CHAPTER TWO

LOGIC

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INTRODUCTION

On the first day of the class for Introduction to Philosophy, your professor tells you that if you keep perfect attendance, complete every homework satisfactorily, participate in class discussion actively, and score 100% in every examination, you will certainly get an *A*+ for the course. You work hard and by the end of the semester, you think you have accomplished all these things. You are pleased. Why? Because you think as follows: "I have kept perfect attendance, completed every homework satisfactorily, participated in class discussion actively, and scored 100% in every exam. This means I get an *A*+ for the course because, as the professor said, if I keep perfect attendance, complete every homework satisfactorily, participate in class discussion actively, and score 100% in every exam, I get an *A*+ for the course."

As you walk to the professor's office to ascertain your course grade, you start wondering about how your classmate Bianca did. So before inquiring about your own grade, you ask the professor, "How did Bianca do for the course?" The professor says, "I am not allowed to discuss any student's grade publicly. All I can tell you is that a majority of mathematics majors

got an *A*- and a majority of philosophy majors got an *A*+.” Hearing this, a smile appears on your face. Why? Because you are happy for your friend as you think as follows: “Bianca is a mathematics major. This means that she probably got an *A*-, because, as the professor informs me, a majority of mathematics majors got an *A*-.”

WHAT IS LOGIC?

Any sustained train of thought on any subject matter is likely to constitute an instance of *reasoning*. **Reasoning** consists of one or more *arguments*. An argument in the sense relevant here is not a quarrel. An **argument** is a set of statements one of which is called the **conclusion** and the others are called the **premises**. Intuitively, the premises are supposed to provide rational grounds for accepting the conclusion. Good arguments do in fact provide such grounds and bad arguments fail to do so. What makes good arguments good and bad arguments bad? Logic is the discipline that is charged to answer this question.

There are two types of argument: **deductive** and **inductive**. The argument implicit in the train of thought described in the first paragraph of this chapter is an instance of a deductive argument. The argument in the second paragraph is an instance of an inductive argument. The difference between the deductive and the inductive lies in the degree of rational grounding of the conclusion intended to be provided by the premises. The premises of a deductive argument are intended to lead to the truth of the conclusion as a matter of absolute necessity, whereas the premises of an inductive argument are intended to lead to the truth of the conclusion as a matter of high likelihood. Thus, a completely successful deductive argument establishes the truth of the

conclusion in a secure manner that is watertight. A completely successful inductive argument, on the other hand, provides an overwhelmingly good support for the truth of the conclusion but does not completely exclude the possibility of its falsity.

We acquire knowledge in a variety of ways. One way of knowledge acquisition is logical reasoning, by means of a deductive argument or an inductive argument. Logical reasoning plays an indispensable role in broadening our knowledge significantly. Without logical reasoning, our knowledge would be confined to an extremely limited realm of what is immediately given to us, either in direct sense perception, introspection, or in some other manner.

We will explore some basic elements of both deductive and inductive arguments. Let us start with deductive arguments.

DEDUCTIVE ARGUMENTS

Validity

The first argument at the beginning of this chapter may be regimented as follows:

The Argument on Your Grade:

1.1 If I have kept perfect attendance, completed every homework satisfactorily, participated in class discussion actively, and scored 100% in every exam, then I get an *A*+.

1.2 I have kept perfect attendance, completed every homework satisfactorily, participated in class discussion actively, and scored 100% in every exam.

So, 3. I get an *A*+.

The premises in this argument provide a rational grounding of its conclusion that is watertight. This means that it is necessarily the case that if 1 and 2 are both true, then 3 *must* be true. That is, the truth of both premises would compel the truth of the conclusion as a matter of necessity. In other words, it is absolutely *impossible* for 1 and 2 to be both true and 3 false at the same time. Thus, we arrive at the definition of our first key term:

> An argument is **valid** if and only if it is impossible for the premises to be true and the conclusion false.

To determine whether a given argument is valid, first, imagine a world in which all premises of the argument in question are stipulated to be true. Next, try to imagine that the conclusion of the argument is false in that same world. If you succeed in doing so without contradicting yourself, you have successfully imagined a possible world in which the premises are true and the conclusion is false. That is, you have successfully shown that the truth of the premises does not necessarily compel the conclusion to be true. So you say that the argument is invalid. If, on the other hand, you find that you cannot avoid contradicting yourself in imagining such a world, you must say that such a world is an impossible world and therefore the argument is valid.

Every successful deductive argument is valid. The Argument on Your Grade is valid. Does validity guarantee the truth of the premises? Does validity guarantee the truth of the conclusion? The answer to both of these questions is "No." In fact, validity is compatible with a variety of combinations of truths and falsities in the premises and the conclusion. Some valid arguments have nothing but true premises and a true conclusion.

Example: If you are human, you are mortal.
You are human.
Therefore, You are mortal.

Some valid arguments have at least one false premise and a true conclusion.

Example: If you are an iguana, you are mortal.
You are an iguana.
Therefore, You are mortal.

Some valid arguments have at least one false premise and a false conclusion.

Example: If you are human, you are immortal.
You are human.
Therefore, You are immortal.

[Give an example of a valid argument all of whose premises are false and whose conclusion is false. Give an example of a valid argument all of whose premises are false and whose conclusion is true. Are there valid arguments with no false premise and a false conclusion? Why?]

The notions of "entailment" and "following from" are definable in terms of validity in obvious ways:

> Statements $S_1, S_2, \ldots, S_n$ **entail** a statement S_k if and only if the argument whose premises are $S_1, S_2, \ldots, S_n$ and whose conclusion is S_k is valid.

> A statement S_k **follows from** statements $S_1, S_2, \ldots, S_n$ if and only if the argument whose premises are $S_1, S_2, \ldots, S_n$ and whose conclusion is S_k is valid.

