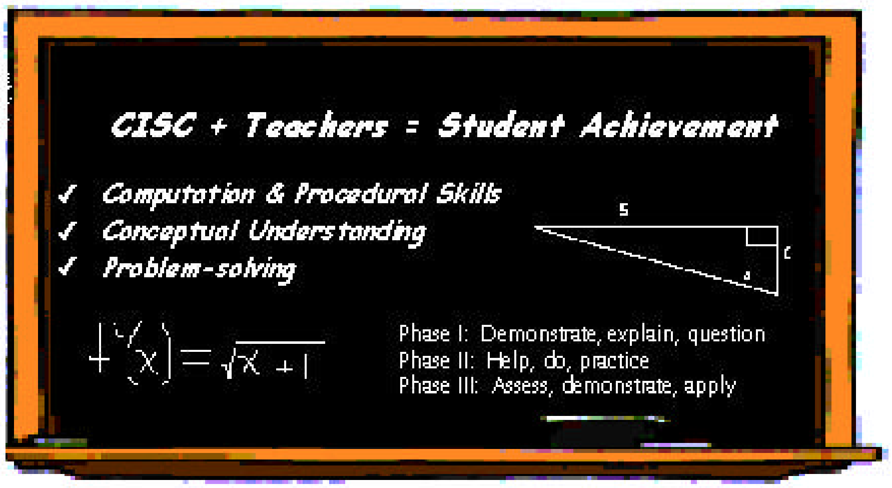
CISC - Curriculum & Instruction Steering Committee

California County Superintendents Educational Services Association

Primary Content Module V   Number Sense: Rational Numbers

The Winning EQUATION

A HIGH QUALITY MATHEMATICS PROFESSIONAL DEVELOPMENT PROGRAM FOR TEACHERS IN GRADES 4 THROUGH ALGEBRA II



STRAND: NUMBER SENSE: Rational Numbers

MODULE TITLE: **PRIMARY CONTENT MODULE V**

MODULE INTENTION: The intention of this module is to inform and instruct participants in the underlying mathematical content in the area of rational numbers.

THIS ENTIRE MODULE MUST BE COVERED IN-DEPTH.

The presentation of these Primary Content Modules is a departure from past professional development models. The content here, is presented for individual teacher's depth of content in mathematics. Presentation to students would, in most cases, not address the general case or proof, but focus on presentation with numerical examples.

In addition to the underlying mathematical content provided by this module, the facilitator should use the classroom connections provided within this binder and referenced in the facilitator's notes.

TIME: 3 hours

PARTICIPANT OUTCOMES:

- Demonstrate understanding of the fractions and rational numbers.
- Demonstrate understanding of equivalent fractions and how to reduce and build fractions.
- Demonstrate how to add, subtract, multiply, and divide rational numbers.
- Demonstrate how to use the basic algorithms for addition and subtraction of fractions that may not have common denominators.

PRIMARY CONTENT MODULE V
NUMBER SENSE - Rational Numbers
Facilitator's Notes

Prerequisite: Familiarity with the arithmetic of the integers and positive and negative whole numbers.

The following are some helpful references that give a foundation for rational numbers:

T-1

Teaching fractions in elementary school: A manual for teachers. (March, 1998). http://www.math.berkeley.edu/-wu/

"Knowing and Teaching Elementary Mathematics," by Liping Ma, www.erlbaum.com.

Pre-Test

Pretest:

Give test on addition, subtraction, multiplication, division, reducing fractions, and solving some word problems. Questions on conceptual understanding are emphasized in post-test.

T-2

1. What is a Fraction?

This is not an easy question. The word fraction comes from the Latin word, "fractio," which means "the act of breaking into pieces." Fractions arise naturally in measurement problems, especially to express a quantity less than a whole unit. Fractions indicate amounts or distances in which a basic unit is subdivided into a whole number of equal parts.

T-3

For example, if a pie is divided into 8 pieces and three of these are eaten, then the fraction, $\frac{3}{8}$ expresses the fraction of the pie that was consumed.

T-4 & T-5

Use T-4 and T-5 to illustrate the concept of fraction further.

There is no universally accepted definition of "fraction," but we will use this one:

T-6

Definition 1: A fraction is an ordered pair of integers *a* and *b*, with *b* not equal to zero, and written as $\frac{a}{b}$. The integer *a* is called the numerator and *b* is called the denominator.

