Applications

The formula y = mx + b sometimes appears with different symbols.

For example, instead of x, we could use the letter C. Instead of y, we could use the letter F.

Then the equation becomes

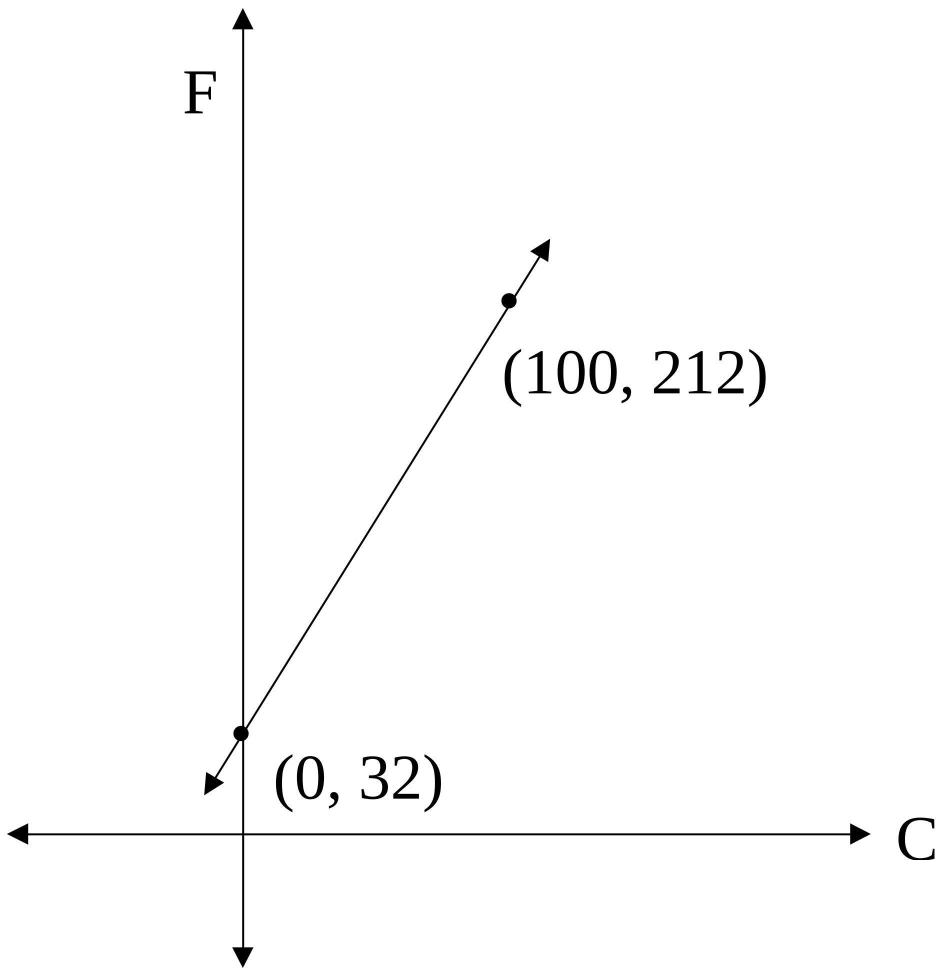
$$F = mC + b.$$

All temperature scales are related by linear equations. For example, the temperature in degrees Fahrenheit is a linear function of degrees Celsius.

Basic Temperature Facts

Water freezes at: 0°C, 32°F

Water Boils at: 100°C, 212°F



Can you solve for m and b in

$$F = mC + b?$$

To find the equation relating Fahrenheit to Celsius we need m and b

$$m = \frac{F_2 - F_1}{C_2 - C_1}$$

$$m = \frac{212 - 32}{100 - 0}$$

$$m = \frac{180}{100} = \frac{9}{5}$$

Therefore $F = \frac{9}{5}C + b$

To find b, substitute the coordinates of either point.

$$32 = \frac{9}{5}(0) + b$$

Therefore b = 32

Therefore the equation is

$$F = \frac{9}{5}C + 32$$

Can you solve for C in terms of F?

Celsius in terms of Fahrenheit

$$F = \frac{9}{5}C + 32$$

$$\frac{5}{9}F = \frac{5}{9}\left(\frac{9}{5}C + 32\right)$$

$$\frac{5}{9}F = C + \frac{5}{9}(32)$$

$$\frac{5}{9}F - \frac{5}{9}(32) = C$$

$$C = \frac{5}{9}F - \frac{5}{9}(32)$$

$$C = \frac{5}{9}(F - 32)$$

Example: How many degrees Celsius is77°F?

