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PARTEE VERBS*

(Received 17 July 1999)

I. INTRODUCTION

Approximately thirty years ago, Barbara H. Partee tried to think of counterexamples to David Lewis's observation that no intransitive verbs appeared to have intensional subject positions. She came up with such verbs as 'rise,' 'change,' and 'increase.' Lewis agreed that they were indeed counterexamples to his observation. He mentioned it to Richard Montague, who incorporated these verbs into his now famous grammatical theory for English.¹

The most frequently discussed sentence with a Partee verb is:

(1) The temperature is rising.

Let us imagine circumstances under which the temperature is 90 (degrees Fahrenheit) and it is getting hotter. That is, (1) and (2) are both true.

(2) The temperature is 90.

Assume that 'the temperature' designates a number. This means that the 'is' in (2) is the 'is' of identity.² By Leibniz's law of indiscernibility of the identical, it apparently follows from (1) and (2) that

(3) 90 is rising.

But (3) is not true. (It may even be unintelligible.) Thus, something must give.

Montague abandons the assumption that 'the temperature' designates (or, as he puts it, "denotes") a number, and consequently rejects the natural claim that the 'is' in (2) is the 'is' of identity. In his view, 'the temperature' designates a function from times to



numbers. The verb ‘rise’ in (1) is a predicate expressing a property of such functions, not numbers, and the ‘is’ in (2) means “has the value that is identical with the value of” (or “has the same value as”).³ This is an elegant way out of the above predicament. Others have since followed him, and it appears that Montague’s treatment of Partee verbs has attained the status of an orthodoxy.⁴

Let us call the view that noun phrases like ‘the temperature’ in sentences like (1) and (2) designate functions, e.g., functions from times to numbers in the case of ‘temperature’ *functionalism*. I wish to propose an alternative to functionalism. Before doing so, however, I would like to dissociate myself from two other suggestions for solving the Partee puzzle.

II. RED HERRING

In deriving (3) from (1) and (2), we assumed that the noun phrase ‘the temperature’ designates a number. It might be suggested that to block the derivation, we should apply Russell’s theory of descriptions and deny that the noun phrase designates anything, i.e., that the noun phrase is a semantically meaningful unit. Would such a suggestion block the derivation of (3) from (1) and (2)? No. It is easy to see that Russell’s theory of descriptions solves nothing, once we formulate the Russellian analyses of (1)–(3):

- (1r) $(\exists!x)Tx \ \& \ (\forall x)(Tx \rightarrow Rx)$;
 (2r) $(\exists!x)Tx \ \& \ (\forall x)(Tx \rightarrow x = n)$;
 (3r) Rn .

‘Tx’ means “x is a temperature,” ‘Rx’ means “x is rising,” and ‘n’ means “90.” Consider

$$(2\frac{1}{2}r) \ (\exists!x)(Rx \ \& \ x = n).$$

$(2\frac{1}{2}r)$ follows from (1r) and (2r), and (3r) follows from $(2\frac{1}{2}r)$. So, (3r) follows from (1r) and (2r). Thus, whether we should take the noun phrase ‘the temperature’ as a semantically significant unit is not the issue. Therefore, for brevity’s sake, I will continue to assume that the noun phrase is a semantically significant unit and speak of its designation.⁵

The second suggestion is due to Ray Jackendoff and consists of two objections to functionalism.⁶ According to the first of Jackendoff's objections, (2) is a truncated version of (4) in which the preposition 'at' is suppressed:

(4) The temperature is at 90.

The inference from (1) and (2), viz., (4), to (3) is analogous to the inference from (5) and (6) to (7):

(5) The airplane is rising.

(6) The airplane is at 6000 feet.

(7) 6000 feet is rising.

The latter inference is obviously invalid. The former inference is deemed invalid in exactly the same way. This treatment of (2) by Jackendoff enables us to avoid the undesirable inference without commitment to any particular view on the designation of 'the temperature.' It only requires that 'the temperature' designate something that can be located on a scale of measure. It does not require any further specific nature of the designation of the noun phrase, *a fortiori* it does not require that it be a function. Thus, the first objection by Jackendoff is that functionalism overreacts to the Partee puzzle. We do not need functionalism to solve it.

Jackendoff's second objection says that 'the temperature' designates something that is subject to perception, as illustrated by the following sentence:

(8) I felt the temperature (of the water).

