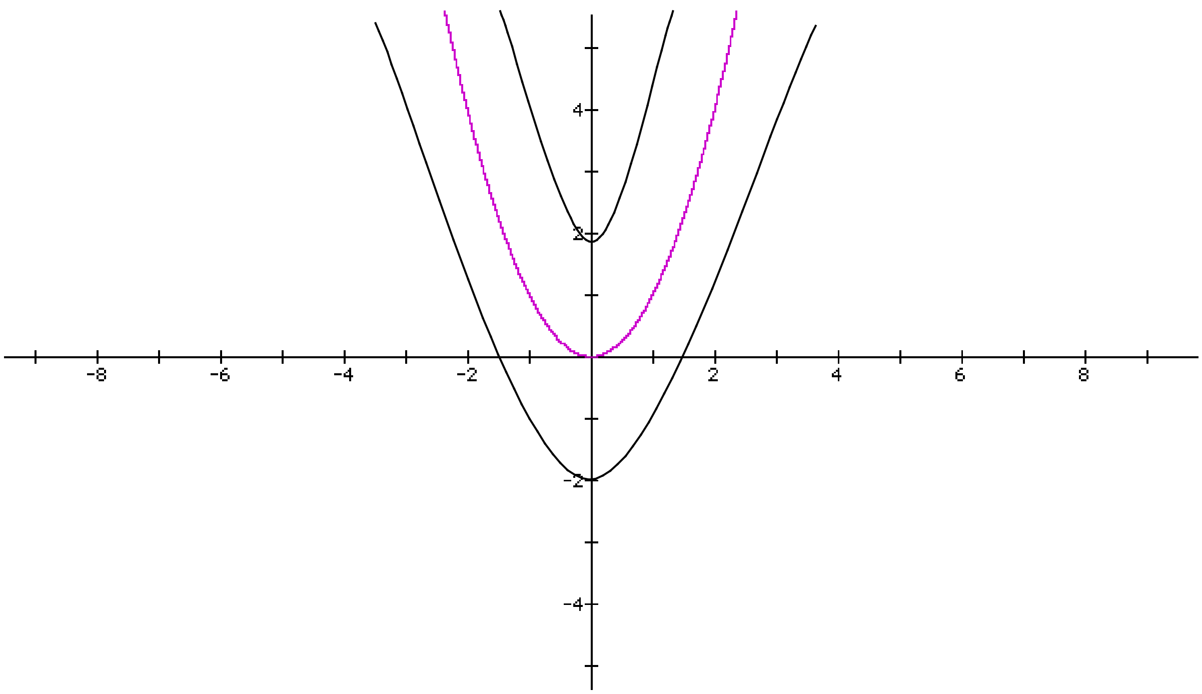
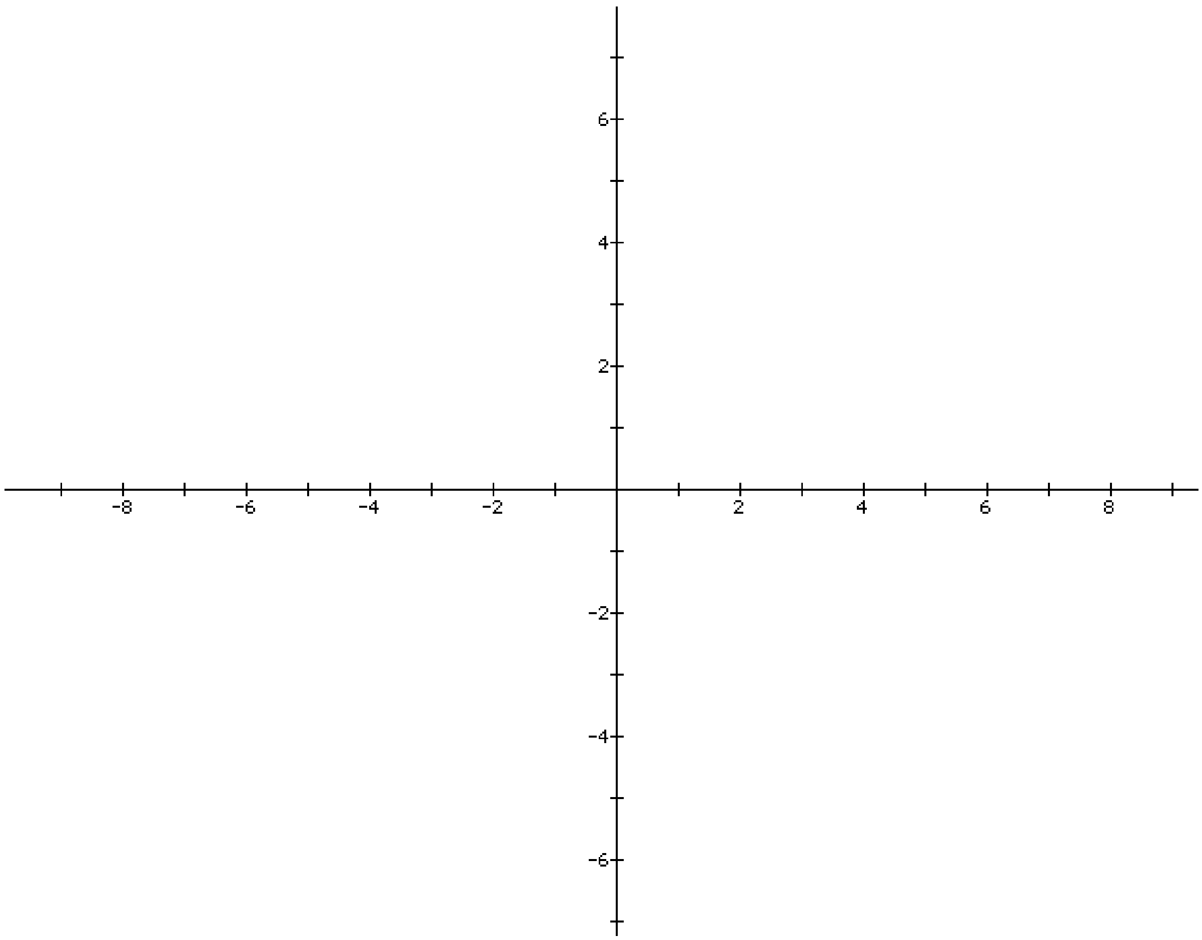
x	$y = x^2$	$y = x^2 + 2$	$y = x^2 – 2$
–2	$(–2)^2 = 4$	$(–2)^2 + 2 = 6$	$(–2)^2 – 2 = 2$
–1	$(–1)^2 = 1$	$(–1)^2 + 2 = 3$	$(–1)^2 – 2 = –1$
0	$(0)^2 = 0$	$(0)^2 + 2 = 2$	$(0)^2 – 2 = –2$
1	$(1)^2 = 1$	$(1)^2 + 2 = 3$	$(1)^2 – 2 = –1$
2	$(2)^2 = 4$	$(2)^2 + 2 = 6$	$(2)^2 – 2 = 2$