$$C = \frac{5}{9}(77 - 32)$$

$$C = \frac{5}{9}(45)$$

$$C = 25^\circ$$

So 72°F = 25°C

Standard 8

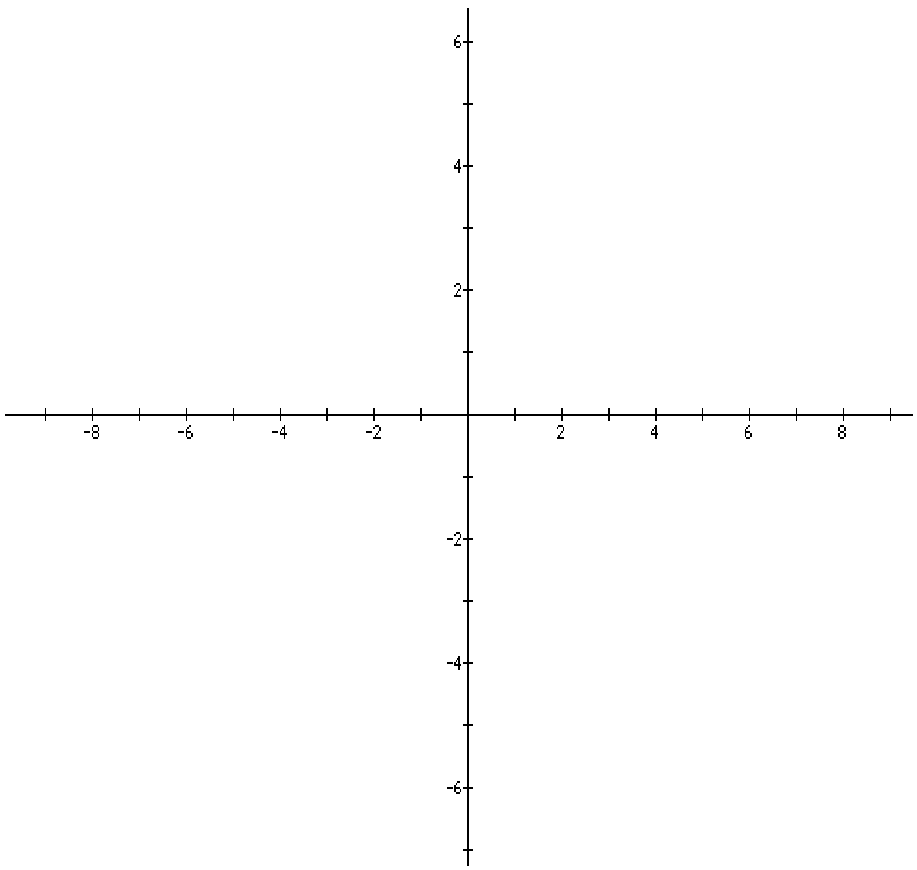
Algebra I, Grade 8 Standards

Students understand the concept of parallel and perpendicular lines and how their slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.

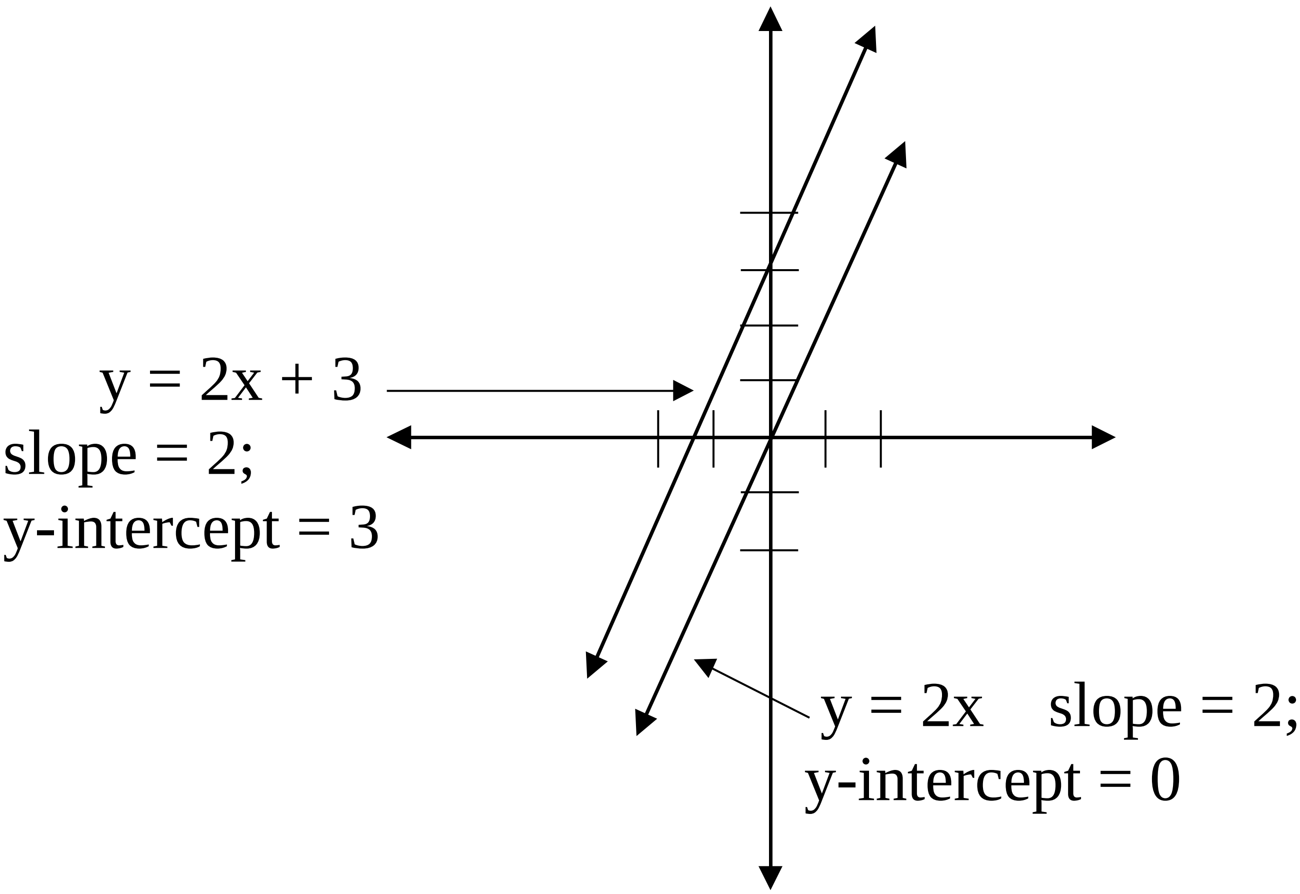
On one clean sheet of graph paper:

1. Graph $y = 2x$

2. Graph $y = 2x + 3$



Solution:



Notice that $y = 2x$ and $y = 2x + 3$ are parallel.

If two lines have the same slope and different y-intercepts, they are parallel.

Remember that a different y-intercept only translates a graph vertically, so the translated graph is parallel to the original.

Theorems

Two non-vertical lines are parallel if and only if they have the same slope.

Two non-vertical lines are perpendicular if and only if the product of their slopes is -1.