Since no function is subject to perception, 'the temperature' does not designate a function. This objection says that functionalism gets the nature of temperatures wrong. Functionalism has temperatures as abstract mathematical objects, but temperatures are perceivable and no abstract mathematical object is perceivable.

I find both of Jackendoff's objections to functionalism concerning 'the temperature' convincing. Why do I not stop here and declare the Partee puzzle solved then? Because Jackendoff's observations about 'the temperature' are parochial. They apply to some noun phrases used in formulations of the Partee puzzle but not others. They merely show that noun phrases like 'the temperature' and 'the price of milk'⁷ are bad examples to use to illustrate the Partee

puzzle, as they tend to divert our attention from the heart of the matter. A better example is something like the following:

(9) The number of sleeping students is increasing.⁸

The matching sentence,

(10) The number of sleeping students is 9,

does not even appear to be synonymous with ‘The number of sleeping students is at 9,’ as the latter sentence is of doubtful intelligibility. To see this clearly, consider:

(11) 9 is the number of sleeping students;

(12) 9 is what the number of sleeping students is at.

Unlike (12), (11) sounds natural and equivalent to (10).⁹ Other candidates along the lines of (12) fare no better:

(13) 9 is that at which the number of sleeping students is;

(14) 9 is where the number of sleeping students is.

Also, note that (14) or any variant of (14) sounds far less natural than

(15) 6000 feet (above the sea level) is where the airplane is.

As for perceivability, it is unacceptable to say things such as

(16) I felt (saw, heard, smelled, tasted) the number of sleeping students.

What ‘the number of sleeping students’ designates is not something we can perceive.¹⁰ Thus, the two objections against functionalism raised by Jackendoff are inapplicable. Yet, (9) and (10) apparently yield (17), which is unacceptable:

(17) 9 is increasing.

The Partee puzzle is back, and functionalism returns as a genuine contender.¹¹

III. MOTIVATION

My motivation for resisting functionalism is ontological parsimony in semantics. Let us call whatever an expression contributes to the truth condition of a sentence in which it occurs the *semantic value*

of the expression as it occurs in the sentence. Functions from times to numbers are respectable objects and may be used freely in appropriate places in semantics. For instance, it may be perfectly natural and acceptable to assign such a function as the semantic value to the phrase ‘the function that maps any time t to the number of sleeping students at t ’ wherever it occurs.¹² On the other hand, it seems unnecessarily cumbersome to assign such a function as the semantic value to another phrase, e.g., ‘the number of sleeping students.’ It seems more intuitive to assign a number instead, namely, the number of sleeping students. Or consider:

(18) The president is running.

The function from times to the presidents at those times is a perfectly respectable object, but it is grotesquely baroque to assign it as the semantic value of ‘the president’ in (18). It is more natural and sleek to assign the person who is the president at the time in question.¹³

Assigning a times-to-numbers function to ‘the temperature’ in (1) and assigning a times-to-persons function to ‘the president’ in (18) are special cases of the “sense”-oriented strategy in semantics. The basic idea of the “sense”-oriented strategy is to postulate for each expression both a “reference” and a “sense.” We may distinguish two versions of the “sense”-oriented strategy: the moderate and the extreme. According to the moderate version, in some sentential contexts the semantic value of an expression is the “reference” of the expression, and in other sentential contexts it is the “sense.” As is widely known, Frege is the originator of this strategy, in its moderate form.¹⁴ In the extreme version, every sentential context is one in which the semantic value of an expression is its “sense.” Montague’s proposal is an instance of the extreme version, where the “sense” of an expression is identified as a function from indices (worlds, times, places, etc.) to the “reference” (the value of the function) at those indices. The function is usually called the *intension* of the expression, and the “reference” at an index is usually called the *extension*. According to Montague, ‘the temperature’ designates (or “denotes”) its intension in this sense. Whether in its Montagovian extreme form or in a more moderate Fregean form, the “sense”-oriented strategy stands in opposition to the “reference”-oriented strategy, according to which it is always the “reference” of an expression that is its semantic value. The “reference”-oriented strategy dispenses

with “senses” and therefore is more parsimonious than the “sense”-oriented strategy.¹⁵ I will discuss some specific advantages of the “reference”-oriented strategy over the “sense”-oriented strategy in the next section.