Some authors require *a* and *b* to be whole numbers (so that they are both positive).

The words "numerator" and "denominator" are appropriate for fractions because the numerator enumerates (i.e., counts) while the denominator specifies what is being counted.

T-7

Point out that fractions can be understood as points on a number line. Make an overhead slide with a number line with some numbers bigger than 1 and less than zero. Identify location of some fractions on this number line. Include some fractions bigger than 1. Tell participants that fractions with which are bigger than one are called "improper fractions."

T-8

Tell participants that a number line is a particularly convenient way to visualize negative fractions, like $-\frac{3}{4}$. Tell participants that it will be explained soon that $-\frac{3}{4} = \frac{-3}{4} = \frac{3}{-4}$ and that similar statements are true for all fractions which have negative numerators or denominators.

Fractions may be thought of as locations or points on a number line and therefore as numbers. So, it is natural to develop rules to add, subtract, multiply, and divide them. The set of fractions is a larger set of numbers than the set of integers. Any integer m can be regarded as a fraction by writing it as $\frac{m}{1}$. For example, $3 = \frac{3}{1}$, and $-5 = \frac{-5}{1}$.

T-8A

Before addition, subtraction, multiplication, and division of fractions can be defined, there is a very important issue which must be addressed. That is, when do two fractions represent the same amount? When do they represent the same point or number on a number line? For example, $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \text{etc.}$ So all of these fractions are really the same number. How does this work for fractions in general?

We will temporarily restrict our attention to fractions with positive numerators and denominators.

What is the easiest way to tell when two fractions are really the same number?

T-9

Use T-9 to explain that fractions such as $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$ represent the same amounts; as well as $\frac{1}{2}$, $\frac{2}{6}$, and $\frac{3}{9}$.

T-10

The "Jumbo Inch" project that follows can be used with T-8 to further reinforce this idea.

Model the making of the "jumbo inch" using T-8, as if it were a narrow strip of paper, to record the steps as you go through the process on the overhead. After modeling for participants, they will do their own "jumbo inch" using H-10.

- •Mark the left side of the strip with 0.
- •Mark the right side of the strip with 1.
- •Fold the strip into halves. Ask, "How many equal pieces do we have?" "What fraction should we put on the fold?" $\left(\frac{1}{2}\right)$
- •Draw a line on the fold about one-third of the distance across.
- •Mark the line with $\frac{1}{2}$ and put $\frac{2}{2}$ over the 1.
- •Refold the strip into halves and fold it into halves again.
- •Fold the strip into halves. Ask, "How many equal pieces do we have?" "What fraction should we put on the folds?" $\left(\frac{1}{4}, \frac{3}{4}\right)$
- •Draw lines on the new folds and label them with $\frac{1}{4}$ and $\frac{3}{4}$.

"What should we put on the middle fold?" $\left(\frac{2}{4}\right)$ Write $\frac{2}{4}$ under the $\frac{1}{2}$ and $\frac{4}{4}$ under the $\frac{2}{2}$.

Continue the procedure through the 16ths.

Discuss what would happen if you continued the process. Talk about what would happen if you had more than one inch. Draw a mark between 2 and 3 and ask how it would be labeled. "How would we write $2\frac{3}{4}$ in sixteenths?"

H-10

T-11

Have participants make their own Jumbo Inch using H-10. This worksheet can be kept and used for other lessons throughout the workshop. T-11 represents what the jumbo inch will look like when the project is finished.

T-12

Indicate the use of fraction strips or paper folding here with some fraction of the paper shaded to represent a fraction.

T-13

T-13A

T-14

By increasing the number of equal pieces of a whole by a multiple of the denominator of a fraction, one can see, using physical models that the number of smaller pieces which are shaded increases by the same factor.

In symbolic form, a fraction like $\frac{2}{3}$ can be rewritten as $\frac{2 \bullet k}{3 \bullet k}$ for any whole number *k*. For example, $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} =$ etc. All of these fractions represent the same number.

Give more examples if needed.

T-15

It follows that the numerator and denominator of a fraction can be divided, or multiplied, by the same common factor to get another fraction that represents the same point on the number line or number. For example, $\frac{6}{9} = \frac{6 \div 3}{9 \div 3} = \frac{2}{3}$

In many cases, two fractions can be seen to be equal in this way; that is, by multiplying or dividing both the numerator and denominator of one fraction by the same whole number to get the other fraction. But this does not always work for whole number values of *k*..

