



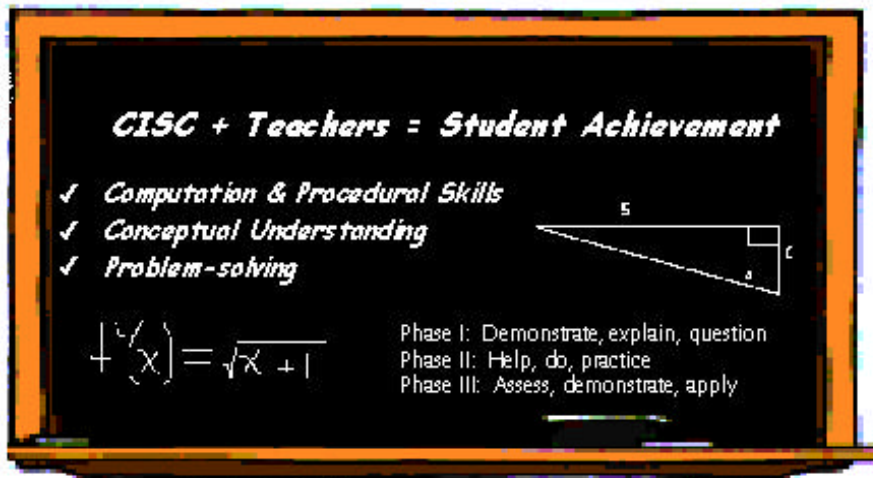
CISC - Curriculum & Instruction Steering Committee

California County Superintendents Educational Services Association

Primary Content Module III NUMBER SENSE: Multiplication & Division of Integers

The Winning EQUATION

A HIGH QUALITY MATHEMATICS PROFESSIONAL DEVELOPMENT PROGRAM FOR TEACHERS IN GRADES 4 THROUGH ALGEBRA II



STRAND: NUMBER SENSE: Multiplication & Division of Integers

MODULE TITLE: PRIMARY CONTENT MODULE III

MODULE INTENTION: The intention of this module is to inform and instruct participants in the underlying mathematical content in the areas of multiplication and division of integers.

THIS ENTIRE MODULE MUST BE COVERED IN-DEPTH.

The presentation of these Primary Content Modules is a departure from past professional development models. The content here, is presented for individual teacher's depth of content in mathematics. Presentation to students would, in most cases, not address the general case or proof, but focus on presentation with numerical examples.

In addition to the underlying mathematical content provided by this module, the facilitator should use the classroom connections provided within this binder and referenced in the facilitator's notes.

TIME: 2 hours

PARTICIPANT OUTCOMES:

- Demonstrate understanding of the base 10 number system.
- Demonstrate understanding of place value and powers of ten.
- Demonstrate how to use the commutative, associative, and distributive laws for addition and multiplication of natural numbers, whole numbers, and integers.
- Demonstrate how to use the basic algorithms for addition, subtraction, multiplication, and division of natural numbers, whole numbers, and integers.
- Demonstrate how the long division algorithm works.



PRIMARY CONTENT MODULE III
NUMBER SENSE: Multiplication and Division of Integers

Facilitator's Notes

Answer key for Pre & Post Test

- | | | | |
|--------|-----------|-----------|------------------|
| 1. -14 | 2. 384 | 3. 11,174 | 4. -24 |
| 5. -16 | 6. 603r.3 | 7. 135r.7 | 8. $q=135$ $r=7$ |

Facilitator should rely on overhead transparencies and study them in advance of the presentation. Guidance for selected transparencies follow:

T-1

Show T-1

T-2/H-2

What is the Commutative Law of Addition?
For any numbers a and b , $a + b = b + a$

T-3/H-3

What is the Associative Law of Addition?
For any numbers a , b , and c , $(a + b) + c = a + (b + c)$

T-4/H-4

Explain the Distributive property

T-5 to T-7

Display and discuss. Show one line at a time for T-7.