For example, "If you are human, you are immortal" and "You are human" entail "You are immortal." "You are immortal" follows from "If you are human, you are immortal" and "You are human." [What do "All walruses swim" and "I am a walrus" entail? What follows from the two statements, "If you are an egg man, you are bald" and "You are not bald"? What follows from the three statements, "No egg man is a walrus," "All of your friends are egg men," and "I am a walrus"?]

A **deductive** argument is an argument that is purported to be valid. Every invalid deductive argument is a failed argument. [Is the conclusion of every failed argument false? Why?]

Soundness

Suppose that you are mistaken about your own performance in the final exam and that in fact you only scored 80%. Is the Argument on Your Grade invalid in that case? Of course not. It is still necessarily the case that if 1 and 2 were both true, then 3 would have to be true. That is, it is still impossible for 1 and 2 to be true and 3 false. Now suppose that scoring 80% in the final exam turns out to be enough for you to get an *A*. Consider the following argument, which you should have made:

The Better Argument on Your Grade:

1. If I have kept perfect attendance, completed every homework satisfactorily, participated in class discussion actively, and scored 100% in every exam except scoring 80% in the final exam, then I get an *A*.

1.1 I have kept perfect attendance, completed every homework satisfactorily, participated in class discussion actively, and scored 100% in every exam except scoring 80% in the final exam.

So, 3. I get an *A*.

This argument is valid. It is also better than the Argument on Your Grade. Why? Because its premises are all true, whereas one of the premises of the Argument on Your Grade is false. [Which one?] Thus, we arrive at the definition of our second key term:

An argument is **sound** if and only if it is valid and its premises are all true.

The Better Argument on Your Grade is sound, while the Argument on Your Grade is valid but unsound.

We observed that validity does not guarantee the truth of the conclusion. Unlike validity, soundness does guarantee that. In other words, the conclusion of every sound argument is true. Here is a proof: Take any sound argument. By the definition of soundness, it is valid and its premises are all true. So by the definition of validity, it is impossible for its premises to be true and the conclusion false. This means that since its premises are in fact true (from the second half of the definition of soundness), its conclusion must also be true.

A sound argument conclusively establishes the truth of its conclusion.

[Suppose we have an unsound argument. Does this mean that the conclusion of that argument must be false? Why?]

ARGUMENTS AND CORRESPONDING CONDITIONALS

It is important to distinguish arguments from *conditional statements*. If Mother says, "Since you eat donuts for breakfast, you care little about your health," she is giving an argument. She is making two statements -- viz., that you eat donuts for breakfast and that you care little about your health -- in such a way that the first statement is her premise and the second statement is her

conclusion. In other words, she is asserting the following: You eat donuts for breakfast, and therefore, you care little about your health. In contrast, suppose Father says, "If you eat donuts for breakfast, you care little about your health." Father is only making one statement, a conditional statement. He is saying that if something is the case, then something else is the case. This is not an argument. He is not asserting some statement as a conclusion on the basis of another statement. Unlike Mother, Father is asserting only one statement. By definition, an argument must consist of at least two statements one of which is a premise and the other is the conclusion. So, what Father says is not an argument. Mother is committed to the truth of the statement that you eat donuts for breakfast and the truth of the statement that you care little about your health, whereas Father is committed to the truth of neither statement. Instead, he is committed to the truth of a different, conditional statement, namely, that if you eat donuts for breakfast, then you care little about your health. Thus, what Mother says and what Father says are different.

But is there not some important connection between what Mother says and what Father says? Yes indeed, there is. Mother's utterance has the form "Since *P*, *Q*," where *P* is the premise and *Q* is the conclusion. So, it can be regimented as "*P*. Therefore, *Q*." Call this *Mother's Argument*. On the other hand, Father's utterance has the form "If *P*, (then) *Q*." Call this *Father's Conditional*. Father's Conditional consists of the premise of Mother's Argument occurring in the **antecedent** position (the "if" part) and the conclusion of Mother's Argument occurring in the **consequent** position (the "then" part). We express this fact by saying that Father's Conditional is the *corresponding conditional* of Mother's Argument. In general:

For any argument "$P_1, P_2, \ldots, P_n$. Therefore C," **the corresponding conditional** of it is the statement "If P_1 and P_2 and ... and P_n, then C."

TRUTH-FUNCTIONALLY VALID FORMS

Compare the following two arguments:

If the dilithium crystal overheats, the warp engine core explodes.
The dilithium crystal overheats.
Therefore, The warp engine core explodes.

If nobody goes to Coney Island, Coney Island is not crowded.
Nobody goes to Coney Island.
Therefore, Coney Island is not crowded.

These arguments are valid. Moreover, they are valid for the same reason. In fact, the Argument on Your Grade and the Better Argument on Your Grade are also valid for that same reason. That is, they all share the following *argument form*:

If P, then Q.
P.
Therefore, Q.

Any argument of this form is valid. [Really? Try to think of an invalid argument of this form.] This is not the only argument form that guarantees validity. There are many such forms. But some validity-guaranteeing argument forms tend to occur more frequently than others in our reasoning. It is therefore useful to become familiar with those frequently occurring valid forms and able to recognize them wherever they occur. They are *Modus Ponens*, *Modus Tollens*, Hypothetical Syllogism, Dilemma, Simplified Dilemma, and Disjunctive Syllogism.

Modus Ponens

If *P*, then *Q*.
P
Therefore, *Q*.

This is the form we have just discussed. *Modus Ponens* is perhaps the most widely used argument form.

Modus Tollens

If *P*, then *Q*.
It is not the case that *Q*.
Therefore, It is not the case that *P*.

Example: If you are an iguana, then you are a reptile.

You are not a reptile.

Therefore, You are not an iguana.

Another Example: If nobody goes to Coney Island, Coney Island is not crowded.

Coney Island is crowded.

Therefore, Somebody goes to Coney Island.

Notice that both *Modus Ponens* and *Modus Tollens* contain one premise that is a conditional statement. In *Modus Ponens* (which means "affirming mode"), the other premise affirms the antecedent, whereas in *Modus Tollens* (which means "denying mode"), the other premise denies the consequent.