T-16

Here's an example:
Consider for example $\frac{3}{6}$ and $\frac{4}{8}$. Each of these is equal to $\frac{1}{2}$, but there is no whole number multiple of 3 that gives 4 and no whole number multiple of 6 that gives 8. With larger numbers involved, it can be harder to tell whether two fractions are equal. For example, $\frac{91}{119} = \frac{143}{187}$ because they can both be reduced to $\frac{13}{17}$, but it is not at all obvious that they are equal by inspection.

Is there a simple way to tell when two fractions are the same number?

YES, it is called "cross multiplication."

Two fractions $\frac{a}{b}$ and $\frac{c}{d}$ are equal, i.e., $\frac{a}{b} = \frac{c}{d}$ if and only if (iff) $ad = bc$.

For example, $\frac{3}{6} = \frac{4}{8}$ because 3 • 8 = 6 • 4.

T-17

And, $\frac{91}{119} = \frac{143}{187}$ because 91 • 187 = 119 • 143.

T-18

Why does this always work?

Answer: To determine whether $\frac{a}{b} = \frac{c}{d}$, find a common denominator for each fraction. One that is sure to work, even without knowing what *a, b, c,* and *d* are is the number *bd*. *bd* is a common denominator for $\frac{a}{b}$ and $\frac{c}{d}$.

As you go through this example: uncover the statements to reveal the numbers example next to the variables on the transparency. This should clarify the concepts presented. Multiplying top and bottom of $\frac{a}{b}$ by *d* and top and bottom of $\frac{c}{d}$ by *b* the two fractions are changed as follows:

$$\frac{a}{b} = \frac{ad}{bd}$$
$$\frac{c}{d} = \frac{bc}{bd}$$

Since the two fractions on the right have the same denominator, they are equal exactly when their numerators are the same, i.e., when $ad = bc$.

This leads to the following important definition:

T-19

Definition 2: Two fractions $\frac{a}{b}$ and $\frac{c}{d}$ are equal, that is, they represent the same number, if and only if $ad = bc$.

T-19A

Notice from the argument leading to *Definition 2* that cross multiplication provides a simple way to tell whether one fraction is larger than another.

Proposition: $\frac{a}{b} < \frac{c}{d}$ if and only if $ad < bc$.

For example, to decide whether $\frac{3}{5}$ is larger than $\frac{4}{7}$, we need to compare 3 • 7 to 5 • 4. Since 5 • 4 < 3 • 7 $\frac{4}{7} < \frac{3}{5}$.

H-19

Use H-19 to practice deciding when two fractions are equal and if not, which is larger.

Call attention of the participants to *Definition 2*, again. Point out that we have been assuming that *a*, *b*, *c*, and *d* are all positive. Tell them even so, this definition includes the cases where some or all of *a*, *b*, *c*, and *d* are negative.

T-20

Use T-20 to explain that $\frac{3}{-4} = \frac{-3}{4}$ by cross multiplication and, also, by multiplying the top and bottom of one fraction by –1 to get the other fraction. Do the same for $\frac{3}{4}$ and $\frac{-3}{-4}$ to show they are equal. Tell participants you will show them why $\frac{-3}{4}$ is the same as $-\frac{3}{4}$ in the next subsection. This justifies the placement of these fractions on the number line.

T-21

T-22

Definition 3: A rational number is any number which can be expressed in the form of $\frac{a}{b}$ where *a* and *b* are integers; *b* does not equal zero.

In other words rational numbers are the numbers represented by fractions. So, fractions and rational numbers are essentially the same thing. But as we have seen above, more than one fraction represents the same rational number, e.g., $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} =$ etc.

T-22A

Sets of numbers. This shows the hierarchy of number systems developed so far.

The Arithmetic of Rational Numbers.

2. Addition and Subtraction of Fractions

T-23

<u>Case 1</u>

Two fractions which are to be added or subtracted have the same denominator.

$$\frac{1}{3}+\frac{1}{3}=\frac{1+1}{3}=\frac{2}{3} \qquad \frac{4}{5}-\frac{1}{5}=\frac{4+1}{5}=\frac{3}{5}$$

In general:

$$\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b} \qquad \frac{a}{b}-\frac{c}{b}=\frac{a-c}{b}$$

T-24

Case 2

Two fractions which are to be added or subtracted have the different denominators.

