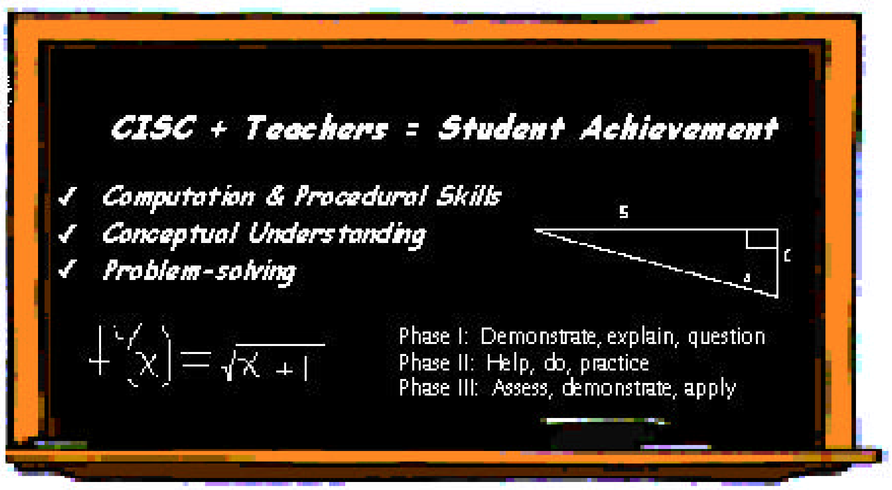
The Winning EQUATION

A HIGH QUALITY MATHEMATICS PROFESSIONAL DEVELOPMENT PROGRAM FOR TEACHERS IN GRADES 4 THROUGH ALGEBRA II



STRAND: NUMBER SENSE: Fractions and Decimals

MODULE TITLE: **PRIMARY CONTENT MODULE VI**

MODULE INTENTION: The intention of this module is to inform and instruct participants in the underlying mathematical content in the areas of fractions and decimals.

THIS ENTIRE MODULE MUST BE COVERED IN-DEPTH.

The presentation of these Primary Content Modules is a departure from past professional development models. The content here, is presented for individual teacher's depth of content in mathematics. Presentation to students would, in most cases, not address the general case or proof, but focus on presentation with numerical examples.

In addition to the underlying mathematical content provided by this module, the facilitator should use the classroom connections provided within this binder and referenced in the facilitator's notes.

TIME: 2 hours

PARTICIPANT OUTCOMES:

- Demonstrate understanding of fractions and decimals.
- Demonstrate understanding of the relationships of fractions and decimals.
- Demonstrate how to convert fractions to decimals and decimals to fractions for repeating and non-repeating decimals.

PRIMARY CONTENT MODULE VI
NUMBER SENSE: Fractions and Decimals

Facilitator's Notes

Pre- Post-Test

Ask participants to take the pre-test. After reviewing the results of the pre-test proceed with the following lesson on fractions and decimals.

Arithmetic provides two ways of representing non-whole numbers: decimals and fractions. An understanding of the connections between these notations is an important part of arithmetic. While the ability to transform decimals into fractions and fractions into decimals is a practical skill of importance in itself, it also leads to some profound insights into the concept of "number."

T-1

Recall the decomposition of whole numbers in powers of ten. An example is

$$428 = 4 \bullet 100 + 2 \bullet 10 + 8 \bullet 1$$

Decimals Into Fractions: By recalling their definition in terms of powers of 10, the problem of transforming decimals into fractions becomes straightforward. Facilitator should guide teachers through each step in this example. For example, to express .428 as a fraction we recall that

$$.428 = 4 \bullet 10^{-1} + 2 \bullet 10^{-2} + 8 \bullet 10^{-3}$$

$$.428 = 4 \bullet \frac{1}{10} + 2 \bullet \frac{1}{100} + 8 \bullet \frac{1}{1000}$$

$$= \frac{4}{10} + \frac{2}{100} + \frac{8}{1000}$$

$$= \frac{4}{10}\left(\frac{100}{100}\right) + \frac{2}{100}\left(\frac{10}{10}\right) + \frac{8}{1000}$$

$$= \frac{428}{1000}$$

One representation of .428 in fractional form is $\frac{428}{1000}$. In order to reduce this fraction to lowest terms, we note that GCD(428, 1000) = 4. Dividing top and bottom by 4 we obtain a fraction in simplified form.

$$.428 = \frac{107}{250}$$

H-1

Have participants do H-1 for practice converting decimals to fractions.

T-2

Fractions Into Decimals: One way of obtaining "$\frac{3}{4}$ = 0.75" is by long division. Recalling that one interpretation of the fraction $\frac{a}{b}$ is "the answer to the division problem $a \div b$." We carry out the division process 3 ÷ 4 as follows:

$$\begin{array}{r} .75 \\ 4\overline{)3.00} \\ \underline{28} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

H-2

Have participants do H-2 for practice converting fractions to decimals.