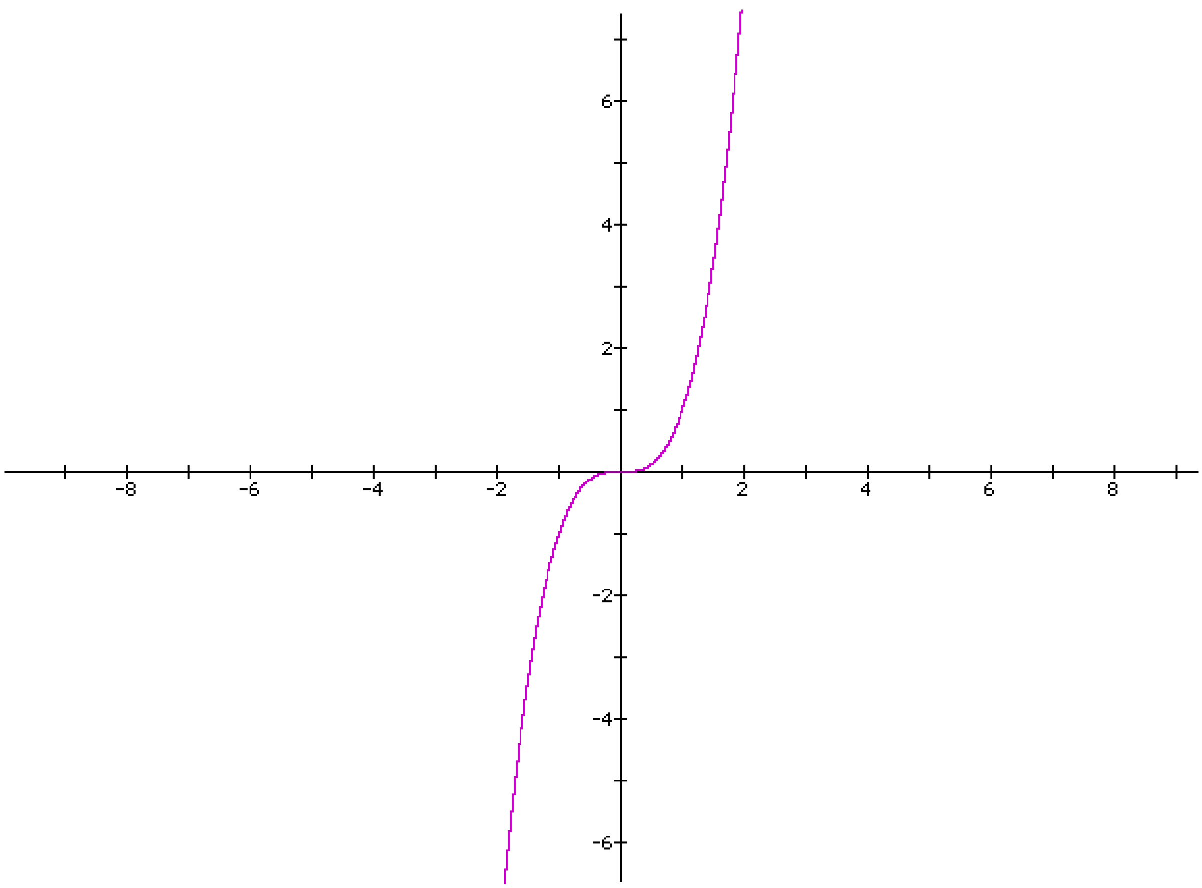


x	$y = x^3$
–2	
–1	
0	
1	
2	

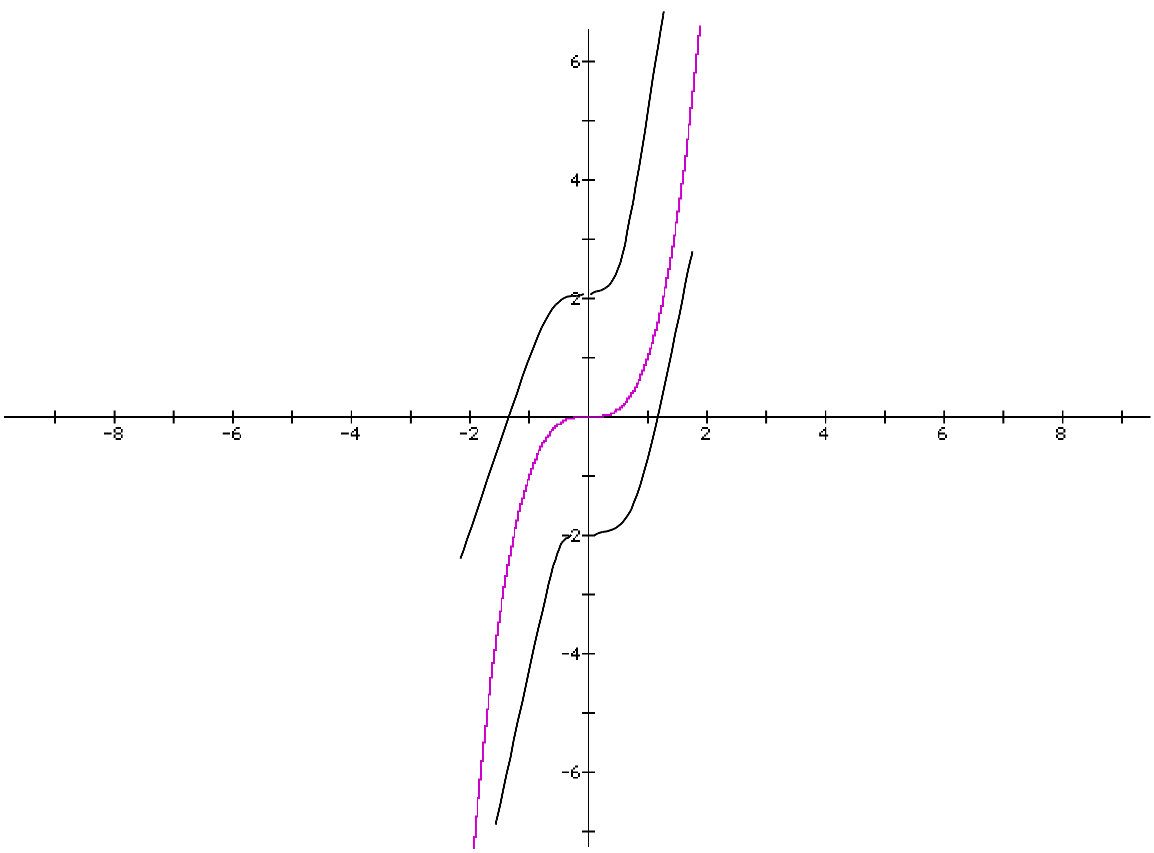


Complete the table and graph $y = x^3$

x	$y = x^3$
–2	$(-2)^3 = 8$
–1	$(-1)^3 = -1$
0	$(0)^3 = 0$
1	$(1)^3 = 1$
2	$(2)^3 = 8$

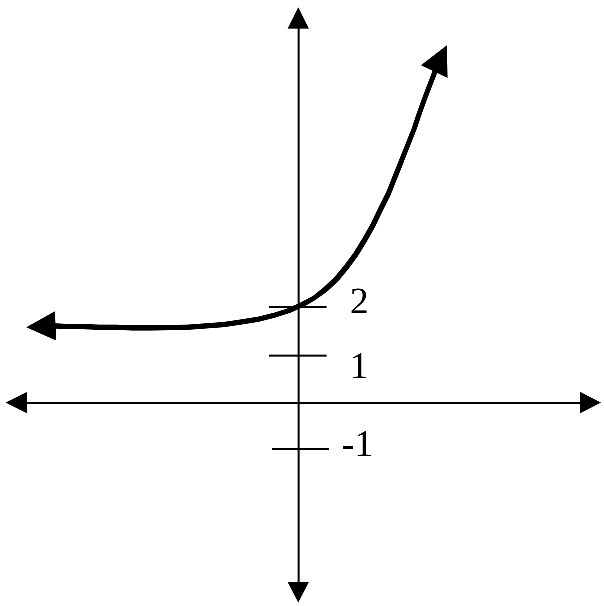


x	$y = x^3$	$y = x^3 + 2$	$y = x^3 - 2$
–2	$(-2)^3 = -8$	$(-2)^3 + 2 = -6$	$(-2)^3 - 2 = -10$
–1	$(-1)^3 = -1$	$(-1)^3 + 2 = 1$	$(-1)^3 - 2 = -3$
0	$(0)^3 = 0$	$(0)^3 + 2 = 2$	$(0)^3 - 2 = -2$
1	$(1)^3 = 1$	$(1)^3 + 2 = 3$	$(1)^3 - 2 = -1$
2	$(2)^3 = 8$	$(2)^3 + 2 = 10$	$(2)^3 - 2 = 6$