This should be defined using the following formula. After explaining the variable process, then go through with the numerical representation for participant understanding.

$$\frac{a}{b}+\frac{c}{d}=\frac{(ad+cb)}{bd} \qquad \frac{2}{3}+\frac{4}{5}=\frac{(2\bullet 5+3\bullet 4)}{3\bullet 5}$$

$$\frac{a}{b}+\frac{c}{d}= \qquad \frac{2}{3}+\frac{4}{5}=$$

$$\left(\frac{a}{b}\bullet\frac{d}{d}\right)+\left(\frac{c}{d}\bullet\frac{b}{b}\right)= \qquad \left(\frac{2}{3}\bullet\frac{5}{5}\right)+\left(\frac{4}{5}\bullet\frac{3}{3}\right)=$$

$$\frac{ad}{bd}+\frac{cb}{db}= \qquad \frac{2\bullet 5}{3\bullet 5}+\frac{4\bullet 3}{5\bullet 3}=$$

$$\frac{(ad+cb)}{bd} \qquad \frac{(2\bullet 5+3\bullet 4)}{3\bullet 5}$$

T-24A

Therefore:

$$\frac{a}{b}+\frac{c}{d}=\frac{(ad+cb)}{bd} \qquad \frac{2}{3}+\frac{4}{5}=\frac{10+12}{15}$$

Which simplifies to: $=\frac{22}{15}$ or $1\frac{7}{15}$

It is important to give this as a definition of fraction addition because it contributes to an understanding of algebra. This formula works in all cases, even when the fractions have the same denominator. This important formula will be used repeatedly in algebra.

Explain that common denominators less than *bd* may be used, including the Least Common Multiple (LCM) of *b* and *d* and this is covered in Number Sense: Module III. Use appropriate overheads already submitted.

T-25

Use the formula for Case 1 to justify the earlier statement that $-\frac{3}{4} = \frac{-3}{4}$. This follows from:

$$\begin{aligned} &\frac{-3}{4} + \frac{3}{4} \\ &= \frac{-3+3}{4} \quad \text{because } -3 + 3 = 0 \\ &= \frac{0}{4} \\ &= 0 \end{aligned}$$

Therefore $\frac{-3}{4} = -\frac{3}{4}$.

T-26

Use T-26 to demonstrate how mixed numbers can be converted to improper fractions using fraction addition. One definition that needs to be clear is: $5\frac{2}{3} = 5 + \frac{2}{3}$.

Using this information, we can convert $5\frac{2}{3}$ to an improper fraction.

$$\begin{aligned} &5\frac{2}{3} \\ &= 5 + \frac{2}{3} \\ &= \frac{15}{3} + \frac{2}{3} \\ &= \frac{17}{3} \end{aligned}$$

Notice the shortcut, $5\frac{2}{3} = \frac{17}{3}$ because 5 • 3 + 2 = 17 (Multiply the whole number times the denominator and then add the numerator).

H-26

H-26 asks participants why these two ways of converting a mixed number to an improper fraction are really the same.

Now, how do we go the other way?

T-27

How would you know to how to write $\frac{17}{3} = \frac{15}{3} + \frac{2}{3} = 5\frac{2}{3}$?

Long division for 17 ÷ 3 will also do this conversion.

More examples should be presented as needed by participants.

3. Multiplication of Fractions

The principal aim of this section is to motivate the formula for multiplying two fractions, i.e. to justify

$$\frac{a}{b} \bullet \frac{c}{d} = \frac{ac}{bd}$$

T-28

Definition 4: The product of $\frac{a}{b}$ and $\frac{c}{d}$ for any *a*, *b*, *b*, and *d* (with denominators not zero) is given by $\frac{a}{b} \bullet \frac{c}{d} = \frac{ac}{bd}$.

The goal of this section is to explain why this is a good definition on a more intuitive level.