Hypothetical Syllogism

If *P*, then *Q*.

If *Q*, then *R*.

Therefore, If *P*, then *R*.

Example: If you watch too much TV, then your brain turns to mush.

If your brain turns to mush, then you get an *F* in philosophy.

Therefore, If you watch too much TV, then you get an *F* in philosophy.

Another Example: If we are not responsible for our actions, morality crumbles.

If our will is not free, we are not responsible for our actions.

Therefore, If our will is not free, morality crumbles.

Observe that in Hypothetical Syllogism, the two premises and the conclusion are all conditionals. Furthermore, the consequent of one premise is identical with the antecedent of the other premise, and the antecedent of the conclusion is identical with the antecedent of the former premise and the consequent of the conclusion is identical with the consequent of the latter premise. Remember that the premises need not be given in any particular order.

Dilemma

Either P or Q

If P, then R.

If Q, then S.

Therefore, Either R or S.

Example: Either I study or I party.

If I study, I will pass the logic exam.

If I party, I will have a headache.

Therefore, Either I will pass the logic exam or I will have a headache.

Another Example: Determinism is true or everything happens by chance.

If determinism is true, our actions are determined.

If everything happens by chance, our actions happen by chance.

Therefore, Our actions are determined or they happen by chance.

Logicians playfully speak of Dilemma as a kind of beast with two horns. "Either *P* or *Q*": Here *P* and *Q* are said to be the two horns of the dilemma. "If *P*, then *R*": If we seize one horn, namely *P*, we get *R*. "If *Q*, then *S*": If we seize the other horn, namely *Q*, we get *S*. Therefore, we get either *R* or *S*.

Simplified Dilemma

Either *P* or *Q*.

If *P*, then *R*.

If *Q*, then *R*.

Therefore, *R*.

Example: Either you get rice or potatoes as your side dish.

If you get rice as your side dish, you are happy.

If you get potatoes as your side dish, you are happy.

Therefore, You are happy.

Another Example: The universe will oscillate forever or end in a Big Crunch.

I am astounded if the universe will oscillate forever.

I am astounded if the universe will end in a Big Crunch.

Therefore, I am astounded.

In Simplified Dilemma, seizing either horn of Dilemma gets us the same thing, namely *R*.

Disjunctive Syllogism

Either *P* or *Q*.

It is not the case that *P*.

Therefore, *Q*.

Example: They are smart or they are gorgeous.

They are not smart.

Therefore, They are gorgeous.

Another Example: Something happens by miracle or everything obeys the laws of nature.

Nothing happens by miracle.

Therefore, Everything obeys the laws of nature.

[Give more example arguments of each of these forms.]

FALLACIES

While some argument forms are valid argument forms, some other argument forms are invalid argument forms. Those invalid argument forms are fallacious argument forms, or *fallacies*. This does not mean, however, that all fallacies are invalid argument forms. Among various fallacies, three in particular deserve special mention. The first two are invalid but easily confused with *Modus Ponens* and *Modus Tollens*. The third fallacy is special in that it is valid but is unacceptable for a different reason.

The Fallacy of Affirming the Consequent

If *P*, then Q.
Q.
Therefore, *P*.

Example: If you are an iguana, then you are mortal.
You are mortal.
Therefore, You are an iguana.

The invalidity of this argument is not hard to see. The premises are both true and the conclusion

is false. No valid argument can have all of its premises true and its conclusion false. [Why?] So, this argument is invalid.

Another Example: We have conscience if God created us.
We have conscience.
Therefore, God created us.

This argument is invalid, for it has the same logical form as the previous example, which is clearly invalid. Remember it is a matter of logical form whether a given argument is valid or not. Notice that like *Modus Ponens* and *Modus Tollens*, the Fallacy of Affirming the Consequent has two premises one of which is a conditional. But unlike *Modus Ponens* or *Modus Tollens*, the other premise of the Fallacy of Affirming the Consequent is the consequent of the conditional, not the antecedent or the negation of the consequent. Also, the conclusion is the antecedent of the same conditional, not the consequent or the negation of the antecedent.

The Fallacy of Denying the Antecedent

If *P*, then *Q*.
It is not the case that *P*.
Therefore, It is not the case that *Q*.

Example: If you are an iguana, then you are mortal.

You are not an iguana.

Therefore, You are immortal.

Another Example: Our will is not free if we do not have the sense of freedom.

We have the sense of freedom.

Therefore, Our will is free.

As before, we can easily see that the first example is invalid because it has all premises true and the conclusion false, and the second example is invalid because it shares the logical form with the first example. Again notice that like *Modus Ponens* and *Modus Tollens*, the Fallacy of Denying the Antecedent has two premises one of which is a conditional. But unlike *Modus Ponens* or *Modus Tollens*, the other premise of the Fallacy of Denying the Antecedent is the negation of the antecedent of the conditional, not the antecedent itself or the negation of the consequent. Also, the conclusion is the negation of the consequent of the same conditional, not the consequent itself or the negation of the antecedent.

Begging the Question

.

.

.

P.

.

.

.

Therefore, *P*.

Example: Killing is not good.

We do not approve of killing.

To say that something is good is to say that we approve of it.

Therefore, Killing is not good.

Another Example: Men box for manly reasons.

Women should not box.

If women box, men will feel violated.

Therefore, Women should not box.

Begging the Question is a fallacy. It is unique among fallacies in being valid. In fact, the very reason for its assured validity is also the reason why it is a fallacy. In Begging the Question, the conclusion appears as one of the premises. This obviously makes it a valid argument but also obviously makes it a mistaken argument. The mistake consists in the fact that what is to be established (the conclusion) is among the statements that are assumed (the premises). Begging the Question assumes what is to be argued for in the conclusion. That is viciously circular. Notice that many people completely misunderstand the verb phrase "to beg the question." They

think it means something like "to raise the question." They are wrong. The verb "to beg" here does not mean "to ask for" but instead it means "to steal." Begging the Question "steals" the conclusion and smuggles it in as a premise. It settles the question at hand right from the start by assuming a particular resolution of it as a premise.