T-8/H-8

Distributive Property

1. Explanation for: $a \cdot (b + c + d) = a \cdot b + a \cdot c + a \cdot d$

$$\begin{aligned}
 \text{Let } E = b + c. \text{ Then } a \cdot (b + c + d) &= a \cdot (E + d) \\
 &= a \cdot E + a \cdot d \\
 &= a \cdot (b + c) + a \cdot d \\
 &= a \cdot (b + c) + a \cdot d \\
 &= a \cdot b + a \cdot c + a \cdot d
 \end{aligned}$$

Standard Algorithm

$$\begin{array}{r}
 53 \\
 \times 4 \\
 \hline
 212
 \end{array}$$



Base Ten Expansion:

$$\begin{aligned}4 \times 53 &= 4 \cdot (50 + 3) \\ &= 4 \cdot 50 + 4 \cdot 3 \\ &= 4 \cdot (5 \cdot 10^1) + 4 \cdot 3 \\ &= (4 \cdot 5) \cdot 10^1 + 4 \cdot 3 \\ &= 20 \cdot 10^1 + 12 \\ &= (2 \cdot 10^1) \cdot 10^1 + 10^1 + 2 \\ &= 2 \cdot (10^1 \cdot 10^1) + 10^1 + 2 \\ &= 2 \cdot (10^2) + 10^1 + 2 \\ &= 2 \cdot (10^2) + 1 \cdot 10^1 + 2 \\ &= 212\end{aligned}$$

Meaning of 53
Distributive Property
Definition of 50
Associative Property of Multiplication
Multiplication Facts
Meaning of 20 and 12
Associative Property of Multiplication
Exponent Law
Multiplicative Identity
Definition of 212

T-9

T-9 and T-10 demonstrate why a negative number times a positive number is a negative number and why a negative number times a negative number is a positive number. Participants need to understand that the distributive property forces these rules. Facilitators should review the concepts in each of these examples. Facilitator should tell participants that we are changing topics to multiplication of integers. Cover slide and expose one line at a time after participants acknowledge that the statement shown is TRUE.

T-10

Ask participants to complete the proof on their own, so cover the answer and allow them to work on completing the proof. Verify participant work by uncovering the slide, in the same manner as T-9. Participants need to un

T-11

General conclusion:
Any negative number raised to an even power is positive.
Any negative number raised to an odd power is negative.

T-12

Expose the question. Cover the rest of the slide. Then review meaning as slide is uncovered.

T-13 to T-16

Change topic to division. Follow slide exposing one line at a time.

T-17

While the long division algorithm poses a challenge to teach, it has important applications in subsequent grades. The grade 7 standards call for knowing that “every rational number is either a terminating or a repeating decimal”, an understanding of which requires the long division algorithm, and the examination of possible remainders.

T-18/H-18

Pre and Post Test.
Participants should complete the problems. Answers are located at the beginning of the facilitator’s notes.

**T-19****Closing Problem: A Very Beady Problem**

Allow participants to work on the problem. Have them discuss how they began to solve the problem. (Many students have difficulty applying division in a problem like this.) Point out that the problem is to “divide” out the block of 6 beads. The remainders correspond to the colors of the beads i.e, R1=red, R2=green and so on. Point out that there will not be an R6, that RØ means that 6 divides without remainder and in this case RØ=blue. Solution: $512 \div 6 = 85$ with remainder of 2; therefore, the color of the bead is green.

Standards covered in the Module:

Grade 3 Number Sense

- 2.2 Memorized to automaticity the multiplication table for numbers between 1 and 10.
- 2.3 Use the inverse relationship of multiplication and division to compute and check results.
- 2.4 Solve simple problems involving multiplication of multidigit numbers by one-digit numbers.
- 2.5 Solve division problems in which a multidigit number is evenly divided by a one digit number.
- 2.6 Understanding the special properties of 0 and 1 in multiplication and division. Algebra and Function.
- 1.5 Recognize and use the commutative and associative properties of multiplication.