Practice

Consider the line whose equation is

$$y = 2x - 5.$$

Questions:

1. If a line is parallel to the graph of the given line, what is its slope?

2. If a line is perpendicular to the graph of the line, what will be its slope?

Practice

Consider the line whose equation is

$$y = 2x - 5.$$

Questions:

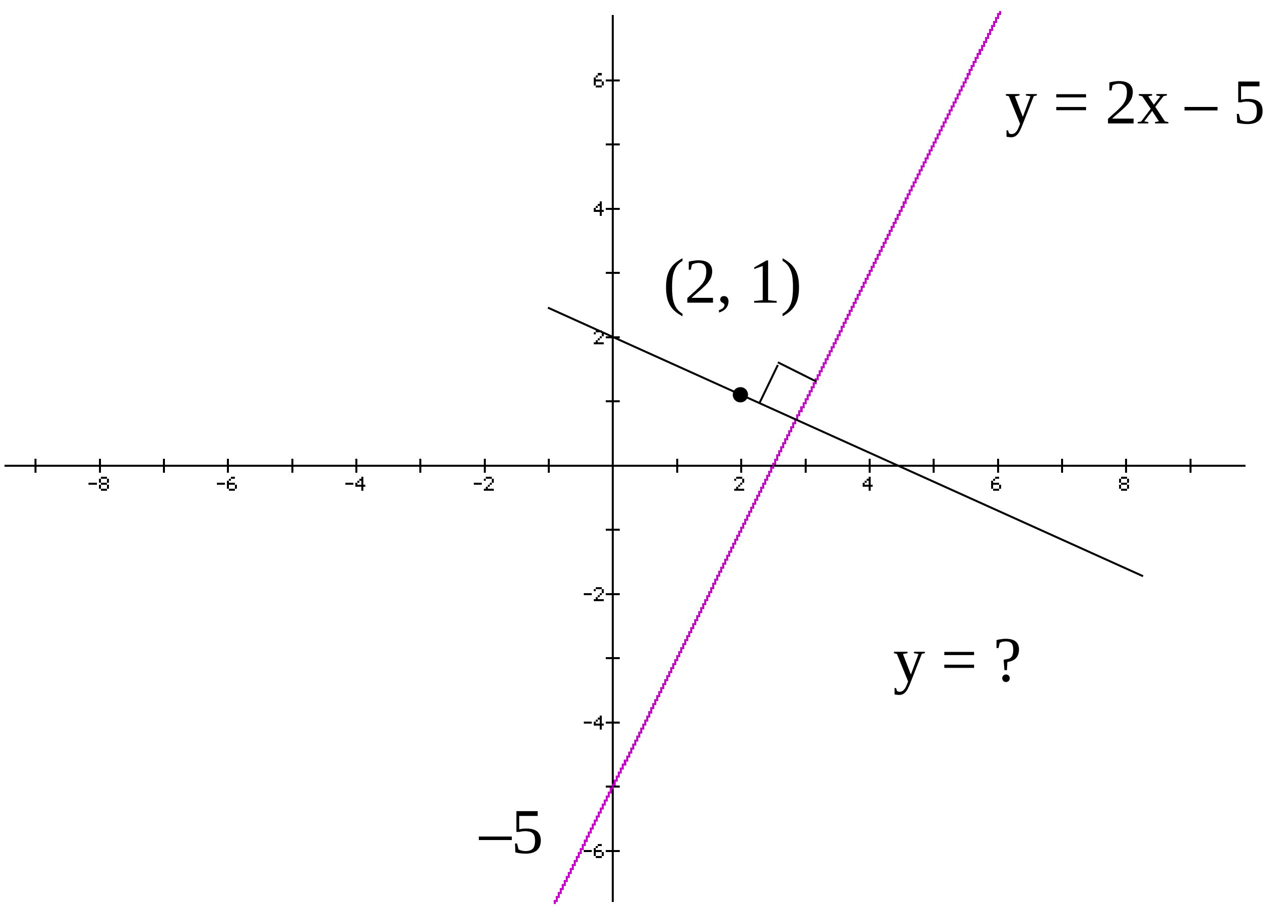
1. If a line is parallel to the graph of the given line, what is its slope?

2. If a line is perpendicular to the graph of the line, what will be its slope?

Answers: 1. $m = 2$ 2. $m = -\frac{1}{2}$

More Practice

Find the equation of the line whose graph contains the point (2,1) and is perpendicular to the line with the equation y = 2x – 5.



Perpendicular Lines

Write the equation of the line that passes through the point (2,1) and is perpendicular to the line whose equation is $y = 2x - 5$.

The slope of the line is 2, so the slope of the line that is perpendicular is $-\frac{1}{2}$.

Point-Slope Formula	Slope-Intercept Formula
$y - y_1 = m(x - x_1)$	$y = mx + b$
$y - 1 = -\left(\frac{1}{2}\right)(x - 2)$	$1 = -\left(\frac{1}{2}\right)(2) + b$
$y - 1 = -\frac{1}{2}(x - 2)$	$1 = -1 + b$
$y - 1 = \frac{-x}{2} + 1$	$2 = b$
$y = -\frac{1}{2}x + 2$	$y = -\frac{1}{2}x + 2$

Practice

Write the equation of the line that passes through the point (5,1) and is perpendicular to the line whose equation is $y = \frac{1}{3}x + 4$.

Solution

The equation of the line $y = \frac{1}{3}x + 4$ has slope of $\frac{1}{3}$ so the slope of a line that is perpendicular is –3. Our point was (5,1).