What Partee noticed is that a simple-minded application of the “reference”-oriented strategy is unacceptable, for it yields (3) from (1) and (2), or (17) from (9) and (10). However, this does not mean that we must accept the “sense”-oriented strategy. I wish to suggest a less simple-minded application of the “reference”-oriented strategy that does not have such an unacceptable consequence.¹⁶

IV. SENTENTIAL OPERATOR THEORY

There is an important area of semantics in which the “reference”-oriented strategy has been remarkably successful as an alternative to the “sense”-oriented strategy. It is modal semantics. The proposal I wish to make takes a hint from modal semantics and applies it to the recalcitrant sentences in question. Consider the following:

- (19) The teacher of Alexander is necessarily a teacher;
- (20) The teacher of Alexander is possibly Macedonian.

The idea is to use sentences like (19) and (20) to understand sentences like (9) and (1). (19) and (20), of course, are ambiguous. The intended reading of (19) is the one that makes it true: “No matter who might be the teacher of Alexander, whoever was the teacher of Alexander, if such existed, would be a teacher.” The intended reading of (20) says that the following could have been the case: the teacher of Alexander be Macedonian. These are the *de dicto* readings. According to the “sense”-oriented strategy, the phrase ‘the teacher of Alexander’ in (19) and (20) designates a function from worlds to individuals in those worlds. It is the function that maps any world in which there is a unique teacher of Alexander to that teacher.¹⁷ It is then said that (19) is true at a world *w* iff the function in question maps any world accessible from *w* to a teacher (an individual who is a teacher in that world), and that (20) is true at *w* iff the function maps some world accessible from *w* to a Macedonian (an individual who is Macedonian in that world).

It is interesting to note how the “sense”-oriented strategy provides the other, *de re* readings. Under the *de re* readings, (19) says that the individual who is in fact the teacher of Alexander could not have failed to be a teacher, and (20) says that the individual who is in fact the teacher of Alexander could have been Macedonian. According to the “sense”-oriented strategy, (19) under the *de re* reading is true iff the value of the function in question at the actual world is a teacher in every accessible world in which he exists, and (20) under the *de re* reading is true iff the value of the function in question at the actual world is Macedonian in some accessible world. The value of the function at a particular world is an individual. So, these truth conditions are unintelligible unless it is intelligible to speak of an individual as being thus and so (e.g., a teacher or Macedonian) in a world. Thus, the “sense”-oriented strategy relies on the notion of a worlds-to-individuals function being thus and so and the notion of an individual being thus and so. As is widely known, the standard modal semantics, which follows the “reference”-oriented strategy and dispenses with talk of functions from worlds to individuals in favor of talk of worlds and individuals, relies on the notion of an individual being thus and so but not on the notion of a worlds-to-individuals function being thus and so. It is therefore more economical.

According to the standard modal semantics, (19) – under the *de dicto* reading – is true at a world w iff for any world w' accessible from w , if the teacher of Alexander in w' exists in w' , the teacher of Alexander in w' is a teacher in w' , or in symbols:

$$(19^*) (w')((Aw'/w \ \& \ Ec-w',w') \rightarrow Tc-w',w'),$$

where ‘A’ expresses the accessibility relation, ‘E’ expresses the relation of existence between an individual and a world, ‘c- w' ’ designates the teacher of Alexander in world w' , and ‘T’ expresses the relation of being a teacher, which holds between an individual and a world. To put it in Russellian terms, the definite description ‘the teacher of Alexander’ is understood to have the narrow scope on this reading. Thus, (19) is regarded as the result of applying the sentential operator ‘necessarily’ to:

(21) The teacher of Alexander, if such exists, is a teacher.

Similarly, (20) is to be regarded here as the result of applying the sentential operator ‘possibly’ to:

(22) The teacher of Alexander is Macedonian.

Thus, (20) is true at a world w iff for some world w' accessible from w , the teacher of Alexander in w' exists in w' ¹⁸ and the teacher of Alexander in w' is Macedonian in w' , or in symbols:

(20*) $(\exists w')(Aw'/w \ \& \ Ec-w',w' \ \& \ Mc-w',w')$,

where ‘M’ expresses the relation of being Macedonian, which holds between an individual and a world. Again, the definite description ‘the teacher of Alexander’ has the narrow scope on this reading. For both (19) and (20), the *de re* reading results when the definite description is understood as having the wide scope and the sentential operator the narrow scope. Thus, the standard modal semantics explains the *de dicto/de re* ambiguity as a distinction of scope. This is a more unified explanation than the “sense”-oriented strategy’s explanation by means of two distinct notions (viz., the notion of a function being thus and so and the notion of an individual being thus and so).