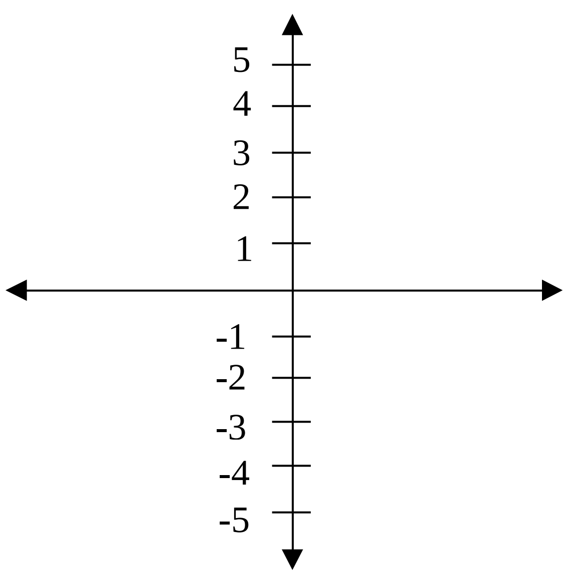


Vertical Translations of Functions

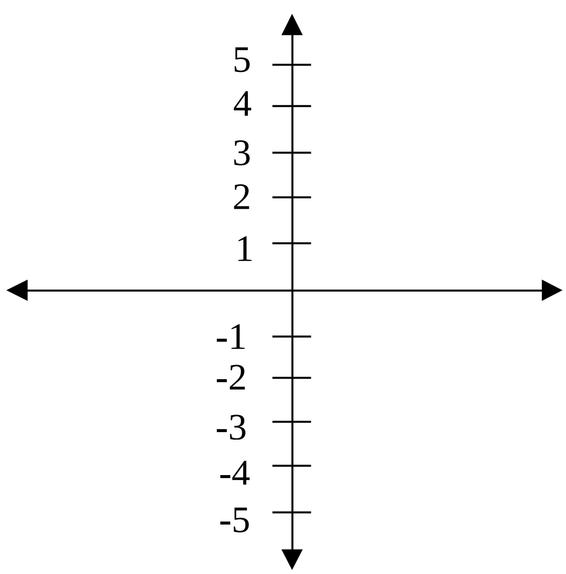
Given the graph of f(x)



Graph f(x) + 2



Graph f(x) – 3

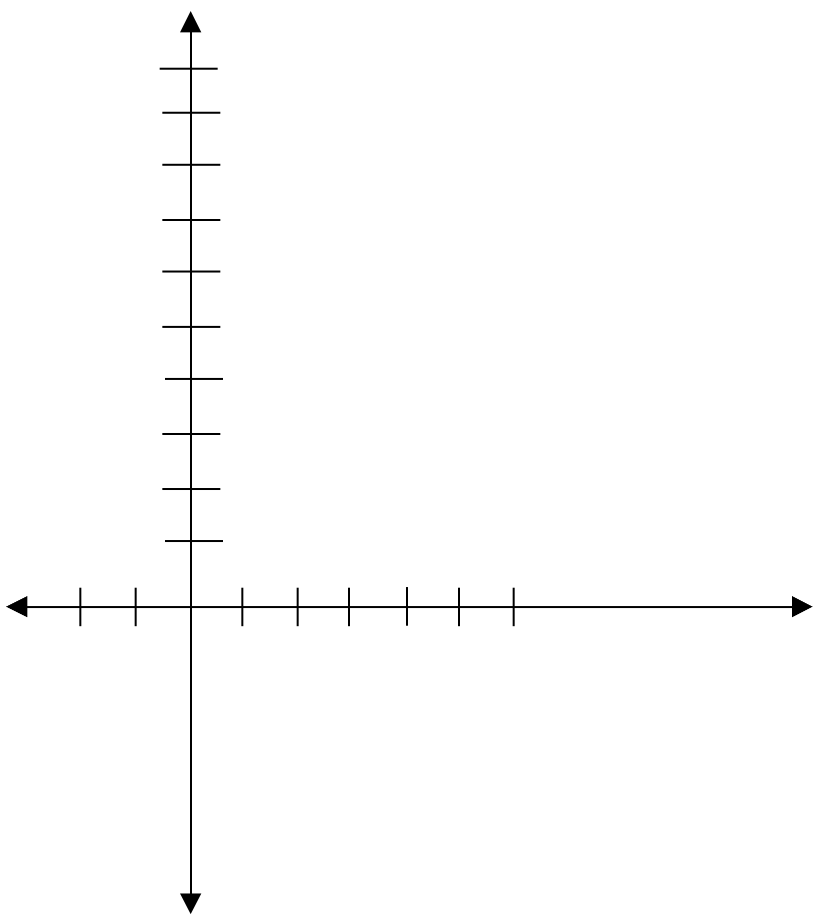


We're now going to focus on linear functions, beginning with the general equation of $y = mx$.

Choose a value for m, say $m = 2$, then $y = mx$ becomes $y = 2x$.

Fill in the table for y then plot the points.

x	y
0	
2	
3	
5	



Points on the graph of $y = 2x$ appear to form a straight line.

The graph of $y = mx$, for any value of m, also appears to be a line.

But how can we be sure that the graph of

$$y = mx$$

is really a straight line?