TRUTH-FUNCTIONAL OPERATORS

The validity of the valid argument forms we have seen so far is the result of the meanings of the expressions "if ..., then ..." (*conditional*), "either ... or ..." (*disjunction*), and "it is not the case that ..." (*negation*). These expressions, as well as "both ... and ..." (*conjunction*), are known as **truth-functional operators**. This means that the truth or falsity of a statement containing such an expression is completely determined by the truth or falsity of the constituent statement(s). For example, the statement "It is not the case that Los Angeles is evil" is true if the constituent statement "Los Angeles is evil" is false, and is false if "Los Angeles is evil" is true. Take another example: "Both Delaware is precious and California is decadent." This statement is true if the two constituent statements "Delaware is precious" and "California is decadent" are both true, and is false if at least one of the two constituent statements is false.

REDUCTIO AD ABSURDUM

There is an especially powerful valid deductive argument form called ***reductio ad absurdum*** ("reduction to absurdity"). Suppose you want to show that *P* but do not know how to argue for

it directly. You may consider using *reductio ad absurdum.* The first thing you do is to assume that it is not the case that *P*. Then combine this assumption with a set of statements that are known to be true, and derive from them by valid argumentation a statement that is a contradiction. Since no contradiction is validly derivable from true statements, you conclude that the initial assumption, that it is not the case that *P*, is not true. That is to say, you conclude that it is the case that *P*. Schematically, we may put this as follows:

"Not-*P* & *A* & *B* & ... & *M*"

entails

"Both *X* and not-*X*"

Here "*P*" is the conclusion you want to establish, and "*A*," "*B*," ..., and "*M*" are known to be true. By the definition of entailment, it is impossible for "Not-*P* & *A* & *B* & ... & *M*" to be true and "Both *X* and not-*X*" false. But "Both *X* and not-*X*" is a contradiction, that is, it is necessarily false, which means that it is impossible not to be false. So, it must be impossible for "Not-*P* & *A* & *B* & ... & *M*" to be true. So, it is impossible for all of "Not-*P*," "*A*," "*B*," ..., and "*M*" to be true. But "*A*," "B," ..., and "*M*" are known to be true. Therefore, "Not-*P*" is not true. That is, "*P*" is true.

Example: Suppose your friend maintains that every opinion is equally correct. You disagree but do not know how to argue against her directly. So you try *reductio ad absurdum*. You say to her, "Let us assume that you are right, that is, that every opinion is equally correct. Now, my

opinion that you are wrong is an opinion, so it is correct. That is, it is correct to say that you are wrong. So, you are wrong. Thus, you are right and you are wrong, which is a contradiction. Therefore by *reductio*, the initial assumption that you are right must be rejected. That is, you are wrong.

Another Example: Suppose your friend insists that nothing exists. You try to persuade her out of this unusual opinion by pointing to a tree nearby and saying, "You see, this tree exists. Therefore, something exists." Your friend is not impressed. She says, "I seem to see a tree there but that is an illusion. There is really no tree there at all." You make your friend touch the tree, but she remains unconvinced, "The sense of touch is just another sense. All of my five senses are subject to deception." At this point you realize the radical nature of her opinion and switch tactics. You will now employ *reductio ad absurdum* against her opinion. "Assume that there is nothing," you begin. "Is it true that even though there is no tree in front of you, you have a sensory impression of a tree?" Your friend replies, "That's right." You say, "So, there is no tree but there is some sensory impression. Right?" Again she says, "Yes." You continue, "So, there is some sensory impression, obviously?" She concurs, "Obviously." You finish your *reductio* as planned; "So, there is something, namely, a sensory impression. So, there is nothing and there is something, which is a contradiction. Therefore by *reductio*, the initial assumption that there is nothing must be rejected. That is, there is something."

Another Example: Here is yet another *reductio* argument you can make against her: "Assume that there is nothing. That is your opinion. That is, you hold the opinion that there is nothing.

So some individual holds that opinion. So, there is something, namely, some individual holder of the opinion. So, there is nothing and there is something; a contradiction. Therefore, there is something. □

NECESSARY CONDITION / SUFFICIENT CONDITION

Let us say that in order to get an *A*+, you need to keep perfect attendance. In that case we say that your keeping perfect attendance is a *necessary condition* for your getting an *A*+. Can we conclude from this validly that if you keep perfect attendance, you get an *A*+? No. Why? Because you might need to do something else, like doing well in the exams. To say that keeping perfect attendance is a necessary condition for getting an *A*+ means nothing more than that if you do not keep perfect attendance, you do not get an *A*+. It does not mean that if you keep perfect attendance, you get an *A*+. Suppose, as we assumed earlier, that you need to complete every homework satisfactorily, participate in class discussion actively, and score 100% in every exam to get an *A*+. Then your completing every homework satisfactorily is another necessary condition for your getting an *A*+. But you must also participate in class discussion actively and score 100% in every exam. That is, if you do not complete every homework satisfactorily, do not participate in class discussion actively, or do not score 100% in every exam, you do not get an *A*+. Suppose further that there is nothing else you need to do to get an *A*+. Then all these four necessary conditions jointly constitute a *sufficient condition* for your getting an *A*+. Can we conclude from this validly that if you keep perfect attendance, complete every homework satisfactorily, participate in class discussion actively, and score 100% in every examination, then

you get an *A*+? Yes. Thus we have the following pair of definitions:

"*P* is a **necessary condition** for *Q*" means "If it is not the case that *P*, then it is not the case that *Q*."

"*P* is a **sufficient condition** for *Q*" means "If *P*, then *Q*."

Another way of saying that *P* is a necessary condition for *Q* is to say, "*Q* only if *P*": e.g., "You get an *A*+ only if you keep perfect attendance." Do not confuse this with "*Q* if *P*," which means the same as "If *P*, then *Q*," which means that *P* is a sufficient condition for *Q*.