According to the standard modal semantics, the subject term in (21) and (22), ‘the teacher of Alexander,’ designates an individual, not a function from worlds to individuals. The main verb phrases in (21) and (22), ‘is a teacher’ and ‘is Macedonian,’ are extensional predicates; they apply to individuals, not functions from worlds to individuals. Yet, as seen above, the truth conditions of (19) and (20) with respect to a world w go beyond whether the predicates apply to the designation of the subject term with respect to w . This is entirely due to the sentential operators ‘necessarily’ and ‘possibly.’ My suggestion is that we apply this well-known maneuver to sentences like (9) and (1).

Superficially (9) is not analogous to (19) or (20), for it does not contain a sentential operator on the surface level. This is significant. If the surface verb phrase cannot be interpreted as consisting of two separable elements, viz., a predicate and a sentential operator, an analog of the modal maneuver is inapplicable. Therefore, if the strategy is to work, the verb phrase ‘is increasing’ in (9) must be understood to have the internal structure consisting of a predicate and a sentential operator. Thus, the proposal postulates

that in (9) the phrase ‘is increasing’ is really (at the level of logical form) a combination of a sentential operator Φ and a predicate Ψ . The account therefore is committed to a kind of lexical decomposition.

The obvious questions are: “What is Φ ?” and “What is Ψ ?” Since the claim is that (9) should be understood along the lines of (19) and (20), let us generalize (19*) and (20*). The truth value of (19) is sensitive to the way each accessible world is that contains a unique teacher of Alexander. In particular, it is sensitive to the relevant individual in each such world (viz., the individual who is a unique teacher of Alexander in that world) having a certain property (viz., being a teacher). In this particular example, whether any such individual has the property in question in any given such world is independent of whether any other such individual has it in any other given such world. But it is easy to conceive a modal operator whose applicability is sensitive to obtainment of a certain relation among relevant individuals in different relevant worlds. Such operators are unobjectionable in principle, for inter-worldly relations in general are evidently respectable. For example, an individual from one world may be more or less tall, heavy, or intelligent than another individual from another world. Generalizing (19*) in view of this, we obtain:

$$(23) (w_1)(w_2) \dots (w_k)((Aw_{1,w} \& Aw_{2,w} \& \dots \& Aw_{k,w} \& Ec_{w_1,w_1} \& Ec_{w_2,w_2} \& \dots \& Ec_{w_k,w_k}) \rightarrow Rc_{w_1,c-w_2, \dots, c-w_k}).$$

The configuration ‘c-x’ (where ‘x’ is replaced with a world variable) stands for a narrow-scope definite description “the such and such in x.” Beyond that, let us dissociate any specific reading from it. The accessibility relation is a two-place relation between worlds, and (23) speaks of its holding between one pair at a time. Let us express this by a single relational schematic predicate ‘Q’ in our next stage of generalization:

$$(24) (w_1)(w_2) \dots (w_k)((Qw,w_1,w_2, \dots, w_k \& Ec_{w_1,w_1} \& Ec_{w_2,w_2} \& \dots \& Ec_{w_k,w_k}) \rightarrow Rc_{w_1,c-w_2, \dots, c-w_k}).$$

In general, the worlds in question may be required not only to be accessible from w but also to stand in a certain other relation,

L, to one another. So, our last step of generalization gives us the following:

$$(25) (w_1)(w_2) \dots (w_k)((Qw, w_1, w_2, \dots, w_k \ \& \ Lw, w_1, w_2, \dots, w_k \ \& \ Ec-w_1, w_1 \ \& \ Ec-w_2, w_2 \ \& \ \dots \ \& \ Ec-w_k, w_k) \rightarrow Rc-w_1, c-w_2, \dots, c-w_k).$$