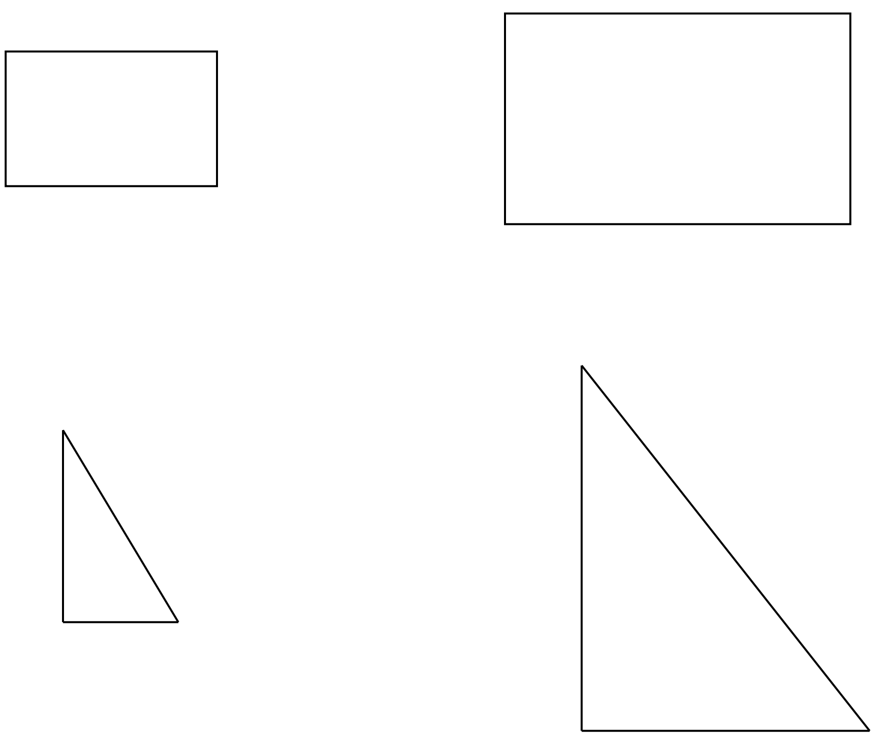
Rewrite $y = mx$ as

$$\frac{y}{x} = m \quad \text{(when } x \neq 0\text{).}$$

Similar Triangles

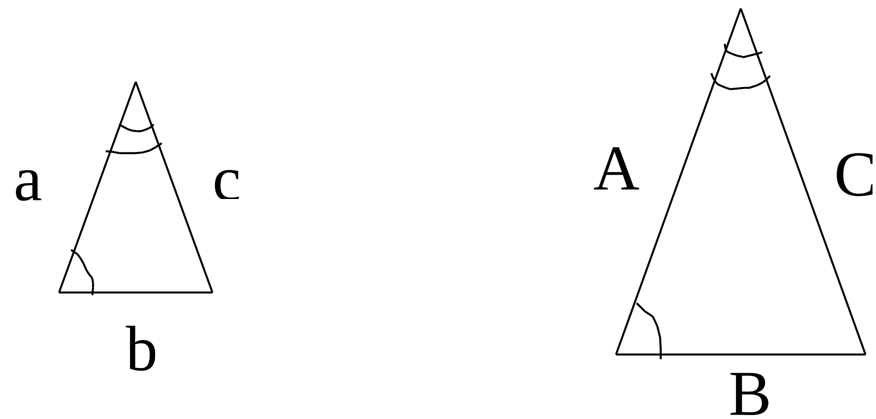
In order to understand why the graph of $y = mx$ is a straight line, we need to understand "similarity."

Roughly speaking, two geometric figures are "similar" if they have the same shape, but possibly different sizes.



Similar Triangles

Two triangles are similar if the measures of all corresponding angles are equal and ratios of corresponding sides of the two triangles are the same.



In the picture measures of corresponding angles are equal AND

$$\frac{a}{A} = \frac{b}{B} = \frac{c}{C}$$

Cross multiplication shows other ratios are the same. E.g.,

$$\frac{a}{b} = \frac{A}{B}$$

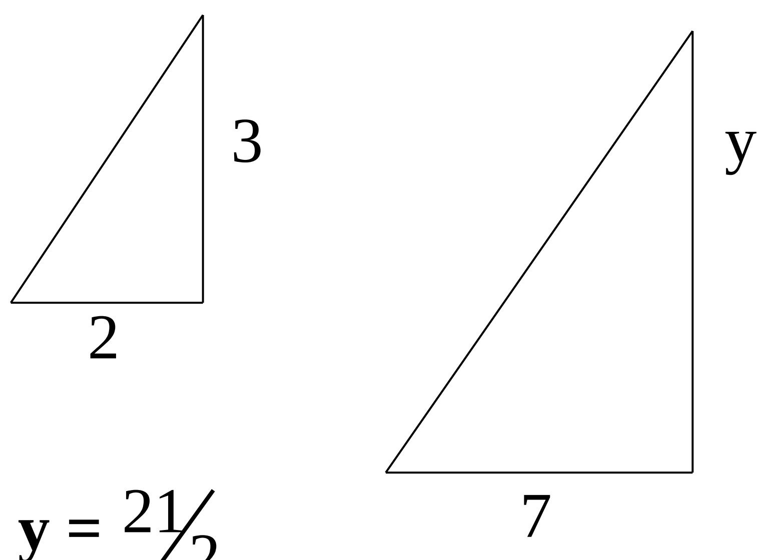
Similar Triangles

Actually it is not necessary to look at BOTH angles and ratios of sides. Either angles or side ratios determine similarity (as well as various other combinations).

A Theorem in geometry says that two triangles are similar if and only if:

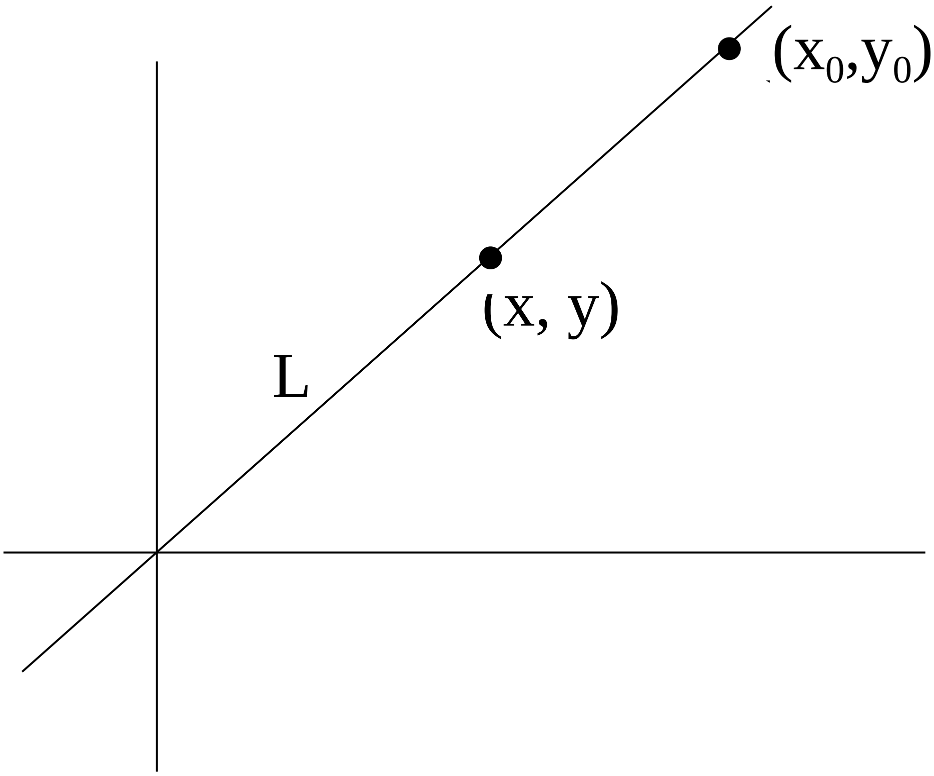
they have equal angle measures
OR
all ratios of corresponding sides are the same