<u>Case 1</u>: an integer times a unit fraction.

$$\begin{aligned} &4 \bullet \frac{1}{5} \\ &= \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} \\ &= \frac{(1+1+1+1)}{5} \\ &= \frac{4}{5} \end{aligned}$$

So, $$4 \bullet \frac{1}{5} = \frac{4}{5}$$

T-29

Since $4 = \frac{4}{1}$ this equation can be rewritten as $\frac{4}{1} \bullet \frac{1}{5} = \frac{4}{5}$. This is also the result that *Definition 4* gives.

Do a few more examples, depending on how the participants react and how they did on the pre-test.

We can look at this result in a different way. $4 \bullet \frac{1}{5} = \frac{1}{5} \bullet 4$ and we can think of this as $\frac{1}{5}$ of 4. In other words, this is 4 ÷ 5.

T-30

In the context of fractions, it is useful, especially in the context of word problems to think of the multiplication symbol "•" as representing the word "of." So, once again,
$\frac{1}{5} \bullet 4 = \frac{1}{5}$ of 4 = 4 divided by 5.

Since $\frac{1}{5} \bullet 4 = \frac{4}{5}$, the fraction $\frac{4}{5}$ may be thought of as "4 divided by 5."

This is a conceptual breakthrough. Fractions may thus be understood as the "answers" to division problems that have no answer if you only had whole numbers or integers. With fractions, division problems, no longer need to have remainders.

T-31

Case 2: The product of two unit fractions, e.g., $\frac{1}{4} \bullet \frac{1}{3}$.

Use an area model to explain this. Use a unit square (one by one) and divide one side into 4 equal subdivision and the other side into 3 equal subdivisions. These generate subrectangles with dimensions $\frac{1}{4}$ by $\frac{1}{3}$ which partition the unit square. Therefore the area of one of these subrectangles is $\frac{1}{4} \bullet \frac{1}{3}$. There are 3 • 4 = 12 of these subrectangles which cover the unit square and they all have the same area. Therefore the area of one of them is the area of the 1 x 1 square divided by 12, or $\frac{1}{12}$. Therefore $\frac{1}{4} \bullet \frac{1}{3} = \frac{1}{12}$ and this is consistent with *Definition 4.*

Do other examples like this one. Use the unit square. The time spent on practice depends on the reactions of participants and the results of the pre-test.

T-32

Case 3: The general case.

Using the associative and commutative properties of multiplication (which we take as axioms).

$$\begin{aligned}&\frac{a}{b}\bullet\frac{c}{d}\\&=(a\bullet\frac{1}{b})\bullet(\frac{1}{d}\bullet c)\\&=a\bullet(\frac{1}{b}\bullet\frac{1}{d})\bullet c\\&=(a\bullet c)\bullet(\frac{1}{b}\bullet\frac{1}{d})\\&=\frac{ac}{bd}\end{aligned}$$

where both Case 1 and Case 2 have been used.

H-32

Have participants do H-32 to practice multiplication of fractions using Case 3.

4. Division of Fractions

The aim of this subsection is to explain and motivate the formula for division of fractions given by:

T-33

Definition 5: For any fractions $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c}$.

Do some examples using this formula. The time spent on examples depends on the understanding of participants. The immediate goal is just to be able to understand what the definition says. E.g.,

$$\begin{aligned}&\frac{3}{4}\div\frac{7}{3}\\&=\frac{3}{4}\bullet\frac{3}{7}=\frac{9}{28}\end{aligned}$$

T-34

Why is *Definition 5* taken as the definition of division of fractions?

There are several ways to understand this rule for division of fractions, including:

recognizing division in relation to multiplication,
first finding common denominators of the two fractions.

Division may be understood in the following way:

$A \div B = C$ means the same as $C \bullet B = A$.

The numbers *A*, *B*, and *C* may represent any numbers including fractions. The above statement may be taken as a general definition of division.

For example, 12 ÷ 4 = 3 because 3 • 4 = 12.