Similarly from (20*) by analogous steps, we obtain the following generalized schema:

$$(26) (\exists w_1)(\exists w_2) \dots (\exists w_k)(Qw, w_1, w_2, \dots, w_k \ \& \ Lw, w_1, w_2, \dots, w_k \ \& \ Ec-w_1, w_1 \ \& \ Ec-w_2, w_2 \ \& \ \dots \ \& \ Ec-w_k, w_k \ \& \ Rc-w_1, c-w_2, \dots, c-w_k).$$

(25) and (26) are the general schemata we want to apply to (9). For obvious reasons, let us shift to temporal variables:

$$(27) (t_1)(t_2) \dots (t_k)((Qt, t_1, t_2, \dots, t_k \ \& \ Lt, t_1, t_2, \dots, t_k \ \& \ Ec-t_1, t_1 \ \& \ Ec-t_2, t_2 \ \& \ \dots \ \& \ Ec-t_k, t_k) \rightarrow Rc-t_1, c-t_2, \dots, c-t_k).$$

$$(28) (\exists t_1)(\exists t_2) \dots (\exists t_k)(Qt, t_1, t_2, \dots, t_k \ \& \ Lt, t_1, t_2, \dots, t_k \ \& \ Ec-t_1, t_1 \ \& \ Ec-t_2, t_2 \ \& \ \dots \ \& \ Ec-t_k, t_k \ \& \ Rc-t_1, c-t_2, \dots, c-t_k).$$

Can we state the condition of truth for (9) at a time t , as uttered at t , along those lines?¹⁹ I think we can. In informal English, we may put the intuitive idea as follows:

(9\$) If we compare the number of sleeping students at any time around t and the number of sleeping students at any later time around t , then we will find that the former number is smaller than the latter.

One problem with this is that it only covers cases of continuous increase. It leaves out cases of step-wise increase, i.e., jumps in number sandwiched between moments of inactivity, or plateaux, around t . We should count such cases of step-wise increase as genuine cases of increase. In fact, step-wise increase is the only type of increase possible in the example at hand. The number of sleeping students at any time is a non-negative integer, hence it can only change discretely, not continuously. Thus, we should modify (9\$) as follows:

(9\$\$) If we compare the number of sleeping students at any time around t and the number of sleeping students at any later time around t , then we will find that the former number is

smaller than or equal to the latter, and we find some time before t but still around t and a later time after t but still around t such that the number of sleeping students at the former time is smaller than the number of sleeping students at the latter.

Eliminating the reference to “us” and the epistemic overtone, we arrive at the following formulation of the truth condition for (9) at a time t , as uttered at t :

- (9#) For the appropriate neighborhood N_t of t , (i) for any pair of times t_1 and t_2 in N_t such that t_1 precedes t_2 , the number of sleeping students at t_1 is smaller than or equal to the number of sleeping students at t_2 , and (ii) for some pair of times t_1 and t_2 in N_t such that t_1 precedes t and t precedes t_2 , the number of sleeping students at t_1 is smaller than the number of sleeping students at t_2 .

A period of time is a *neighborhood* of t if it includes t . The context of utterance determines what makes a neighborhood of t *appropriate*. In the case of the current example, no period that is one microsecond long or one century long is likely to be appropriate. Other constraints on the appropriateness of a neighborhood may be understood from the context of utterance.²⁰ The clause (i) assures that the number of sleeping students never drops in N_t , and the clause (ii) assures that the number does not stay flat in N_t , either. In symbols:

$$(9^*) (t_1)(t_2)((Bt,t_1,t_2 \ \& \ t_1 \{ t_2 \ \& \ En-t_1,t_1 \ \& \ En-t_2,t_2) \rightarrow (n-t_1 < n-t_2 \vee n-t_1 = n-t_2)) \ \& \ (\exists t_1)(\exists t_2)(Bt,t_1,t_2 \ \& \ t_1 \{ t_2 \ \& \ En-t_1,t_1 \ \& \ En-t_2,t_2 \ \& \ n-t_1 < n-t_2),$$

where ‘ Bt,t_1,t_2 ’ means “ t_1 and t_2 are in N_t ,” ‘ $\{$ ’ means “precedes,” and ‘ $En-t_1,t_1$ ’ (‘ $En-t_2,t_2$ ’) means “the number of sleeping students at t_1 (t_2) exists at t_1 (t_2).”²¹ (9*) is a conjunction of an instance of (27) and an instance of (28). Thus, we have given (9) a treatment that is along the lines of (19) and (20).