Example. The triangles below are similar. Find the length y of the side of the larger triangle.



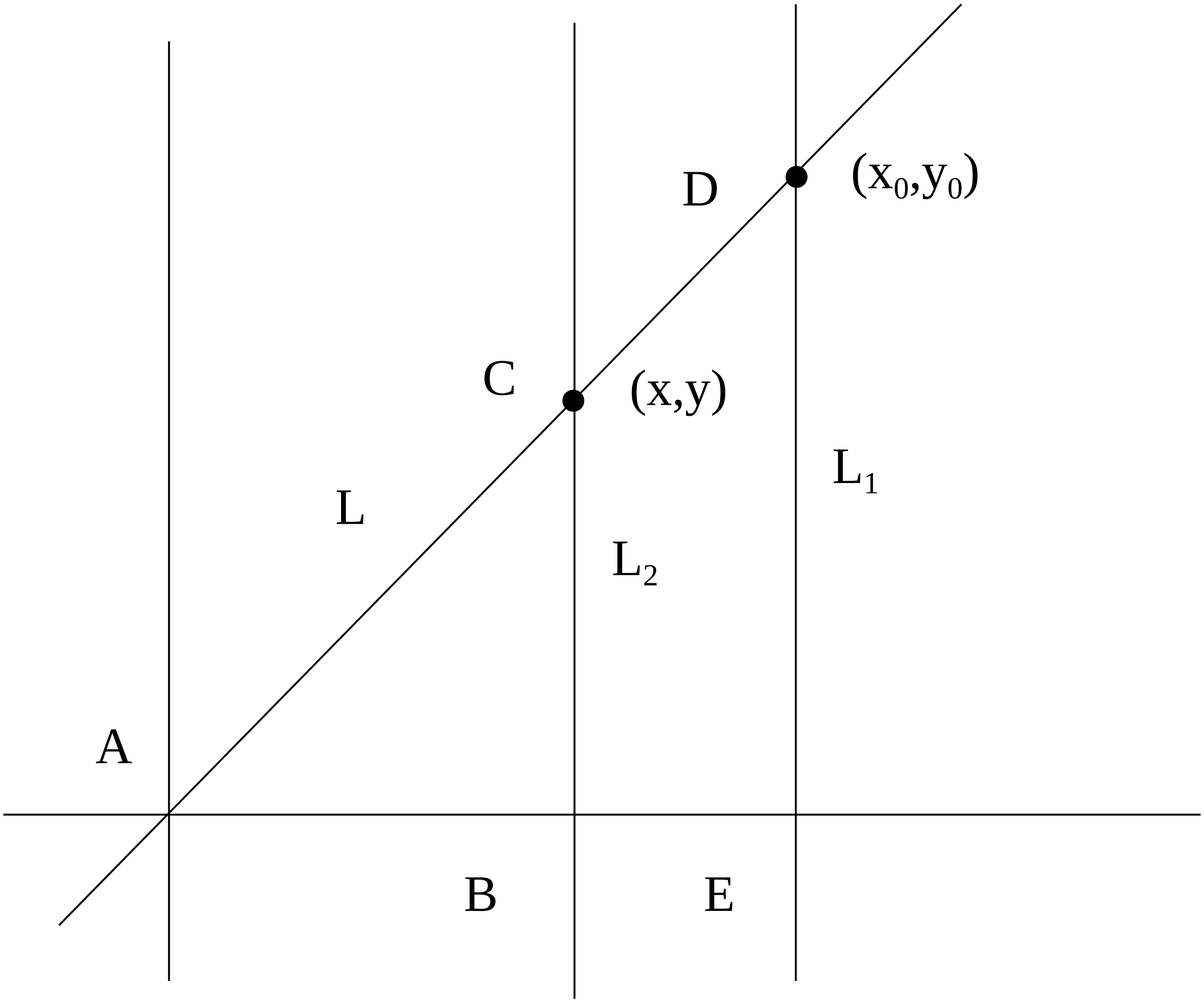
Answer: y = $21/2$

How Do We Know That The Graph of $y = mx$ is a Straight Line ($m \neq 0$)?



Choose a point (x_0, y_0) on the GRAPH of $y = mx$. Let (x, y) be an arbitrary point on the LINE L from the origin to (x_0, y_0). We are NOT assuming here that (x, y) is on the graph of $y = mx$. We will deduce that it is. For simplicity, assume all coordinates are positive as the picture indicates.

Construct vertical lines L_1 through (x_0, y_0) and L_2 through (x, y).