Grade 4 Number Sense

- 5.2 Demonstrate an understanding of, and the ability to use standard algorithms for multiplying a multidigit number by a two-digit number and for dividing a multidigit number by one-digit number; use relationships between them to simplify computations and to check results.

Grade 5 Number Sense

- 2.2 Demonstrate proficiency with division, including division with positive decimals and long division with multidigit divisors.



Grade 6 Number Sense

- 2.1 Solve addition, subtraction, multiplication and division problems, including those arising in concrete situations, that use positive and negative integers and combinations of these operations.

Grade 7 Number Sense

- 1.3 Simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse, distributive, associative, commutative) and justify the process used.

Grade 3, 4, 5, 6, 7

Mathematical Reasoning

- 2.0 Students use strategies, skills, and concepts in finding solutions:
- 2.1 Use estimation to verify the reasonableness of calculated results.
- 2.2 Apply strategies and results from simpler problems to more complex problems.
- 2.4 Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.

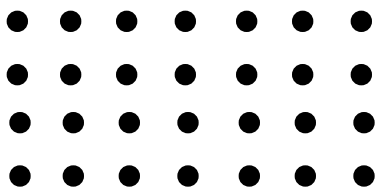
Multiplication

Multiplication can be defined as repeated addition.

$$4 \cdot 7 = 7 + 7 + 7 + 7$$

"seven added to itself four times."

A rectangular array for this problem:

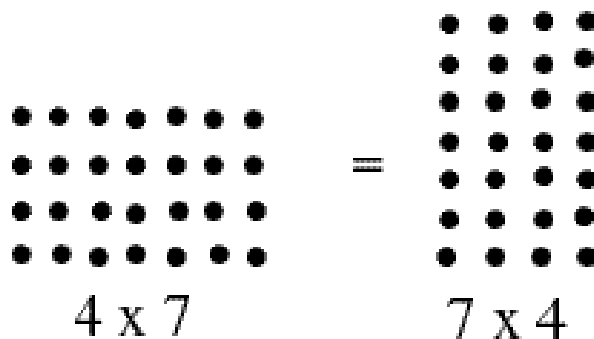


$$4 \times 7$$

Is there another way to represent this?