But what does (9*), or its English version (9#), tell us about Φ and Ψ ? It tells us that Φ is in effect a combination of a pair of universal quantifiers and a pair of existential quantifiers, each pair quantifying over a pair of times in an appropriate neighborhood of t . Ψ , on the other hand, is in effect a combination of the smaller-than-or-equal-to relation and the smaller-than relation. There is no

easy way to express Φ or Ψ separately in English, but it is clear from (9#)/(9*) that (9) is analyzed in terms of temporal quantifiers and extensional predicates in a way analogous to a conjunctive combination of (19) and (20).²² Φ quantifies over temporal indices, and Ψ relates numbers. Φ is as intensional as ‘necessarily’ and ‘possibly,’ whereas Ψ is as extensional as ‘is a teacher’ and ‘is Macedonian.’

It is now easy to see that the inference from (9) and (10) to (17) is invalid, just as the inference from (19) and

(29) Aristotle is the teacher of Alexander.

to

(30) Aristotle is necessarily a teacher

is invalid. The case of (1) is exactly parallel.

It is a mistake to be skeptical about using modal sentences as a guide in analyzing Partee sentences. The latter crucially involve temporal indices, and a significant amount of parallel between modality and temporality in general is well known. If we need an explicit bridge between (19)/(20) and (9), consider:

(31) The president has always been male.

(32) The president has sometimes been bearded.

(31) and (32), like (19) and (20), are ambiguous, and we are only interested in the readings according to which (31) is true at t (as uttered at t) iff for any time before or simultaneous with t , the president at that time, if such exists at that time, is male at that time, and (32) is true at t (as uttered at t) iff for some time before or simultaneous with t , the president exists at that time and is bearded at that time:

(31*) $(t)((t\{t \vee t' = t\} \& Ep-t',t') \rightarrow Gp-t',t')$;

(32*) $(\exists t')((t\{t \vee t' = t\} \& Ep-t',t' \& Dp-t',t')$,

where ‘ $Ep-t',t'$ ’ means “the president at t' exists at t' .” ‘ $Gp-t',t'$ ’ means “the president at t' is male at t' ,” and ‘ $Dp-t',t'$ ’ means “the president at t' is bearded at t' .” The definite description here, as before, is understood to have the narrow scope. The structural parity between (31*)/(32*) and (19*)/(20*) is evident. Modal semantics is thus plausibly transferred to the temporal cases: the apparent sensitivity to intensions shown by the apparent verb phrases ‘has

always been male' and 'has sometimes been bearded' is entirely due to the sentential operators 'has always been' and 'has sometimes been,' and the verbs 'is male' and 'is bearded' are perfectly extensional. It is a small step from (31*) and (32*) to (9*).

V. CONCLUSION

One virtue of the sentential operator theory is that it helps us avoid creating a new category of simple predicates. It is a mistake to think that we already have simple intensional predicates outside Partee sentences, in sentences such as:

- (33) The function from times to the number of sleeping students at those times is increasing.

Suppose (33) is true. Then the simple predicate 'increase' is indeed true of the function in question. Does this not make the predicate an intensional predicate? No. It is not an intensional predicate in the sense that it is true of the intension of the subject term. The subject term is 'the function from times to the number of sleeping students at those times' and it "refers to" a function. The predicate 'increase' is true of its "reference," that is, its extension. So it is an extensional predicate.

Fans of functionalism might point out that functionalism makes 'increase' univocal between (9) and (33); it is invariably a predicate of functions. On the other hand, they might continue, the sentential operator theory makes 'increase' ambiguous; in (9) it consists of predicates of numbers and temporal quantifiers, but in (33) it is a predicate of functions. To this I would say that it is peculiar for functionalists to appeal to avoidance of ambiguity as a theoretical advantage. It is integral to functionalism to postulate a new meaning of 'is' for (2) (Montague) or interpret 'the temperature' ambiguously (Thomason). Also, in order to treat 'increase' invariably as a predicate of functions, some functionalists (of a non-Montagovian moderate kind) classify 'increase' sometimes as intensional – as in (1) and (9) – and sometimes as extensional – as in (33) – whereas the sentential operator theory always classifies it as extensional. And Montagovian functionalists have to introduce different complications to explain how (33)

can be true, given that the straightforward application of their move with (1) and (9) would make the subject term in (33) designate a constant function from indices to a function from times to numbers.