Now try this with fractions.

$$\frac{a}{b} \div \frac{c}{d} = \frac{e}{f}$$

means the same as

$$\frac{e}{f} \bullet \frac{c}{d} = \frac{a}{b}$$

T-35 Now solve for $\frac{e}{f}$ by multiplying both sides by $\frac{d}{c}$:

$$\left(\frac{e}{f} \bullet \frac{c}{d}\right) \bullet \frac{d}{c} = \frac{a}{b} \bullet \frac{d}{c}$$

$$\frac{e}{f} \bullet \left(\frac{c}{d} \bullet \frac{d}{c}\right) = \frac{a}{b} \bullet \frac{d}{c}$$

$$\frac{e}{f} \bullet \frac{cd}{cd} = \frac{a}{b} \bullet \frac{d}{c}$$

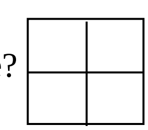
$$\frac{e}{f} \bullet 1 = \frac{a}{b} \bullet \frac{d}{c}$$

$$\frac{e}{f} = \frac{a}{b} \bullet \frac{d}{c}$$

Since $\frac{e}{f} = \frac{a}{b} \div \frac{c}{d}$, this means, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c}$ and the definition is verified.

T-36 Use T-36 to work through this definition with $\frac{2}{3} \div \frac{5}{7} =$

T-37 $1 \div \frac{1}{4}$ = means how many one-fourths are in 1 whole?



Each subrectangle is one-fourth of the unit square. There are 4 of the one-quarter units in the unit square. So: $1 \div \frac{1}{4} = 1 \bullet \frac{4}{1} = 4$

T-38

Common denominators approach.
Start with an example. "$\frac{6}{7} \div \frac{2}{7}$" is analogous to 6 apples divided by 2 apples." The answer in either case is 3. Think of "$\frac{6}{7} \div \frac{2}{7}$" as "6 groups of $\frac{1}{7}$ of something divided by 2 groups of $\frac{1}{7}$ of that thing."

T-39

Illustration

Dividing fractions with the same denominator can be done by just dividing the numerators.

T-40

T-40 presents another method for division of fractions.

To do the general case, $\frac{a}{b} \div \frac{c}{d}$, first find a common denominator for both of these fractions. A common denominator is *bd*. Rewrite

$$\frac{a}{b} = \frac{ad}{bd} \quad \text{and} \quad \frac{c}{d} = \frac{bc}{bd}$$

Then

$$\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bd} \div \frac{bc}{bd}$$

Since these last two fractions have the same denominator, we can just divide the numerators. So the answer is *ad* divided by *bc* or $\frac{ad}{bc}$.

Putting this altogether says that $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c}$, and this is exactly *Definition 5*.

T-41
T-42

This might be called the "fractions of fractions" approach.

T-43

A special note:

Many students through the years complain about not understanding fractions. They will often avoid problems involving fractions. Traditional sequence of adding, subtracting, multiplying, and dividing natural numbers, whole numbers, and integers leads us to do the same with rational numbers. However, when we get to Algebra, the order is often reversed when working with polynomials involving rational expressions.

Teachers might give students greater security when working with fractions by capitalizing on their successes. Students tend to find reducing of fractions an easier task than finding common denominators when adding or subtracting them. Some students may find greater success with fractions by multiplying and dividing them first and then getting to one of the most difficult concepts to learn; addition and subtraction of fractions with unlike denominators.

Some teachers may wish to teach addition and subtraction of fractions with Case 1 conditions, move to multiplication and division of fractions as an extension of reducing or building fractions, and then conclude with adding and subtracting fractions with unlike denominators as in Case 2 conditions when they are more experienced in working with fractions.

T-44 and T-44A

Conceptual Problem. Before showing T-44A have participants share their problem solving strategies. The general understanding should be emphasized: C and D are both less than 1 and positive so their product has to be less than either C or D and positive. B is the only possibility.

T-45

Optional Word Problems

Post-Test

Participants should take the Post-Test at the end of this module. The Post-Test has more conceptual questions than the Pre-Test. Problems should be discussed with participants to clear up any confusions.

Standards for Rational Numbers

Grade 3 Number Sense

3.0 Students understand the relationship between whole numbers, simple fractions, and decimals.
3.1 Compare fractions represented by drawings or concrete materials to show equivalency and to add and subtract simple fraction in context.
3.2 Add and subtract simple fractions.

Grade 4 Number Sense

1.5 Explain different interpretations of fractions, for example, parts of a whole, parts of a set, and division of whole numbers by whole numbers; explain equivalents of fractions.
1.7 Write the fractions represented by a drawing of parts of a figure; represent a given fraction by using drawings; and relate a fraction to a simple decimal on a number line.