If *P* is a necessary condition for *Q* and also a sufficient condition for *Q*, then *P* is a *necessary and sufficient* condition for *Q*. In the above example, your keeping perfect attendance, completing every homework satisfactorily, participating in class discussion actively, and scoring 100% in every examination is a necessary and sufficient condition for your graduation.

"*P* is a **necessary and sufficient condition** for *Q*" means "*P* if and only if *Q*."

[Why?] If *P* is a necessary and sufficient condition for *Q*, then *Q* is a necessary and sufficient condition for *P*. [Why?]

QUANTIFICATIONALLY VALID FORMS

Not all valid argument forms are truth-functional. The validity of some valid argument forms is guaranteed by the meanings of such expressions as "every" (the *universal quantifier*) and "some" (the *existential quantifier*), in addition to the truth-functional operators. Here are two of the standard valid argument forms of that type, Universal Instantiation and Universal Syllogism:

Universal Instantiation

Every *F* is *G*.
Therefore, If α is *F*, then α is *G*.

Example: Every whale is a mammal.
Therefore, If Walt is a whale, Walt is a mammal.

Simplified Example: Everything is in space and time.
Therefore, Bianca is in space and time.

Any statement of the form "Every *F* is *G*" (or its simplified kin "Everything is *G*") is called a **universal quantification**. Here are some more example universal quantifications: "Every philosophy major is a student," "Every mother is a woman," "Everything is blue," "Everything is a number." Because of the validity of Universal Instantiation, any universal quantification is subject to refutation by a *counterexample.* Intuitively, a counterexample is an example that refutes the statement in question.

A **counterexample** to a statement of the form "Every *F* is *G*" is an item that is *F* and not *G*.

A **counterexample** to a statement of the form "Everything is *G*" is an item that is not *G*.

For instance, anything that is a romance novel and is not banal is a counterexample to the statement "Every romance novel is banal." Anything that is not blue is a counterexample to the statement "Everything is blue." [Give a counterexample to the universal quantification "Every act performed with a good intention is morally good." Give a counterexample to the universal quantification "Everything is in space and time."]

Any statement of the form "Some *F* is *G*" (or its simplified kin "Something is *G*") is called an **existential quantification**. Here are some example existential quantifications: "Some mammal is a whale," "Some student is a philosophy major," "Some woman is a mother," "Something is blue," "Something is an abstract entity." Existential quantifications are not subject to refutation by a counterexample. The reason is that existential quantifications yield no entailments concerning any given item. Take "Some philosopher is weird" as an example. We cannot refute this claim by producing a particular philosopher who is not weird, for the claim does not entail that *that* particular philosopher is weird. On the other hand, if we examined all philosophers and concluded none of them was weird, then we would have refuted the claim. Another way to refute the same claim is to use *reductio* and argue by assuming that some philosopher is weird and deriving a contradiction from this assumption coupled with some known truths. (We are highly unlikely to succeed in this particular case.)

Universal Syllogism

Every *F* is *G*.

α is *F*

Therefore, α is *G*.

Example: Every whale is a mammal.

Walt is a whale.

Therefore, Walt is a mammal.

The validity of Universal Syllogism is assured by the validity of Universal Instantiation and *Modus Ponens*. Here is how: Suppose that (1) every whale is a mammal and (2) Walt is a whale. Then from (1) by Universal Instantiation, it follows that if Walt is a whale, then Walt is a mammal. From this and (2) by *Modus Ponens*, it follows that Walt is a mammal.

Universal Quantification and Negation

Be careful about sentences of the form "Every *F* is not *G*." Take, for example, "Every student is not a woman." This sentence is ambiguous. It could mean either "Every student is a non-woman" or "It is not the case that every student is a woman." The first reading makes the sentence true just in case no student is a woman, whereas the second reading makes it true just in case there is at least one student who is not a woman. The same kind of ambiguity is present in

the simplified form "Everything is not *G*." The sentence "Everything is not blue" may be interpreted as "Everything is non-blue" or as "It is not the case that everything is blue." The first is true just in case nothing is blue, while the second is true just in case there is at least one thing that is not blue.

"Some" and "Not all"

It is a common mistake to think that "some" entails "not all," that is, to think that the following argument form is a valid argument form:

Some *F* are *G*.
Therefore, Not all *F* are *G*.

Example: Some students are philosophy majors.
Therefore, Not all students are philosophy majors.

The easiest way to see why this is **not** a valid argument form is by *reductio ad absurdum*. Assume for *reductio* that the above is a valid argument form. Then for any argument of that form, it is impossible for the premise to be true and the conclusion false. So, it is impossible for the premise of the following argument to be true and its conclusion false:

Some students are women.

Therefore, Not all students are women.

But it is possible for the premise of this argument to be true and the conclusion false. Therefore, by *reductio*, the above argument form is not a valid argument form. Here is why it is possible for the premise of this argument to be true and the conclusion false: Suppose you visit a small college about which you know little. You look around and discover quickly that the few students you see in the quad are women. So you write on page one of your notebook, "Some students are women." You continue to explore the campus and somehow manage to meet every single one of the students. Unexpectedly they all turn out to be women. So you write on page two of your notebook, "All students are women." Does this mean that you now need to deny your previous statement on page one? Of course not. Your statement on page two is true, but your statement on page one is true, too. You just did not know it was a women's college at first. It simply means that you only had a partial picture of the student population and now you have the whole picture.