Associated with this is the problem of interpreting the metalanguage of a thorough-going Montagovianism. (33), or some sentence like (33), is needed to express what is meant by the sentence in the Montagovian intensional language that translates (9). But (33) does not mean what it appears to mean, if we apply Montague's semantics to it. A thorough-going Montagovian would have the phrase 'the function from times to the number of sleeping students at those times' designate a constant function from indices to a function from times to numbers. After all, (33) is English and Montague's semantics is intended for English. But if (33) is to be interpreted along those lines, it is mysterious how we could ever understand Montague's semantics properly when explicated in English. Remember that this note itself is written in English. It is unhelpful to object that the official specification of the truth condition of (9) is not given by (33) but by a formal semantics for the intensional language into which (9) is translated. This is unhelpful because any formal theory needs to receive an explication in a natural language sooner or later; otherwise, it would remain merely formal, i.e., without content.

NOTES

* A distant ancestor of this paper was presented at the American Philosophical Association Pacific Division Annual Meetings in Seattle in April and at Stockholm University in June, 1996. This paper was partially supported by a research grant from California State University, Northridge.

¹ I thank Barbara Partee for the information in this paragraph. Also see Lewis, 1983, p. 199.

² I am also assuming that numerals like '90' designate a number.

³ The numeral '90' designates a constant function from times to numbers. Montague summarizes his theory: "[The phrase] **the temperature** "denotes" an individual concept, not an individual; and **rise**, unlike other verbs, depends for its applicability on the full behavior of individual concepts, not just on their extensions with respect to the actual world and (what is more relevant here) moment of time. Yet the sentence, **the temperature is ninety**, asserts the identity not of

two individual concepts but only of their extensions” (Montague, 1974, pp. 267–268).

⁴ Richmond H. Thomason, in Thomason (1979), construes ‘the temperature’ as ambiguous between designating a function and designating a number, and avoids a non-standard reading of ‘is’ in (2). Also, see Hacking (1975), Barwise et al. (1983, pp. 158–159), and Recanati (1993, pp. 293–296). I consider all these proposals more or less Montagovian in spirit.

⁵ It is worth noting that Russell’s theory of descriptions is perfectly consistent with the functionalist reading of ‘the temperature,’ for one may consistently hold that what uniquely satisfies ‘Tx’ in (1r) is a function and that function also satisfies ‘Rx.’ (1r) is indeed a straightforward Russellian analysis of (1) for a functionalist to have. Of course, one will then need to modify (2r) to ‘ $(\exists!x)Tx \ \& \ (\forall x)(Tx \rightarrow x \text{ is } n)$,’ where the ‘is’ is interpreted in a Montagovian manner. (It is optional to interpret ‘n’ as Montague does.) But Russell’s theory of descriptions certainly allows it. It is therefore a mistake to say that Russell’s theory of descriptions “does not allow one to account for functional uses in any straightforward manner” (Recanati, 1993, p. 293).

M.J. Cresswell, in Cresswell (1975), assumes, in effect, that functionalists are committed to the claim that phrases such as ‘the temperature’ in (1) are names. As is clear from the foregoing, his assumption is incorrect.

⁶ Jackendoff (1979) offers an additional, third objection against functionalism. I find this third objection, which is based on the distinction between expressions of motion and expressions of extent, less compelling. I therefore refrain from discussing it.

⁷ ‘The price of milk is rising’ is an example used in Lewis (1983, p. 199).

⁸ This example is due to Barwise et al. (1983, p. 158). I changed the tense of the main verb.

⁹ This confirms that the ‘is’ in (10) expresses a symmetric relation.

¹⁰ We may perceive numerals, or at least, numeral tokens, and utter the word ‘number’ when we mean “numeral.” But this is irrelevant, as numerals are not numbers and it is numbers that are in question.

¹¹ We could even stick to the temperature example, modifying (1) and (2) only slightly:

(1’) The measure of the temperature (in Fahrenheit) is increasing;

(2’) The measure of the temperature (in Fahrenheit) is 90.