Commutative Law of Multiplication

For any numbers a and b , $a \cdot b = b \cdot a$



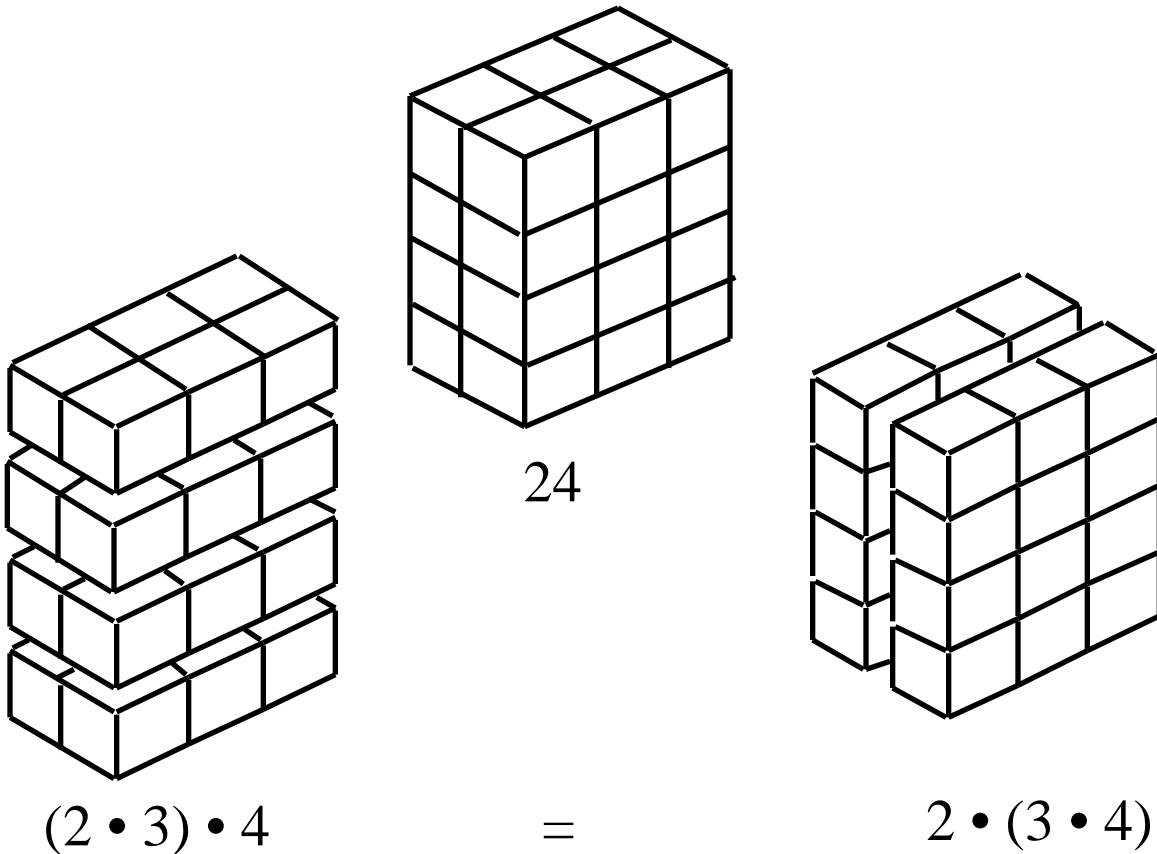
Review Question:

What is the Commutative Law of Addition?

Associative Law of Multiplication

For any numbers a , b , and c ,

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c$$



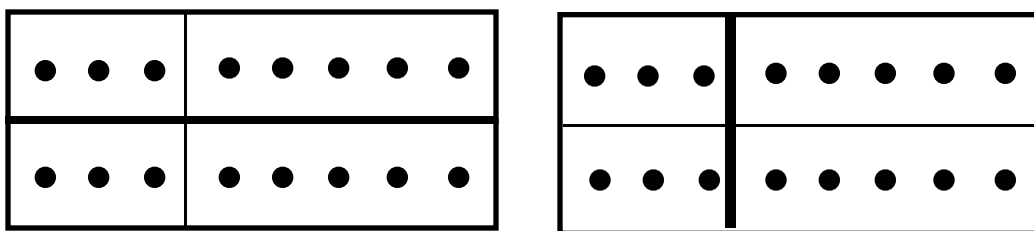
Review Question:

What is the Associative Law of Addition?

Distributive Law of Multiplication over Addition

For any numbers a , b , and c ,

$$a \cdot (b + c) = a \cdot b + a \cdot c$$



$$2 \cdot (3 + 5) = (2 \cdot 3) + (2 \cdot 5)$$

Using the base ten place value system, multidigit multiplication can be shown with single digit multiplication facts and the Commutative, Associative, and Distributive Laws.

The multiplication algorithm can be explained using the base ten structure of numbers and the CAD Laws.

Because of this powerful algorithm, any multiplication problem can be reduced to single digit multiplication facts. Memorizing the multiplication tables is essential for understanding. (Grade 3 Standard)

The standard algorithm for calculation 3×32 is:

$$\begin{array}{r} 32 \\ \times 3 \\ \hline 96 \end{array}$$

Why does this work?

$$3 \cdot 32 = 3 \cdot (30 + 2)$$

$$= 3 \cdot 30 + 3 \cdot 2$$

$$= 3 \cdot (3 \cdot 10^1) + 3 \cdot 2$$

$$= (3 \cdot 3) \cdot 10^1 + 3 \cdot 2$$

$$= 9 \cdot 10^1 + 6$$

$$= 96$$

Meaning of 32

Distributive property

Definition of 30

Associative Property

Multiplication Facts

Definition of 96

The standard algorithm for 3×38 is:

$$\begin{array}{r} 38 \\ \times 3 \\ \hline 114 \end{array}$$

Why does this work?