Point-Slope Formula	Slope-Intercept Formula
$y - y_1 = m(x - x_1)$	$y = mx + b$
$y - 1 = -3(x - 5)$	$1 = -3(5) + b$
$y - 1 = -3x + 15$	$1 = -15 + b$
$y = -3x + 16$	$16 = b;\ \ b = 16$
	$y = -3x + 16$

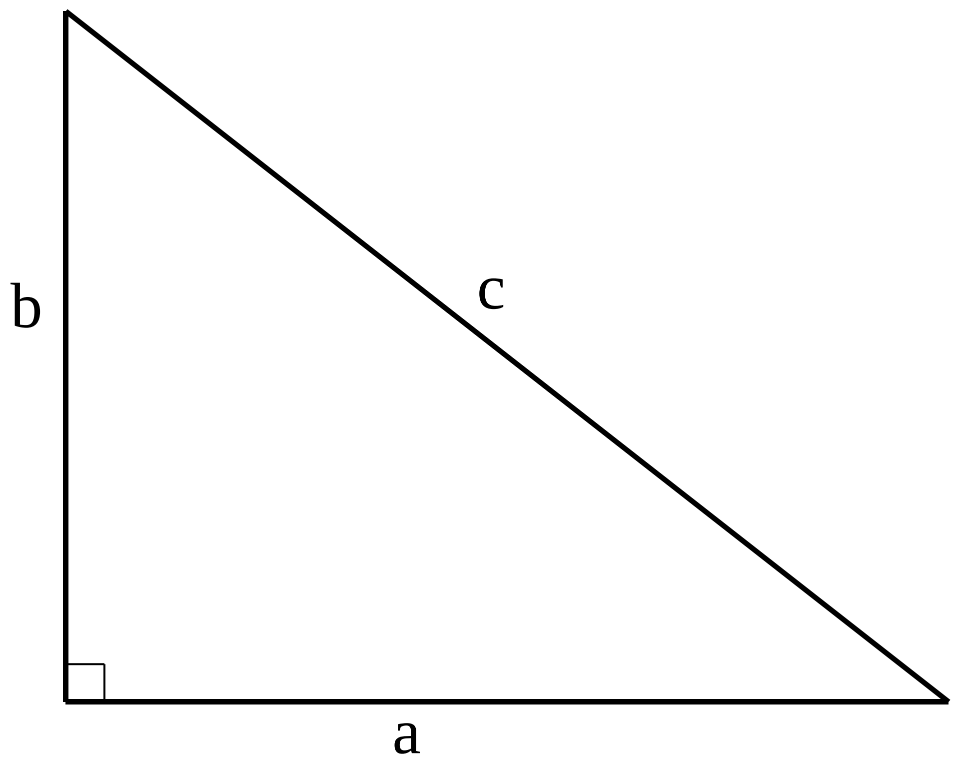
In-Depth Understanding

Why is it that two non-vertical lines are perpendicular if and only if the product of their slopes is –1?

The explanation uses the Pythagorean Theorem and its converse.

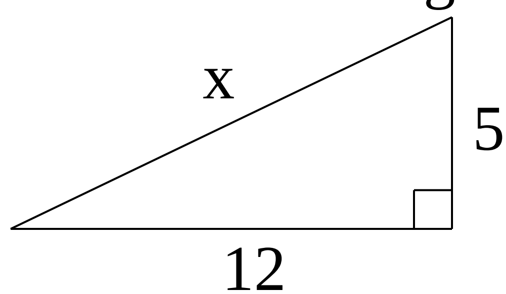
What is the Pythagorean Theorem?

PYTHAGOREAN THEOREM



$$a^2 + b^2 = c^2$$

Example: Find the missing length.



$x^2 = 5^2 + 12^2$

$x^2 = 25 + 144$

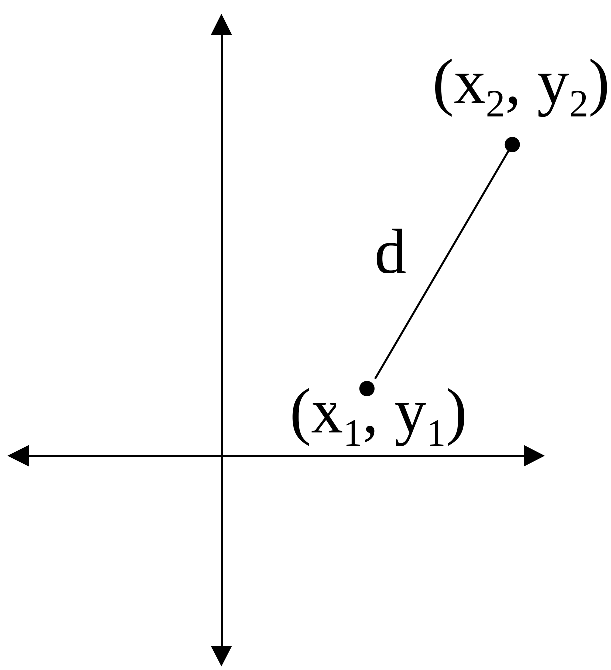
$x^2 = 169$; $x = 13$

An Important Tool:

The Distance Formula

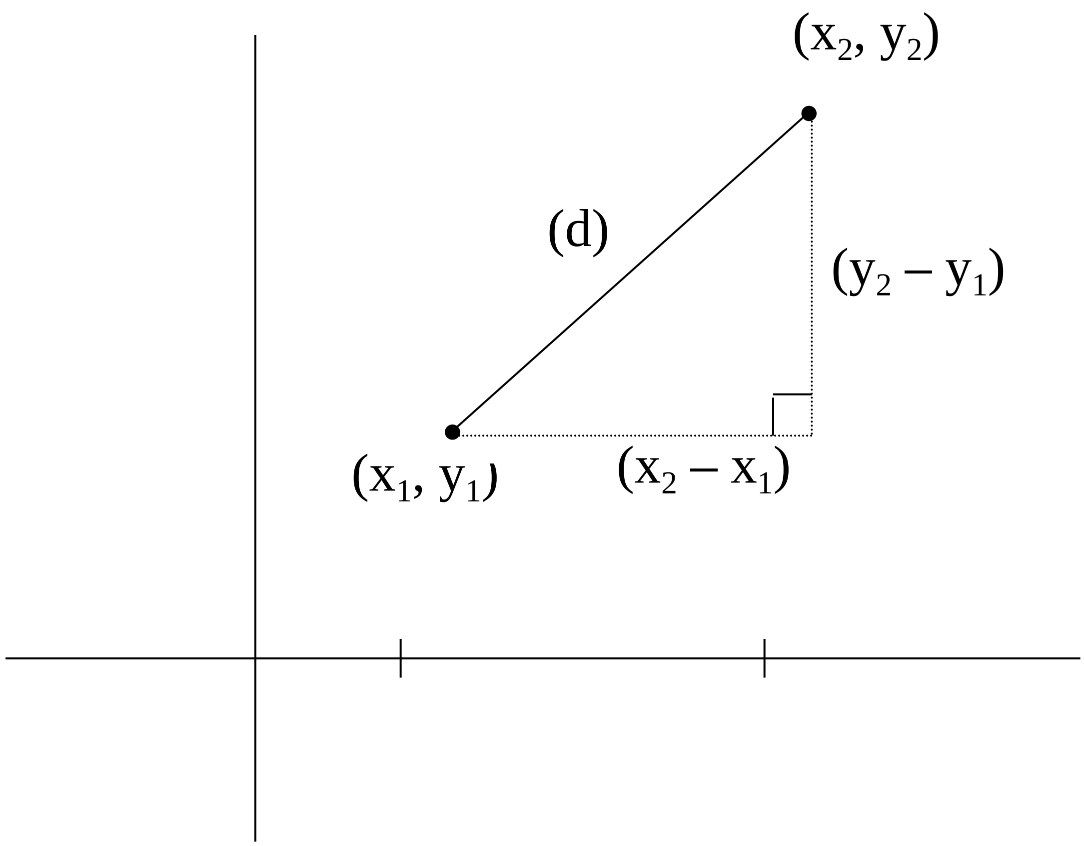
The distance between two points (x_1, y_1) and (x_2, y_2) is given by the distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



This comes from the Pythagorean Theorem.

The Distance Formula



From Pythagoras' Theorem

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Determining the Length of a Segment

So for example given (2,5) and (3,9) the distance between these points is

$$d = \sqrt{(2-3)^2 + (5-9)^2}$$

$$d = \sqrt{(-1)^2 + (-4)^2}$$

$$d = \sqrt{1+16}$$

$$d = \sqrt{17}$$

We will need the converse of the Pythagorean Theorem to understand why lines whose slopes have a product of –1 must necessarily be perpendicular.

Converse of the Pythagorean Theorem:
If the lengths of the sides of a triangle, a, b, and c, satisfy $a^2 + b^2 = c^2$, then the triangle is a right triangle.

One More Piece of the Puzzle

A little algebra:

$$
\begin{aligned}
(a + b)^2 &= (a + b)(a + b) \\
&= A(a + b) \text{ (think of } A = (a + b)) \\
&= Aa + Ab \\
&= (a + b)a + (a + b)b \\
&= a^2 + ba + ab + b^2 \\
&= a^2 + ab + ab + b^2 \\
&= a^2 + 2ab + b^2
\end{aligned}
$$

Now try:

$$(a - c)^2 =$$

$$(b - d)^2 =$$

One More Piece of the Puzzle

A little algebra:

$$
\begin{aligned}
(a + b)^2 &= (a + b)(a + b) \\
&= A(a + b) \text{ (think of } A = (a + b)) \\
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&= (a + b)a + (a + b)b \\
&= a^2 + ba + ab + b^2 \\
&= a^2 + ab + ab + b^2 \\
&= a^2 + 2ab + b^2
\end{aligned}
$$

Now try:

$(a - c)^2 =$

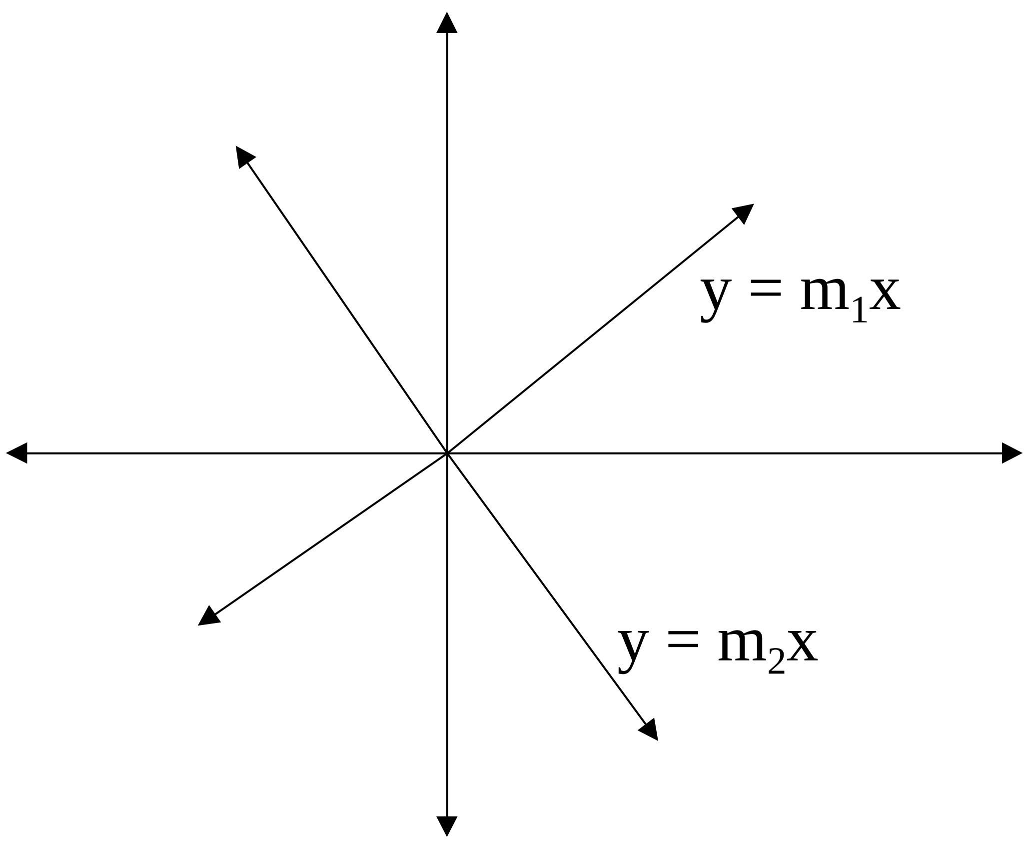
$(b - d)^2 =$

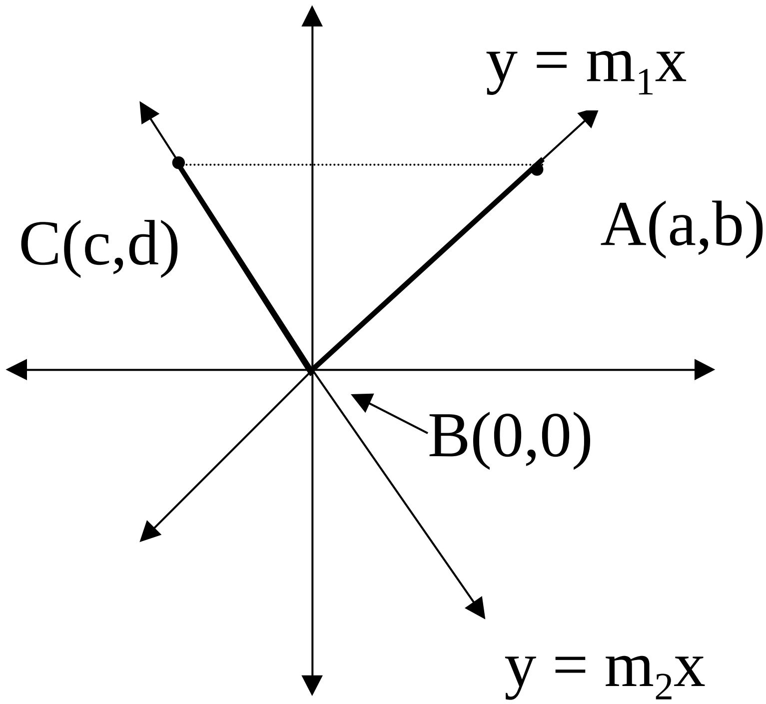
Answer: $(a - c)^2 = a^2 - 2ac + c^2$

$(b - d)^2 = b^2 - 2bd + d^2$

Goal: Show that two non-vertical lines are perpendicular if and only if the product of their slopes is –1.

For simplicity, assume that both intersecting lines have y-intercepts equal to zero.





Let (a,b) be a point on the graph of $y = m_1x$ and (c,d) be a point on the graph of $y = m_2x$.

Then $m_1 = \dfrac{b-0}{a-0} = \dfrac{b}{a}$ and $m_2 = \dfrac{d-0}{c-0} = \dfrac{d}{c}$.

Assume that triangle ABC is a right triangle. We want to show that $m_1m_2 = \dfrac{b}{a} \bullet \dfrac{d}{c} = -1$.

The Pythagorean Theorem says

$(AB)^2 + (BC)^2 = (AC)^2$

$AB = \sqrt{a^2 + b^2}$ $\qquad AC = \sqrt{(a-c)^2 + (b-d)^2}$

$BC = \sqrt{c^2 + d^2}$ $\qquad$ Now substitute

Perpendicular Lines Continued

$$\left(\sqrt{a^2 + b^2}\right)^2 + \left(\sqrt{c^2 + d^2}\right)^2 = \left(\sqrt{(a-c)^2 + (b-d)^2}\right)^2$$

$$\begin{bmatrix} \text{Remember our goal is to show that} \\ m_1 m_2 = \frac{b}{a} \bullet \frac{d}{c} = -1 \end{bmatrix}$$

$$a^2 + b^2 + c^2 + d^2 = (a - c)^2 + (b - d)^2$$
$$a^2 + b^2 + c^2 + d^2 = a^2 - 2ac + c^2 + b^2 - 2bd + d^2$$

Now subtract a^2, b^2, c^2, and d^2 from both sides.

$$\begin{aligned} 0 &= -2ac - 2bd \\ 2bd &= -2ac \\ bd &= -ac \\ \frac{bd}{ac} &= -1 \\ \frac{b}{a} \bullet \frac{d}{c} &= -1 \\ m_1 m_2 &= -1 \quad \text{(Whew!!)} \end{aligned}$$

What have we done?

We have shown that if two lines are perpendicular, then the product of their slopes is –1.

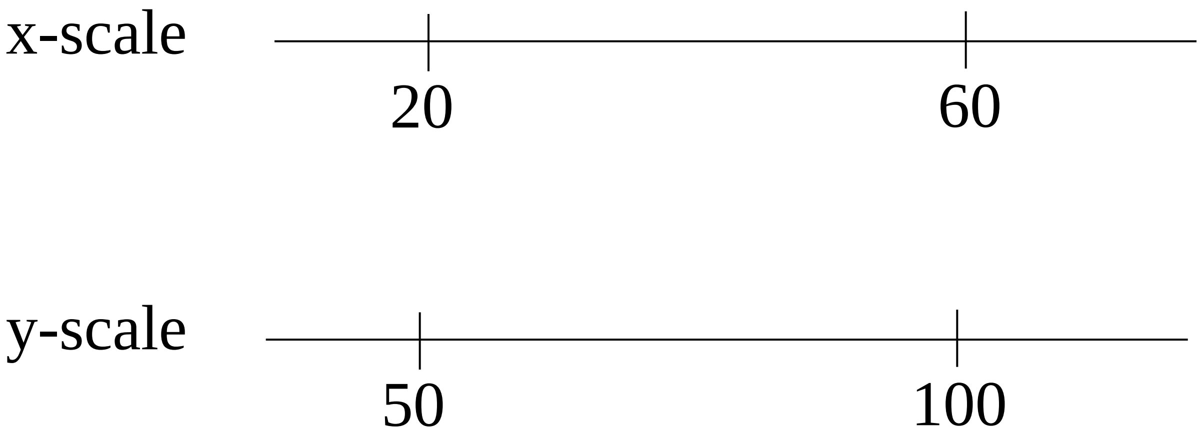
Now suppose the product of the slopes is –1. Can we deduce that the lines are perpendicular?