IDENTITY

One particular expression occurring in quantificational logic deserves a special treatment. It is "is identical with," which is customarily abbreviated as "=." The identity relation expressed by "=" holds between *x* and *y* just if *x* and *y* are one and the same thing. For this reason, philosophers sometimes call this relation **numerical identity**, and distinguish it from qualitative similarity. Identical twins are not identical, i.e., not numerically identical. In Lewis Carroll's

Through the Looking-Glass, Tweedledum and Tweedledee are identical twins but they are not identical. If they were identical, they would be one and the same, hence they would be one, not twins. Your copy of this textbook and my copy of this textbook are not identical. They, like identical twins, are merely extremely similar. Thus, extreme similarity does not entail identity. However, identity entails extreme similarity. In fact, it entails absolute indiscernibility. If $x = y$, then x and y are completely indiscernible, because x and y share all traits. This is a famous principle named after Gottfried Wilhelm Leibniz (1646-1716):

Leibniz's Law of Indiscernibility of the Identical:

For every x, for every y, if $x = y$, then x and y have exactly the same properties.

If Lewis Carroll = Charles Dodgson, then Carroll and Dodgson share exactly the same properties. But is that really so? Isn't it true to say that Carroll wrote *Through the Looking-Glass* but false to say that Dodgson wrote it? Let us be careful here. If Carroll and Dodgson are one and the same individual, how could Carroll do anything without Dodgson also doing that very thing? He could not. When Carroll was writing *Through the Looking-Glass*, Dodgson was writing the same story at the same time at the same location in the same way. There was only one individual doing that, namely Carroll, viz., Dodgson. That man wrote the story under the name "Lewis Carroll." That is, Carroll wrote the story under the name "Lewis Carroll" and Dodgson wrote it under the name "Lewis Carroll." There is nothing Carroll did but Dodgson did not.

Leibniz's Law of Indiscernibility of the Identical is useful in demonstrating the

distinctness of two easily confused things. For example, a certain type of materialists claim that all mental phenomena are identical with some neurophysiological phenomena. Their opponents typically attempt to show that some mental phenomenon has a certain property no neurophysiological phenomenon has. If such an attempt is successful, Leibniz's Law of Indiscernibility of the Identical will entail, via Universal Instantiation and Modus Tollens, that the mental phenomenon in question is not identical with any neurophysiological phenomemon. [How so?]

There is a converse principle, which also bears the name of Leibniz and appears equally plausible:

Leibniz's Law of Identity of the Indiscernible:

For every x, for every y, if x and y have exactly the same properties, then $x = y$.

This principle may appear to entail that identical twins are identical, but in fact it does not entail that. There are many properties identical twins fail to share. One such property is spatial location. No matter how similar they may be, identical twins will occupy different spatial locations. Thus, among actually existing objects, there do not seem to be two of them that have exactly the same properties. But could there possibly be such objects? If there could be, the above principle is not necessarily true. That is, it is not a principle of logic. [Can you imagine a logically possible world in which two distinct objects share exactly the same properties?]

INDUCTIVE ARGUMENTS

Strength

The second argument at the beginning of this chapter may be regimented as follows:

The Argument on Bianca's Grade:

1.1 A majority of mathematics majors got an *A*-.

1.2 Bianca is a mathematics major.

So, 3. Bianca got an *A*-.

In this argument, the rational grounding of the conclusion by the premises is less than watertight. It is not necessarily the case that if 1 and 2 are both true, then 3 must be true. Even if 1 and 2 are both true, it is still possible for 3 to be false. Bianca might be among the few mathematics majors who did not get an *A*-. Such an eventuality is not impossible, even though it may be *improbable* under the circumstances. This means that the Argument on Bianca's Grade is invalid. But this does not mean that it is a bad argument. It is unlikely that the Argument on Bianca's Grade is intended as a deductive argument in the first place. We should judge it as an inductive argument instead. Inductive arguments, unlike deductive arguments, are not expected to meet the high standard of validity but a somewhat lower standard of *strength*, defined as follows:

An argument is **strong** if and only if it is improbable for the premises to be true and the conclusion false.[1]

Unlike validity, strength is a matter of degree, as improbability is a matter of degree. An argument is weak to the degree to which it is not strong. The Argument on Bianca's Grade is strong, i.e., it is strong to a degree greater than it is weak. An **inductive** argument is one that is purported to be strong rather than valid. Thus, not every invalid inductive argument is a failed argument. An inductive argument that is weak to a degree is a failed argument to that degree.

Reliability

Suppose that you are mistaken about Bianca's major and that in fact she majors in philosophy, not mathematics. Is the Argument on Bianca's Grade weak then? No. It is still the case that if 1 and 2 were both true, then 3 would probably be true. That is, it is still improbable for 1 and 2 to be true and 3 false. The argument is exactly as strong as before. Remember what the professor told you about the philosophy majors in the class: "A majority of them got an *A*+." Now consider the following argument, which you should have made:

The Better Argument on Bianca's Grade:

1. A majority of philosophy majors got an *A*+.
2. Bianca is a philosophy major.

So, 3. Bianca got an *A*+.

This argument is strong. It is also better than the Argument on Bianca's Grade. Why? Because its premises are all true, whereas one of the premises of the Argument on Bianca's Grade is false. [Which one?] Thus we arrive at the second key definition of inductive logic:

An argument is **reliable** if and only if it is strong and its premises are all true.

Notice that reliability is to strength what soundness is to validity. A reliable argument is reliable to the same degree to which it is strong. The Better Argument on Bianca's Grade is reliable, while the Argument on Bianca's Grade is strong but unreliable.

STRONG FORMS

There are four major types of inductive argument: Enumeration, Statistical Syllogism, Analogical Syllogism, and Inference to the Best Explanation.

Enumeration

This is the most elementary form of induction.

A particular *F* is *G*.

Another *F* is *G*.

Yet another *F* is *G*.

.

.

.

Yet another *F* is *G*.

Therefore, Every *F* is *G*.

Example: Raven #1 is black.

Raven #2 is black.

Raven #3 is black.

.

.

.

Raven #k is black.

Therefore, Every raven is black.

We are said to *generalize* from particular cases (k-many ravens) to obtain the conclusion, which is a universal quantification (about all ravens). It is obvious that the more particular cases we enumerate as constituting the basis of generalization, the stronger the argument is.

Statistical Syllogism

This is a probabilistic version of Universal Syllogism.