It is considerably less plausible to suggest that (2’) is a truncated version of ‘The measure of the temperature (in Fahrenheit) is at 90,’ which hardly sounds intelligible. Also, the measure of the temperature (of the water) is not something that we can feel. So, Jackendoff’s objections fail to apply.

¹² Except between direct quotation marks. Henceforth, I shall ignore direct quotational contexts.

¹³ This is not intended as a denial of Russell’s theory of descriptions. See the first paragraph of section II.

¹⁴ Frege, 1892.

¹⁵ David Kaplan says, “An alternative to Montague’s solution would take ‘the temperature’ and ‘ninety’ both to designate a number (the unit, degrees Fahrenheit, is tacit in the terms); the name rigidly and the description flaccidly. The ‘is’ of the first premise [‘the temperature is ninety’] then *is* the ‘is’ of identity. The predicate ‘is rising’ must be regarded as producing an intensional context, but it receives the now standard treatment” (Kaplan, 1973, p. 517, note 28, emphasis his). The account I propose in the next section is not incompatible with this Kaplanian spirit, even though its implementation of what he calls “the now standard treatment” may not be what Kaplan had in mind.

¹⁶ Some might say that there is an obvious alternative to functionalism that uses the “reference”-oriented strategy. Suppose that we are strolling along Elm Street. We start at the east end of the street and walk westward, observing the street numbers of the houses as we pass by. And we say:

- (A) The street numbers on the south side of Elm Street are increasing from east to west.

Suppose also that the following is true:

- (B) The street numbers on the south side of Elm Street from east to west are 100, 102, 104, . . . , 896, and 898 in that order.

(A) and (B) do not even appear to generate an absurdity the way (9) and (10) do. What appears to follow from them is:

- (C) 100, 102, 104, . . . , 896, and 898 in that order are increasing.

This does not seem absurd. It certainly does not seem as absurd as (17). It is intelligible to regard increasing as a property of an ordered tuple of numbers, or perhaps, a relation among numbers. This impression is strengthened by replacing (C) with:

- (D) 100, 102, 104, . . . , 896, and 898 in that order are increasing numbers.

(D) seems obviously intelligible and evidently true in the envisioned situation. This suggests a provocative possibility, namely, that (9) under the intended reading is equivalent to:

- (E) The numbers of sleeping students are increasing.

The order among the numbers is indicated by the present progressive tense, to be the same as the temporal order of the corresponding times. Perhaps, (9) is to be understood as (E) at a deep, logical level. Or perhaps, (9) is to be replaced with (E); perhaps, whenever people utter sentences like (9), they really mean to utter sentences like (E), and whenever people accept sentences like (9), they really mean to accept sentences like (E). In any case, if (9) is somehow a misleading or confused version of (E), no absurdity threatens, and the extensionality of ‘increase’ is safe and sound; ‘increase’ is a simple extensional predicate.

This is an attractive “reference”-oriented proposal for (E) and other sentences like it. Moreover, apparently among some ordinary Americans, locutions like (E) are rapidly replacing locutions like (9). But, for many native English speakers,

(E) is still unacceptable as a paraphrase of, or replacement for, (9). We need a separate analysis of (9) that does not assume its equivalence with (E).

¹⁷ As for those worlds in which there is no such teacher, the function may be undefined or map them to some object specifically picked for this purpose. We need not concern ourselves with this particular detail.

¹⁸ This clause may well be redundant. I leave it in for the sake of uniformity with (19*).

¹⁹ Cognoscenti may notice that we are collapsing the time of utterance and the time of evaluation into one, viz., t . Our modal examples are coarse enough for us to suppress the world of utterance.

²⁰ For example, only the second half of a particular class period may be in question rather than the entire class period. The uniqueness of N_t is an ideal, just like the uniqueness of an accessibility relation appropriate on a typical occasion of utterance of a modal sentence with no occurrence of an explicit modifier of a modal word (such as 'physically'). When uniqueness fails, we may simply take N_t to be any appropriate neighborhood of t , just as we may take 'A' in (23) to express any appropriate accessibility relation.

²¹ Suppose that the number of sleeping students in a particular room is in question. Then, if the intended room did not exist, no number, not even zero, would number sleeping students in it.

²² It may be open to debate whether the proposed analysis of (9) is *exactly* right. But, details aside, if *something like* (9#)/(9*) can be regarded as the correct analysis of (9), the sentential operator theory stands.

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