Yes!

Start at the bottom of the previous slide and work backward. The last step is to apply the converse of the Pythagorean Theorem.

Theorem: Two non-vertical lines are perpendicular if and only if the product of their slope is –1.

SAT Problem



On the linear scales above, 20 and 60 on the x-scale correspond to 50 and 100, respectively, on the y-scale. Which of the following linear equations could be used to convert an x-scale value to a y-scale value?

A) $y = x + 30$

B) $y = x + 40$

C) $y = 1.25x + 25$

D) $y = 1.25x + 30$

E) $y = 0.8x + 34$

Solving Inequalities

Solving inequalities is similar to solving equations, except that instead of an equal sign one has an inequality. The main fact that you need to remember is that when you multiply or divide both sides of the inequality by a negative number, it reverses the inequality.

Let's examine this number sentence

$$3 > -5$$

If I multiply both sides by –1

$$3(-1) \underline{\quad ? \quad} (-5)(-1)$$

$$-3 \quad \underline{\quad < \quad} \quad 5$$

You can see that if we put the original inequality (greater than) in the blank, the sentence is no longer true. The inequality sign must change to less than (<).

How do we graph linear inequalities?

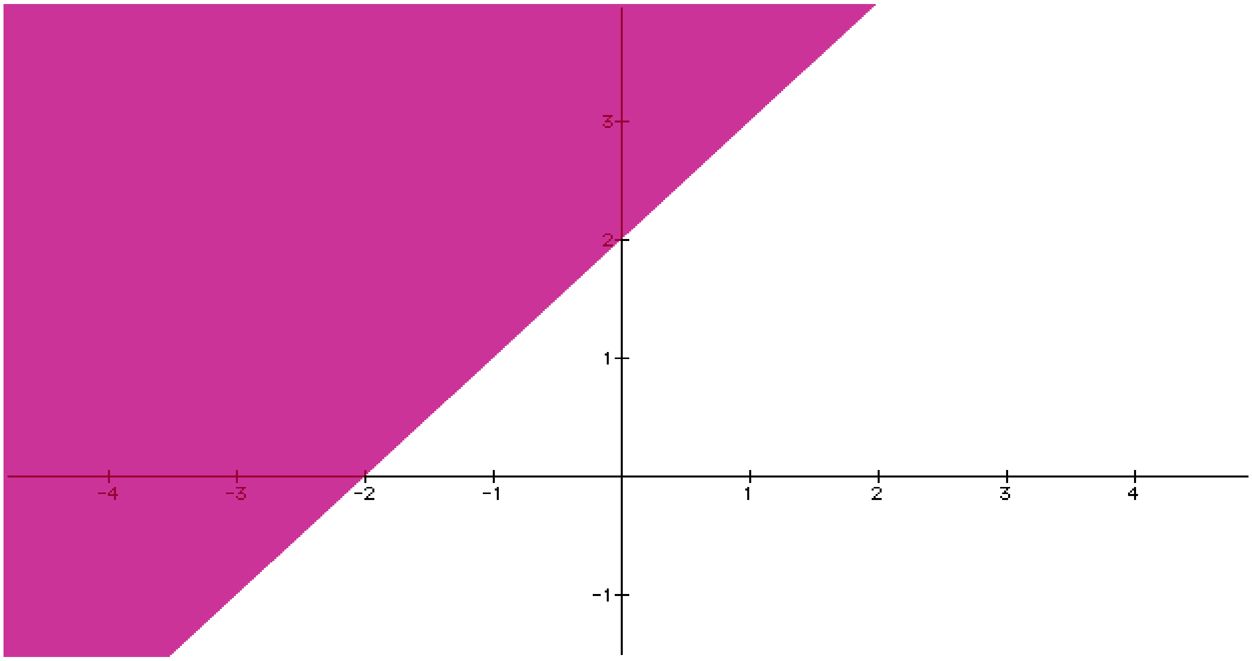
Example

Graph $y > x + 2$

First: Identify the equation of the line $y = x + 2$.