Most *F* are *G*.

α is *F*.

Therefore, α is *G*.

Example: Most students are under 40 years old.

Chad is a student.

Therefore, Chad is under 40 years old.

Given the truth of the second premise, the more *F*s (students) we ascertain to be *G* (under 40 years old), the higher the probability is for the conclusion to be true.

Analogical Syllogism

This is an argument based on a comparison between two items.

Most of the attributes belonging to α also belong to β.

α is *F*.

Therefore, β is *F*.

Example: Most of the attributes belonging to Ginger also belong to Cinnamon.

Ginger is good at logic.

Therefore, Cinnamon is good at logic.

Is it the case that, given the truth of the second premise, the more attributes of α we ascertain to belong to β, the more probable the conclusion is? No. Ascertaining that Ginger and Cinnamon are both blonde will not increase (or decrease) the probability of the truth of the conclusion. The attributes in question must be *relevant* to the conclusion. Being blonde is not relevant in this case but, say, being meticulous is.

INFERENCE TO THE BEST EXPLANATION

Inference to the best explanation is an important type of inductive argument. It is hard to classify in relation to the other forms of reasoning, but it is prevalent and powerful.

P.

The best explanation for the fact that *P*, entails that *Q*.

Therefore, *Q*.

Example: The sun appears to move across the sky.

The best explanation for the fact that the sun appears to move across the sky entails that the earth rotates.

Therefore, The earth rotates.

What is it for an explanation to be the best explanation for a given fact? It is for the explanation to be better than any other explanation for that fact. So, the size of the set of rival explanations determines the strength of the inference to the best explanation. The larger the set, the stronger the inference.

A particular explanation for a given fact may be the best explanation for that fact at one time but not at another time. Consider:

The sun appears to move across the sky.
The best explanation for the fact that the sun appears to move across the sky entails that the sun circles the earth.
Therefore, The sun circles the earth.

This argument was fairly reliable when uttered in antiquity; the best explanation for the apparent movement of the sun at that time was geocentric. It is, however, unreliable when uttered now, as the second premise is false when uttered now; the best explanation now is no longer geocentric.

Prediction

Any explanation of a given phenomenon usually gives rise to **predictions** about future phenomena of the same type. If we explain the apparent movement of the sun across the sky by reference to the rotation of the earth, we are in a position to predict that as long as the earth

continues to rotate, the sun will likely continue to appear to move across the sky. If we explain the appearance of thin tracks of vapor inside a cloud chamber by postulating the existence of electrons traveling through the chamber, we are in a position to predict that whenever a cloud chamber is placed in the midst of traveling electrons, thin tracks of vapor will likely appear. If we explain the onset of a certain disease by means of a combination of the presence of a particular genetic material and a particular environmental trigger, we are in a position to predict that whenever the genetic material and the environmental trigger are present, the disease will likely be present. Thus, explanation and prediction go *in tandem*. They are really two sides of the same coin. As a general rule, an explanation that gives rise to poor predictions is a poor explanation, and a prediction that is only supported by a poor explanation is not trustworthy.

Causal Inference

A particular kind of inference to the best explanation deserves to be singled out. It is **causal inference**. It is an inference from a phenomenon to its cause. If it is successful, it allows us to give a **causal explanation** of the phenomenon. Suppose Jane has a back pain. She wants to eliminate it. She tries everything she can think of: massage, yoga, shiatsu, acupuncture, medication, meditation. Nothing works. Finally in desperation she goes to see a chiropractor specialized in applied kinesiology, who determines that her left leg is longer than her right leg by half an inch. Jane then theorizes and concludes that the leg-length discrepancy is the cause of her back pain. Jane has just made a causal inference. The reliability of this inference can be tested by checking whether the predictions it gives rise to are confirmed. The crucial prediction

says that if Jane corrects the leg-length discrepancy by means of an appropriate shoe insert, her back pain will disappear. Suppose she does this. Suppose further that after a month of wearing the insert, the pain persists as before. This disconfirms the prediction and weakens the causal inference in question. She may then try a different causal inference, e.g., that the pain is caused by her wearing high-heels on the job every day. This new inference gives rise to the new prediction that if she stops wearing high-heels, the pain will cease. If this prediction is confirmed, then the new causal inference is vindicated and her back pain may finally be causally explained. If not, it is not causally explained. [Is the inference to the best explanation of the apparent motion of the sun across the sky by reference to the rotation of the earth a causal inference? If so, what is the inferred cause and what is the inferred effect? If not, why not?]

CONCLUSION

The importance of logic was widely known and appreciated in antiquity but Aristotle (384-322 B.C.) was the first to study logic systematically. He discovered logical forms, classified arguments according to their logical forms, and investigated the properties of the logical forms and their relationships to one another. Thus, Aristotle is justifiably known as the Father of Logic. His logic is often called Categorical Logic or Syllogistic Logic. Aristotelian logic was as dominant as Euclidian geometry for more than two thousand years. But in the late nineteenth century, Aristotelian logic was beginning to be replaced by a more powerful system known as First-Order Predicate Logic. First-Order Predicate Logic is the logic which philosophers use today and which we have studied in this chapter under Deductive Arguments. Unlike

Aristotelian logic, no single person was wholly responsible for the development of this new logic. But two logicians among the non-Aristotelian pioneers deserve to be mentioned: Gottlob Frege (1848 - 1925), who played a major role in creating quantificational logic, and Ludwig Wittgenstein (1889 - 1951), who proposed the idea of truth-function. The history of the development of inductive arguments is considerably more complicated and overlaps significantly with the history of the development of scientific methods.[2]

NOTES

For the instructor: I ignore the objection that this definition makes any argument with improbable premises strong. I ignore the distinction between conditional probability and probability of conditional.

[2] I thank Leemon McHenry and Ron McIntyre for useful comments on earlier drafts.

