.210 - 46.240 - 29.710 - 22.540 - 23.140 = 18.510$$

Source of variation	Sum of squares	df	Mean square (variance estimate)	F
Mode (A)	1.210	1	1.210	$1.210 \div 2.504 = 0.483$
Media (B)	0.610	1	0.610	$0.610 \div 2.571 = 0.237$
Interaction (A×B)	46.240	1	46.240	$46.240 \div 2.057 = 22.479$
Subjects (S)	29.710	9	3.301	
AS	22.540	9	2.504	
BS	23.140	9	2.571	
ABS	18.510	9	2.057	
Total	141.960	39		

Critical Value for Mode = 5.12 or 10.56

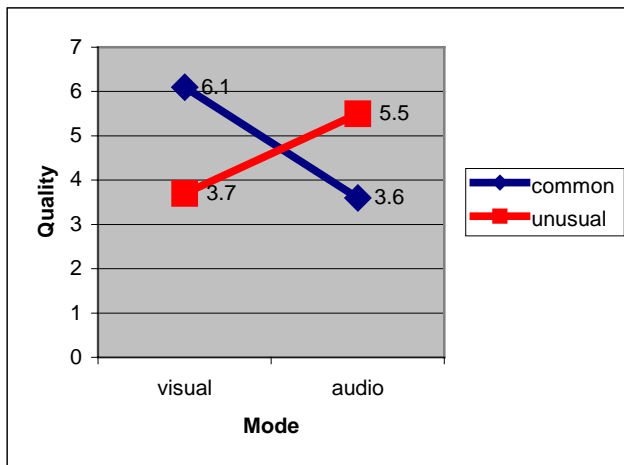
Critical Value for Media = 5.12 or 10.56

Critical Value for Interaction = 5.12 or 10.56

For Mode: Reject  $H_0$  if  $F >$  critical value, otherwise do not reject  $H_0$ . Since  $F = 0.483 < 5.12$ , do not reject  $H_0$ . There is insufficient evidence that mode of presentation affected perception of quality..

For Nut Type: Reject  $H_0$  if  $F >$  critical value, otherwise do not reject  $H_0$ . Since  $F = 0.237 < 5.12$ , do not reject  $H_0$ . There is insufficient evidence that media type affected perception of quality..

For Interaction: Reject  $H_0$  if  $F >$  critical value, otherwise do not reject  $H_0$ . Since  $F = 22.579 > 10.56$ , reject  $H_0$ . There is evidence that an interaction between mode and media had an effect on perceived quality of a product..



9. For gender:  $H_0: \mu_F = \mu_M$                        $H_1: \mu_F \neq \mu_M$   
 For layout:  $H_0: \mu_1 = \mu_2$                        $H_1: \mu_1 \neq \mu_2$   
 For interaction:  $H_0: \mu_{G \times L} = 0$                        $H_1: \mu_{G \times L} \neq 0$

$M = 5.550$        $M^2 = 30.803$        $M^2(N) = 616.060$        $S = 2.856$        $S^2 = 8.157$   
 $SS_{TOT} = 154.983$

	F	M	
L1	38 (5)	41(5)	79 (10)
L2	20 (5)	12 (5)	32 (10)
	58 (10)	53 (10)	

S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
10	7	15	14	12	12	9	9	14	9

$$SS_A = \left[ \frac{T_F^2}{n_F} + \frac{T_M^2}{n_M} \right] - M^2(N) = \left[ \frac{58^2}{10} + \frac{53^2}{10} \right] - 616.060 = 617.300 - 616.060 = 1.240$$

$$SS_B = \left[ \frac{T_{L1}^2}{n_{L1}} + \frac{T_{L2}^2}{n_{L2}} \right] - M^2(N) = \left[ \frac{79^2}{10} + \frac{32^2}{10} \right] - 616.060 = 726.500 - 616.060 = 110.440$$

$$SS_{AB} = \left[ \frac{T_{FL1}^2}{n_{FL1}} + \frac{T_{FL2}^2}{n_{FL2}} + \frac{T_{ML1}^2}{n_{ML1}} + \frac{T_{ML2}^2}{n_{ML2}} \right] - SS_A - SS_B - M^2(N)$$

$$= \left[ \frac{38^2}{5} + \frac{41^2}{5} + \frac{20^2}{5} + \frac{12^2}{5} \right] - 1.240 - 110.440 - 616.060 = 733.800 - 1.240 - 110.440 - 616.060 = 6.060$$

$$SS_S = \left[ \frac{T_{S1}^2}{n_{S1}} + \frac{T_{S2}^2}{n_{S2}} + \frac{T_{S3}^2}{n_{S3}} + \frac{T_{S4}^2}{n_{S4}} + \frac{T_{S5}^2}{n_{S5}} + \frac{T_{S6}^2}{n_{S6}} + \frac{T_{S7}^2}{n_{S7}} + \frac{T_{S8}^2}{n_{S8}} + \frac{T_{S9}^2}{n_{S9}} + \frac{T_{S10}^2}{n_{S10}} \right] - M^2(N)$$

$$= \left[ \frac{10^2}{2} + \frac{7^2}{2} + \frac{15^2}{2} + \frac{14^2}{2} + \frac{12^2}{2} + \frac{12^2}{2} + \frac{9^2}{2} + \frac{9^2}{2} + \frac{14^2}{2} + \frac{9^2}{2} \right] - 616.060$$

$$= 648.500 - 616.060 = 32.440$$

$$SS_{E:BS} = SS_S - SS_A = 32.440 - 1.240 = 31.200$$

$$SS_{WS} = SS_{TOT} - SS_S = 154.983 - 32.440 = 122.543$$

$$SS_{E:WS} = SS_{WS} - SS_B - SS_{AB} = 122.543 - 110.440 - 6.060 = 6.043$$

Source of variation	Sum of squares	df	Mean square	F
Between Subj.	32.440	s-1 = 9		
Gender (A)	1.240	a-1 = 1	1.240	0.318
Error B-S	31.200	s-a = 8	3.900	
Within Subj	122.543	a(b-1) = 10		
Layout (B)	110.440	b-1 = 1	110.440	146.278
Interaction (AB)	6.060	(a-1)(b-1) = 1	6.060	8.026
Error W-S	6.043	(b-1)(s-a) = 8	0.755	
Total	154.983	n-1 = 19		

Critical Value for Gender = 5.32 or 11.26

Critical Value for Layout = 5.32 or 11.26

Critical Value for Interaction = 5.32 or 11.26

For Mode: Reject  $H_0$  if  $F >$  critical value, otherwise do not reject  $H_0$ . Since  $F = 0.318 < 5.32$ , do not reject  $H_0$ . There is insufficient evidence of a gender difference in preference.

For Nut Type: Reject  $H_0$  if  $F >$  critical value, otherwise do not reject  $H_0$ . Since  $F = 8.026 > 5.32$ , reject  $H_0$ . There is evidence that layout affected people's preference.

For Interaction: Reject  $H_0$  if  $F >$  critical value, otherwise do not reject  $H_0$ . Since  $F = 146.278 > 11.26$ , reject  $H_0$ . There is evidence that an interaction between gender and layout had an effect on people's preference..