x-intercept: When $y = 0$ what is x? -2
So coordinates of one point $(-2,0)$

y-intercept: When $x = 0$ what is y? 2
So coordinates of second point $(0,2)$

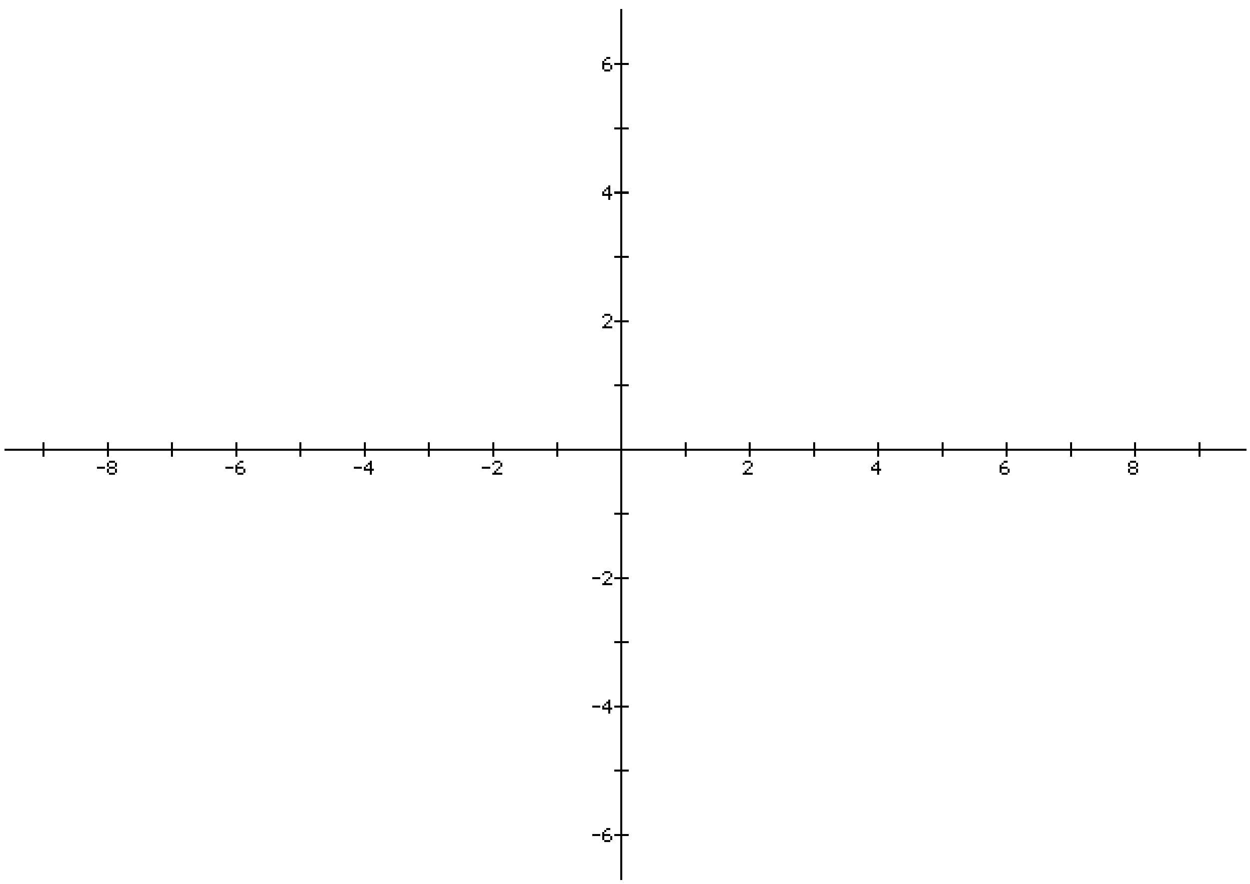


(The graph does not include the line y = x + 2)

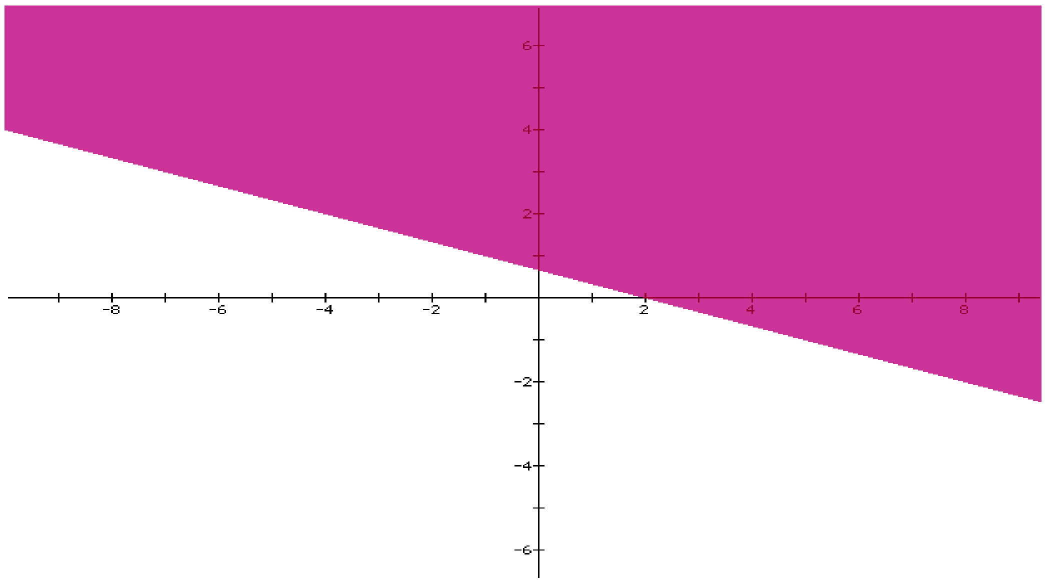
For a linear inequality of the form y>mx+b, the y-values satisfying the inequality are greater than the corresponding y-values which satisfy the equation y=mx+b. It follows that the half-plane above the graph is shaded.

NOTE: The equation needs to be in slope-intercept form for the above assumption to be made. Also the coefficient of y should be +1.

What would the graph of $2x + 6y > 4$ look like?



How would the graph change if the problem was $2x + 6y \geq 4$?



From the inequality 2x + by > 4, we can isolate y to get $y > -\frac{1}{3}x + \frac{2}{3}$. The graph of this inequality is the set of all points of the form (x,y) where $y > -\frac{1}{3}x + \frac{2}{3}$.

If the problem were instead, 2x + 6y ≥ 4, then the graph would include the line.

Graph the inequality:

$$2x - 3y \le 9$$

First isolate y:

$$2x - 3y \le 9$$
$$-3y \le -2x + 9$$
$$y \ge \frac{2}{3}x - 3 \text{ (Why did } \le \text{ switch to } \ge\text{?)}$$