EXERCISE PROBLEMS

1. Is every sound deductive argument valid?
2. Is the conclusion of every valid deductive argument true?
3. Is every strong inductive argument reliable?
4. Is every valid deductive argument sound?
5. Is it possible to make a valid argument invalid by adding a premise? Why? [Hint: Remember that an argument is valid if and only if it is impossible for the premises to be true and the conclusion false. This means that in a valid argument, one can not have true premises and a false conclusion. In view of this, consider the following analogy: Mother tells you, "You can't have ice cream and cookies," to which you respond, "In that case, can I have ice cream, a milkshake, and cookies?" What would Mother say?]
6. Is it always possible to make an invalid argument valid by adding a premise? Why?
7. Is it always possible to make a sound argument unsound by adding a premise? Why?
8. Is it always possible to make an unsound argument sound by adding a premise? Why?
9. Is it always possible to make an unsound argument sound by deleting a premise? Why?
10. Assume the following: (i) If you pass philosophy and physics, you graduate; (ii) If you fail literature or chemistry, you do not graduate; (iii) If you graduate, that means that you passed philosophy or physics. List all necessary conditions for graduation. List some sufficient condition for graduation.
11. Superman = Clark Kent. So by Leibniz's Law of Indiscernibility of the Identical, Clark Kent has every property Superman has. Lois Lane adores Superman. That is, Superman

has the property of being adored by Lois. It then follows that Clark Kent has the property of being adored by Lois. But Lois does not adore Clark Kent. She thinks he is a nerd. Does this refute Leibniz's Law of Indiscernibility of the Identical? Why?

12. For any *x*, *x* is *x*, that is, *x* has the property of being *x*. So, anything that has exactly the same properties as *x* will have the property of being *x*. But to have the property of being *x* is to be identical with *x*. This seems to prove Leibniz's Law of Identity of the Indiscernible. Does it? If it does, does it make the principle uninteresting?

13. Are the following items arguments? If so, should they be considered deductive or inductive? If deductive, are they valid? Can you tell whether they are sound? What argument forms do they instantiate? If inductive, are they strong? Can you tell whether they are reliable? What argument forms do they instantiate?

(1) April will win the race, for she has been undefeated so far this season.

(2) You ate the ice cream. Why did you eat the ice cream? I told you not to eat the ice cream. You are bad. You are really bad.

(3) Every property is possessed by some concrete individual. Unicornhood is possessed by some concrete individual, because it is a property.

(4) You will graduate in June if you pass the exam. You don't pass the exam. That means you won't graduate in June.

(5) If God does not exist, life is meaningless. God does not exists. So, life is meaningless.

(6) Children master their first language, whatever it may be, very quickly under normal circumstances irrespective of their intelligence level. So, we conclude that children have innate knowledge of the grammatical rules common to all humanly possible natural languages, for there is no better way to explain it.

(7) Nim Chimpsky is smart because most chimpanzees are smart. [Hint: An obvious premise is suppressed.]

(8) Get out quickly because there is a fire in the building.

(9) Everything I see is my sensation. I see the chair I am sitting on. So, I am sitting on my sensation.

(10) I have consciousness. You look and behave like me. So you have consciousness.

(11) Either the existentialists are right or the positivists are right. If the existentialists are right, life is absurd. If the positivists are right, metaphysics is nonsensical. It follows that either life is absurd or metaphysics is nonsensical.

(12) The difference between a crazy man and Salvador Dali is that a crazy man is not Salvador Dali. (Salvador Dali)

(13) Since the soul is devoid of matter, it cannot be divided into smaller bits. Nothing can fall apart unless it can be divided into smaller bits. So, the soul cannot fall apart. But all material beings can fall apart. Therefore, the soul is not a material being.

(14) There must be physical objects external to me. Otherwise, how could I have such organized and predictable sensory impressions?

(15) If Plato is right, concrete particulars are not ultimately real. The world we perceive by our five senses is an illusion if concrete particulars are not ultimately real. Hence, if Plato is right, the world we perceive by our five senses is an illusion

(16) I know nothing if I am certain of nothing. I do know something. Therefore, I am certain of something.

(17) Either there is an objective standard for judging art, or it is all a matter of subjective attitude. No objective standard exists for judging art. This means that judging art is purely a matter of subjective attitude.
(18) Every student takes logic. Stu is a student. Therefore, Stu takes logic.
(19) If every student takes logic and Stu is a student, then Stu takes logic.
(20) Santa Claus exists. Here is why: If Santa Claus does not exist, it is false to say that Santa Claus is loved by many; but it is true to say that Santa Claus is loved by many.
(21) Exercising regularly is a necessary condition for being healthy. So you must be healthy, because you exercise regularly.
(22) Captain Jean-Pierre Lucard of the star ship USSOS Undertaking is a person. Commander Datum behaves very much like Lucard. So Datum is a person.
(23) If God exists, evil does not exist. God exists. Therefore, evil does not exist.
(24) If God exists, evil does not exist. Evil exists. Therefore, God does not exist.
(25) God exists. Evil exists. Therefore, it is false that if God exists, evil does not exist.
(26) This apple from the barrel is rotten. That apple from the barrel is rotten. This other one from the barrel is also rotten. So I say all of the apples from the barrel must be rotten.
(27) (In 1846, when the farthest known planet was Uranus, Urbain Jean Joseph Leverrier argued as follows.) The observed orbit of Uranus deviates from the theoretical orbit predicted by the Newtonian physics. Therefore, there is an unknown planet outside the orbit of Uranus exerting its gravitational influence on Uranus. [Can you name the "unknown planet"?]
(28) (In 1845 Leverrier argued as follows.) The observed orbit of Mercury deviates from the theoretical orbit predicted by the Newtonian physics. Therefore, there is an unknown planet inside the orbit of Mercury exerting its gravitational influence on Mercury. [Can you name the "unknown planet"? Can you offer a better account of the discrepancy between the observed orbit and the predicted orbit of Mercury?]

FOR FURTHER READING

Aristotle	*Prior Analytics* *Sophistical Refutations*
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