$3 \times 38 = 3 \cdot (30 + 8)$	Meaning of 38
$= 3 \cdot 30 + 3 \cdot 8$	Distributive Property
$= 3 \cdot (3 \cdot 10^1) + 3 \cdot 8$	Definition of 30
$= (3 \cdot 3) \cdot 10^1 + 3 \cdot 8$	Associative Property of Multiplication
$= 9 \cdot 10^1 + 24$	Multiplication Facts
$= 9 \cdot 10^1 + (2 \cdot 10^1 + 4)$	Definition of 24
$= (9 \cdot 10^1 + 2 \cdot 10^1) + 4$	Associative Property of Addition
$= (9 + 2) \cdot 10^1 + 4$	Distributive Property
$= (11) \cdot 10^1 + 4$	Addition Fact
$= (10^1 + 1) \cdot 10^1 + 4$	Definition of 11
$= (10^2) + (1 \cdot 10^1) + 4$	Distributive Property
$= (1 \cdot 10^2) + (1 \cdot 10^1) + 4$	Multiplicative Identity
$= 114$	Definition of 114

“CAD Worksheet”

1. Given the distributive property:

$$a \cdot (b + c) = a \cdot b + a \cdot c,$$

Explain why:

$$a \cdot (b + c + d) = a \cdot b + a \cdot c + a \cdot d$$

2. Use base ten expansion to explain the standard algorithm for:

$$4 \times 53$$

Why does $(-3) \cdot 7 = -21$?

$$\begin{aligned}0 &= 0 \cdot 7 \\0 &= [3 + (-3)] \cdot 7 \\0 &= (3 \cdot 7) + [(-3) \cdot 7] \\0 &= 21 + [(-3) \cdot 7] \\-21 + 0 &= -21 + 21 + [(-3) \cdot 7] \\-21 &= (-3) \cdot 7\end{aligned}$$

The Distributive Property forces $(-3) \cdot 7 = -21$

This shows that:

A negative times a positive is a negative.

Because of the communicative property of multiplication, it follows also that

A positive times a negative is a negative.

Why does $(-3) \cdot (-7) = 21$?

Hint: Replace 7 by (-7) in the preceding argument and recall that $(-3) \cdot 7 = -21$.

$$\begin{aligned} 0 &= 0 \cdot (-7) \\ 0 &= [3 + (-3)] \cdot (-7) \\ 0 &= [3 \cdot (-7)] + [(-3) \cdot (-7)] \\ 0 &= -21 + [(-3) \cdot (-7)] \\ 21 + 0 &= 21 + -21 + [(-3) \cdot (-7)] \\ 21 &= (-3) \cdot (-7) \end{aligned}$$

Again, the Distributive Property forces $(-3) \cdot (-7) = 21$

This shows that:

A negative times a negative is a positive.

Signs and Exponents

What patterns do you notice?

$$(-3)^3 = (-3) (-3) (-3) = -27$$

$$(-3)^4 = (-3) (-3) (-3) (-3) = 81$$

$$(-2)^5 = (-2) (-2) (-2) (-2) (-2) = -32$$

$$(-2)^6 = (-2) (-2) (-2) (-2) (-2) (-2) = 64$$

$$(-4)^2 = (-4) (-4) = 16$$

$$(-4)^3 = (-4) (-4) (-4) = -64$$

$$\text{Is } -3^2 = (-3)^2 ?$$

No!

$$\begin{aligned} -3^2 &= \text{the opposite of } 3^2 \\ &= \text{the opposite of } 9 \\ &= -9 \end{aligned}$$

$$\begin{aligned} (-3)^2 &= (-3) \cdot (-3) \\ &= 9 \end{aligned}$$

$$-3^2 \neq (-3)^2$$

In general, for any number x , $x \neq 0$,

$$-x^2 \neq (-x)^2$$

Division

corresponds to "repeated subtractions"