Triangle ABC and triangle AED are both right triangles with the same vertex A. Therefore all three corresponding angles have equal measure, and the two triangles are SIMILAR. Therefore ratios of corresponding sides must be equal.

Since $\overline{AB} = x$, $\overline{AE} = x_0$, $\overline{BC} = y$, and $\overline{DE} = y_0$, this tells us that

$$\frac{y}{x} = \frac{y_0}{x_0}$$

GRAPH OF $y = mx$ IS A STRAIGHT LINE

But $\frac{y_0}{x_0} = m$ because (x_0, y_0) lies on the graph of $y = mx$.

Therefore $\frac{y}{x} = \frac{y_0}{x_0} = m$

Therefore $y = mx$

Therefore (x, y) lies on the graph of $y = mx$

We have shown that any point (x, y) on the LINE from the origin to ANY point on the GRAPH of $y = mx$ lies on the GRAPH.

Therefore the graph of $y = mx$ contains the straight line from the origin to any point on the graph of $y = mx$.

Therefore the graph of $y = mx$ is a straight line, since if the graph contained any points in addition to those on the line, it would violate the vertical line test.

Converse: Any Straight Line Through the Origin is the Graph of $y = mx$ for Some Value of m

What about the Converse? We have shown that the graph of any equation of the form $y = mx$ is a straight line. Is it necessarily true that any nonvertical straight line containing the point (0, 0) is the graph of an equation of the form $y = mx$ for some value of m?

Yes!

Here is one way to deduce this result:

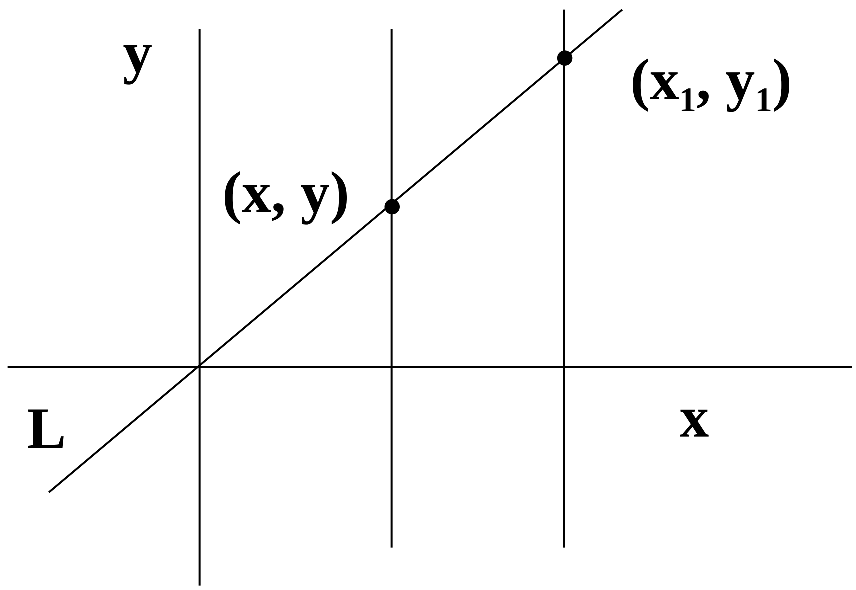
Let L be a line through the origin. Choose any point (x_0, y_0) on L other than (0,0) and define

$$m = \frac{y_0}{x_0}.$$

If (x, y) is any other point on the line, all we have to do is show that

$$y = mx.$$

Converse Argument



As in the previous argument, by similar triangles,

$$\frac{y}{x} = \frac{y_0}{x_0}$$

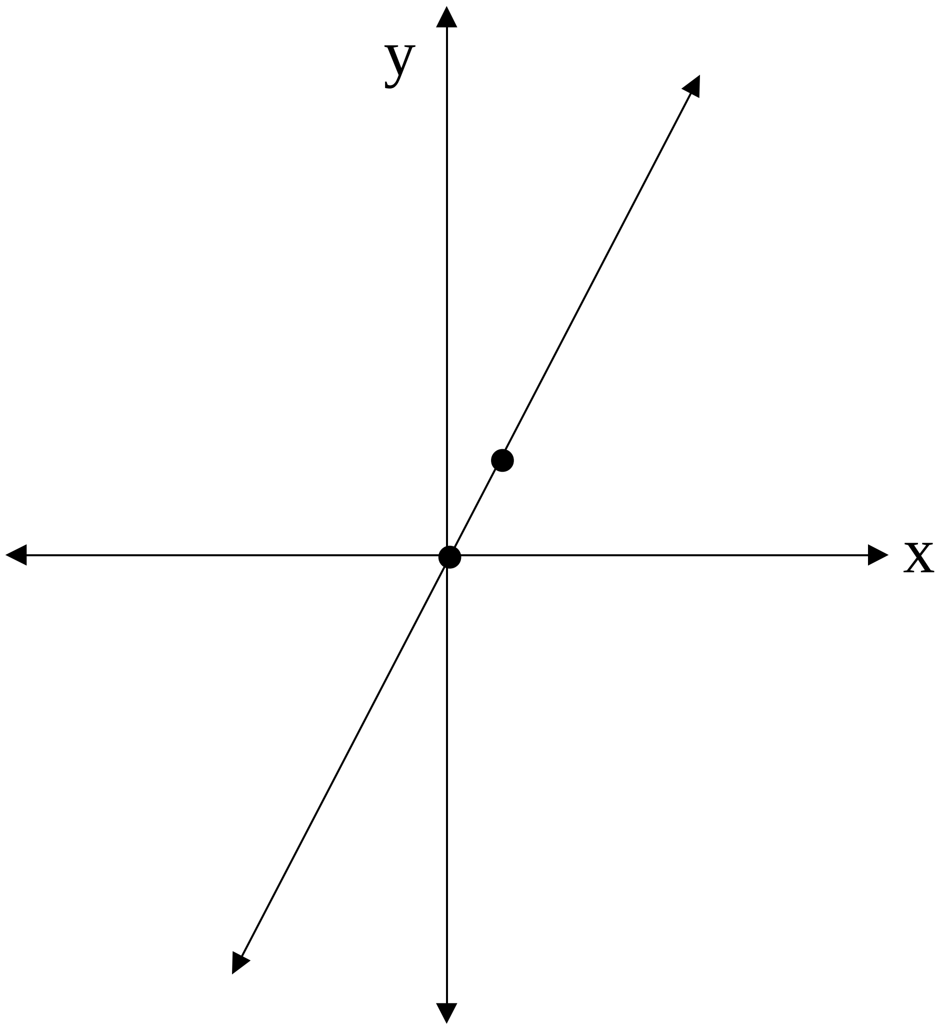
Since $m = \frac{y_0}{x_0}$ it follows that $\frac{y}{x} = m$. Therefore, for any point (x,y) on the line L, y = mx. Therefore any line is the graph of an equation of the form *y* = m*x*, where m is the ratio of the y coordinate to the x coordinate for any point on the line L (except (0,0)).