Grade 5 Number Sense

1.5 Identify and represent on a number line decimals, fractions, mixed numbers, and positive and negative integers.
2.0 Students perform calculations and solve problems involving addition, subtraction, and simple multiplication and division of fractions and decimals.
2.3 Solve simple problems, including ones arising in concrete solutions, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form.
2.4 Understand the concept of multiplication and division of fractions.
2.5 Complete and perform simple multiplication and division of fractions and apply procedures to solving problems.

Grade 6 Number Sense

2.0 Students calculate and solve problems involving addition, subtraction, multiplication, and division.
2.1 Solve problems involving addition, subtraction, multiplication, and division of positive fractions and explain why a particular operation was used for a given situation.
2.2 Explain the meaning of multiplication and division of positive fractions and perform the calculations.
2.4 Determine the least common multiple and the greatest common divisor of whole numbers; use them to solve problems with fractions (e.g., to find a common denominator to add two fractions or to find the reduced form for a fraction).

References:

"Teaching Fractions in Elementary School: A Manual for Teachers." (March, 1998).

http://www.math.berkeley.edu/~wu/

"Knowing and Teaching Elementary Mathematics," by Liping Ma,

www.erlbaum.com.

Pre-Test

1. $\frac{1}{5} + \frac{2}{5} =$

2. $\frac{5}{3} - \frac{1}{3} =$

3. $\frac{3}{4} \bullet \frac{1}{3} =$

4. $\frac{25}{12} \div \frac{3}{4} =$

5. $\frac{2}{3} + \frac{3}{5} =$

6. $3\frac{5}{12} - 2\frac{4}{15} =$

7. $124 \div 3\frac{1}{2} =$

8. $9\frac{2}{3} - 4\frac{1}{2} =$

9. $1\frac{1}{2} \bullet 1\frac{2}{5} =$

10. There are 25 children in the class; $\frac{3}{5}$ of the children in the class are boys. How many girls are in the class?

11. Half of the children in our school watch television every night. Three-fourths of those children watch for more than an hour. What fraction of the total children watch for more than an hour a night?

Pre-Test Key

1. $\frac{3}{5}$

2. $\frac{4}{3}$

3. $\frac{1}{4}$

4. $\frac{25}{9}$

5. $\frac{19}{15}$

6. $1\frac{3}{20}$

7. $35\frac{3}{7}$

8. $5\frac{1}{6}$

9. $\frac{21}{10}$

10. 10 girls

11. $\frac{3}{8}$

WHAT IS A FRACTION?

The word fraction comes from the Latin word,

“fractio”

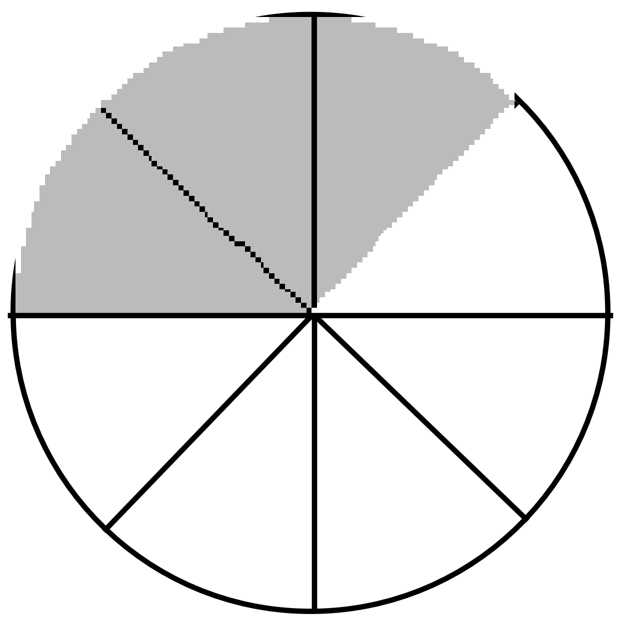
which means “the act of breaking into pieces.”

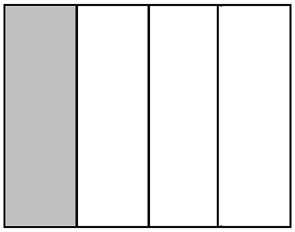
Fractions

- arise naturally in measurement problems
- proper fractions express a quantity less than a whole unit
- indicate amounts or distances in which a basic unit is subdivided into a whole number of equal parts

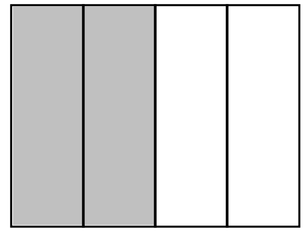
An Example

A pie is divided into 8 pieces and three of them are eaten. The fraction $\frac{3}{8}$ expresses the relative amount of the pie that was consumed.