$27 \div 6$ is the number of times 6 can be subtracted from 27 with a remainder less than 6.

$$\begin{array}{r}
 27 \\
 \underline{-6} \quad 1 \\
 21 \\
 \underline{-6} \quad 2 \\
 15 \\
 \underline{-6} \quad 3 \\
 9 \\
 \underline{-6} \quad 4 \\
 3
 \end{array}$$

$27 \div 6 = 4$ with 3 remaining
 q is the quotient and
 r is the remainder

$$27 = 6 \cdot q + r \quad \text{where } 0 \leq r < 6$$

$$27 = 6 \cdot 4 + 3$$

In general, for any two whole numbers a and b ,

$$b \neq 0,$$

$$a \div b$$

is the search for whole numbers q and r for which

$$a = b \cdot q + r, \text{ where } 0 \leq r < b$$

An efficient way of doing successive subtractions when a and b get large is provided by the

Long Division Algorithm.

Solve:

$$5739 \div 17$$

Think of 17 boys dividing 5739 baseball cards.

The long division algorithm poses the problem in this form

$$17 \overline{)5739}$$

An easier question is:

"How often does 17 go into 57?"

$$3 \cdot 17 < 57 \quad \text{and} \quad 4 \cdot 17 > 57$$

leads to:

$$\begin{array}{r}
 3 \\
 17 \overline{)5739} \\
 \underline{51} \\
 6
 \end{array}
 \rightarrow 300$$

Because the digit 3 is in the hundred's column, this first step corresponds to "allocating" 300 baseball cards to each boy

$$\begin{aligned}
 5739 - 17 \cdot 300 &= 5739 - 5100 \\
 &= 639
 \end{aligned}$$

Continuing this process for the tens column:

$$3 \cdot 17 < 63 \quad \text{and} \quad 4 \cdot 17 > 63$$

$$\begin{array}{r}
 \overline{)5739} \\
 \underline{51} \\
 63 \\
 \underline{51} \\
 12
 \end{array}$$

$\begin{array}{r} \rightarrow 300 \\ \rightarrow +30 \\ \hline 330 \end{array}$

Because this 3 is written in the tens column, it corresponds to "allocating an additional 30 baseball cards to each boy" or to

$$\begin{aligned}
 639 - 17 \cdot 30 &= 639 - 510 \\
 &= 129
 \end{aligned}$$

The final step,

$$\begin{array}{r}
 17 \overline{)5739} \\
 \underline{51} \\
 63 \\
 \underline{51} \\
 129 \\
 \underline{119} \\
 10
 \end{array}$$

$$\begin{array}{r}
 300 \\
 +30 \\
 \hline
 330 \\
 +7 \\
 \hline
 337
 \end{array}$$

is based on

$$7 \cdot 17 < 129 \quad \text{and} \quad 8 \cdot 17 > 129$$

and corresponds to "allocating a final 7 baseball cards to each boy" or

$$129 - 17 \cdot 7 = 129 - 119 = 10$$

Each boy received 337 baseball cards, with 10 left over. The condition $0 \leq 10 < 17$ assures only one choice for q and r .

$$5739 = 17 \cdot 337 + 10$$

$$5739 = 17 \cdot q + r$$

Pre- and Post- Test

Perform the indicated operations.

1. $7 \times (-2) =$

2. $(-8)(-4)(12) =$

3. $37 \times 302 =$

4. $3[4 \cdot (-2)] =$

5. $(-16)[(-8) + 9] =$

6. $4 \overline{)2415}$

7. $2842 \div 21 =$

8. Find whole numbers q and r , so that
 $2842 = 21 \cdot q + r$.

A VERY BEADY PROBLEM

A long string of beads is arranged in the order red, green, orange, purple, yellow, and blue. The beads repeat in that arrangement. What is the color of the 512th bead?