From geometry, we know that two points determine a line. Since the graph of y = mx is always a line, it is enough to plot two different points and connect them with a straightedge to draw the graph.

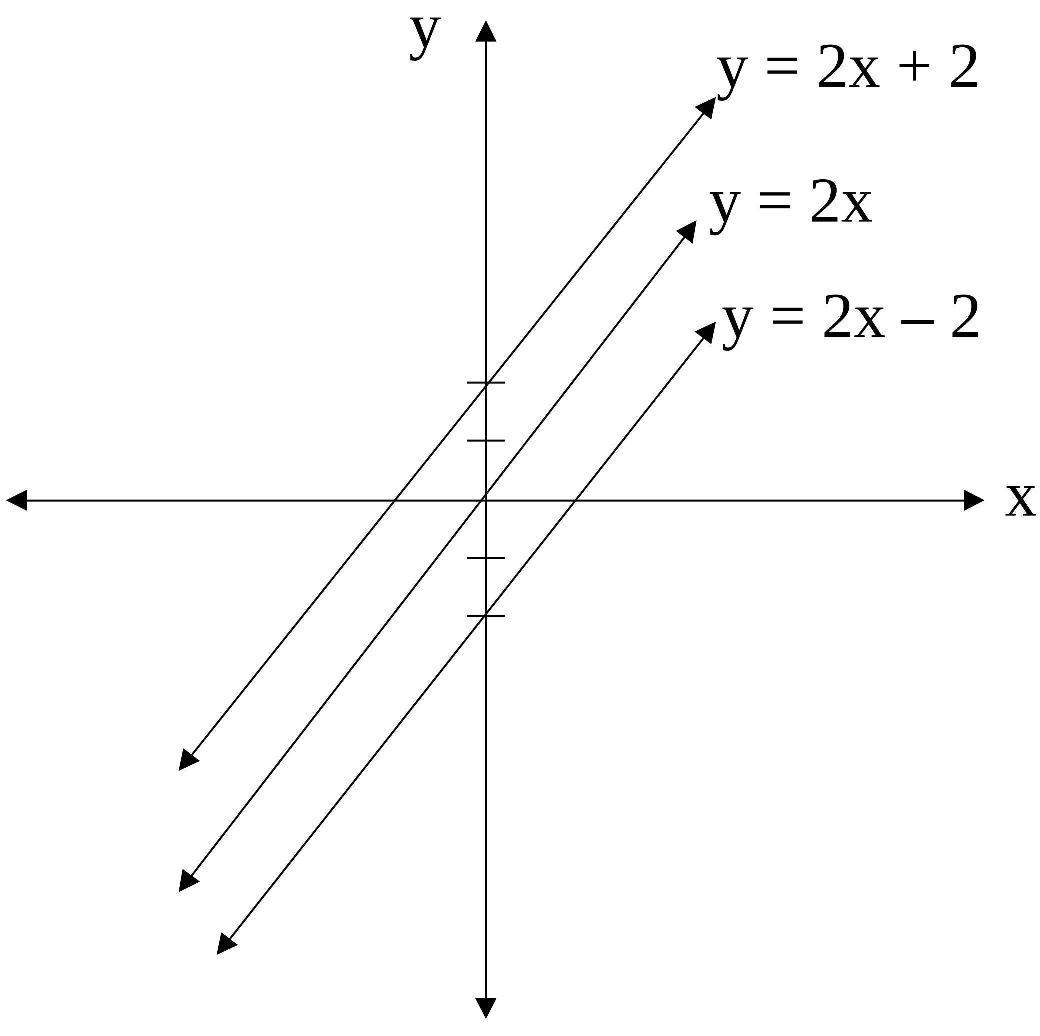
Try this for y = 2x.

Choose any two values for x, and find the y-values.

x	y = 2x
0	0
1	2



x	y = 2x	y = 2x + 2	y = 2x –2
0	0	0 + 2 = 2	0 – 2 = –2
1	2	2 + 2 = 4	2 – 2 = 0



What happens to the graph when y = mx is replaced by y = mx + b, if:

b = 2?

b = –2?

How does the number b affect the graph of y = mx + b?

Answer: It translates the graph of y = mx vertically.

Since the graph of y = mx is a line, it follows that the graph of y = mx + b is always a line too.