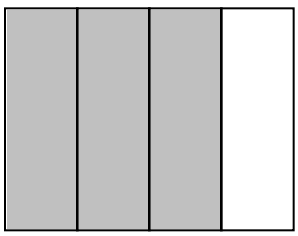




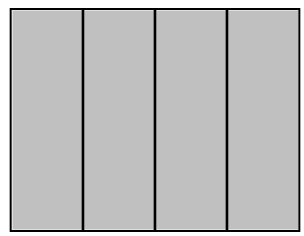
$$\frac{1}{4}$$



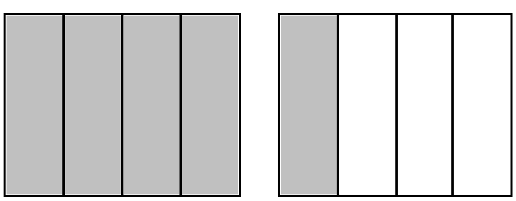
$$\frac{2}{4}$$



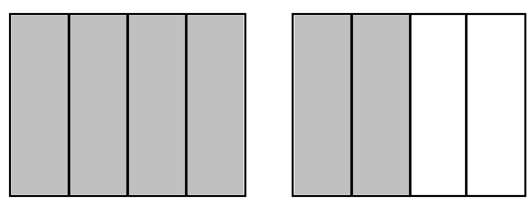
$$\frac{3}{4}$$



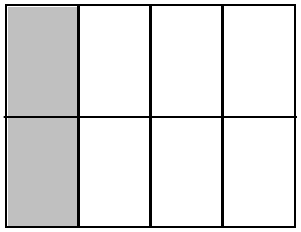
$$\frac{4}{4}$$



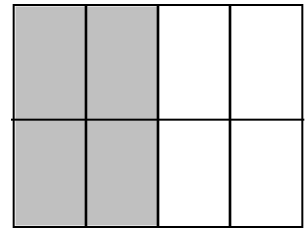
$$\frac{5}{4}$$



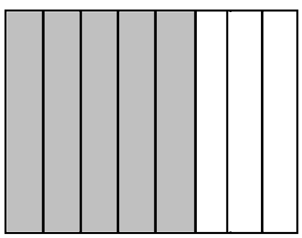
$$\frac{6}{4}$$



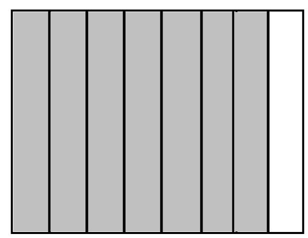
$\frac{2}{8}$



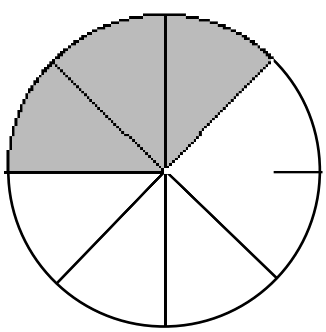
$\frac{4}{8}$



$\frac{5}{8}$

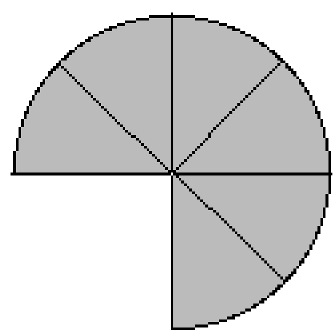


$\frac{7}{8}$



$\frac{3}{8}$

color in $\frac{3}{8}$ of the circle



$\frac{6}{8}$

color in $\frac{6}{8}$ of the circle

Definition 1:

A fraction is a ratio of integers *a* and *b*,

- $b \neq 0$
- written as $\frac{a}{b}$
- *a* is the <u>numerator</u>
- *b* is the <u>denominator</u>

the **numerator**;
<u>counts</u>
or <u>enumerates</u>

the **denominator**;
specifies what is <u>being counted</u>
or gives the <u>denomination</u>