T-3

But why does dividing numerator by denominator give the correct decimal? When converting a fraction into a decimal by dividing numerator by denominator, we essentially find an equivalent fraction whose denominator is a power of 10, i.e. a fraction of the form:

$$\frac{N}{10^n}$$

T-4

In the case of $\frac{3}{4}$, we are to find $\frac{3}{4} = \frac{N}{10^2}$ (i.e, N=2), or $\frac{3}{4} = \frac{N}{100}$

(How do we know to use 100 for the searched equivalent fraction? We'll come back to this question.) Solve N, we use cross multiplication.

$$\mathbf{4N = 3 \bullet 100 = 300}$$
$$\mathbf{N =} \frac{300}{4} = \frac{300 \div 4}{4 \div 4} = \frac{300 \div 4}{1} = \mathbf{300 \div 4}$$

Then using long division, we obtain N=75. Long division shows

$$\frac{3}{4} = \frac{75}{100} = \mathbf{.75}$$

T-5

Note, however, not all fractions may have an equivalent fraction whose denominator is a power of 10. Fractions like

$$\frac{2}{5} = \frac{4}{10^1} = \frac{4}{10} = .4$$
$$\frac{1}{16} = \frac{625}{10^4} = \frac{625}{10000} = .0625$$

have corresponding terminating decimal forms.

Other fractions can only be converted into decimals with an infinite number of decimal places.

$$\frac{1}{3} = .3333... = .\overline{3}$$

$$\frac{3}{11} = .272727... = .\overline{27}$$

T-6

How can we tell if a fraction can convert into a terminating decimal? Answer: For any fraction $\frac{a}{b}$ that is written in the lowest terms, it can be shown that the corresponding decimal terminates **only** when $b = 2^m \bullet 5^n$ for m, n = 0, 1, 2, … Note: m or n could be zero.

As shown above, a terminating fraction equals a fraction whose denominator is a power of 10. Also recall that $(cd)^n = a^n b^n$ from Module 1. Therefore, $10^n = (2 \bullet 5)^n = 2^n \bullet 5^n$. This means that the only prime factors of 10^n are 2 and 5.

T-7

In order for $\frac{a}{b} = \frac{N}{10^n}$, we must multiply **a** and **b** by some whole number, let's say, ***k*** so that the answer is ***bk*** $= 10^n$. Since ***bk*** $= 10^n = (2 \bullet 5)^n = 2^n \bullet 5^n$, we can conclude, that ***b*** (as well as ***k***) has only prime factors of 2 and/or 5.

For example,

$$\frac{1}{8} = \frac{1\bullet 125}{8\bullet 125} = \frac{125}{1000}$$

or $$\frac{1}{8} = \frac{1}{2^3} = \frac{1\bullet 5^3}{2^3\bullet 5^3} = \frac{125}{1000}$$

$$\frac{3}{40} = \frac{3\bullet 25}{40\bullet 25} = \frac{75}{1000}$$

or $$\frac{3}{40} = \frac{3}{2^3\bullet 5} = \frac{3\bullet 5^2}{(2^3\bullet 5)\bullet 5^2} = \frac{3\bullet 5^2}{2^3\bullet 5^3} = \frac{75}{1000}$$

T-8

This also explains why, in the case of $\frac{3}{4}$, we chose to convert it to $\frac{N}{10^2} = \frac{N}{100}$ because $\frac{3}{4} = \frac{3}{2^2} = \frac{3\bullet 5^2}{2^2\bullet 5^2} = \frac{75}{100}$

T-9

Fractions Into Repeating Decimals: The second method described above made use of the fact that, when written in lowest terms, the denominators of $\frac{1}{16}$, $\frac{3}{8}$, $\frac{1}{50}$, and $\frac{12}{75} = \frac{4}{25}$ all have prime factorizations consisting only of 2s and 5s. For fractions such as $\frac{1}{3}$, $\frac{4}{13}$, $\frac{5}{12}$, it is impossible to transform them into terminating decimal form.

It is possible to use long division to relate such fractions to a repeating form in decimal notation. Here we use long division to show that $\frac{1}{3} = .3333\ldots = .\overline{3}$ by

$$\begin{array}{r} .3333\ldots \\ 3\overline{)1.0000\ldots} \\ \underline{9} \\ 10 \\ \underline{9} \\ 10 \text{ etc.} \end{array}$$

T-10

Indeed, long division shows that it is possible to convert **any** fraction $\frac{a}{b}$ into a terminating or a repeating decimal. Regarding $\frac{a}{b}$ as the solution of a division problem of the form $a \div b$, we note that long division eventually involves "bringing down zeros." These zeros are appended to remainders that are whole numbers smaller than the divisor b. Thus, in the course of at most b divisions we must have a remainder that appeared before. When this occurs, we fall into a repeating pattern that leads to a repeating decimal representation for $a \div b$.

By way of example, $\frac{3}{11}$ corresponds to the long division problem

$$\begin{array}{r} .2727\ldots \\ 11\overline{)3.0000\ldots} \\ \underline{22} \\ 80 \\ \underline{77} \\ 30 \\ \underline{22} \\ 8 \text{ etc.} \end{array}$$

The fact that we alternate between remainders of 8 and 3 shows that the pattern .272727 ... will repeat indefinitely and that $\frac{3}{11} = .\overline{27}$.

T-11
H-3

Ask participants to find the decimal for $\frac{1}{7}$ and explain why it must have a repeating block. The repeating decimal pattern corresponding to $\frac{4}{13}$ is not revealed by the first eight digits of its decimal expansion. However, even without knowing what the repeating decimal is, long division assures the existence of a pattern of length at most 12.

T-12

$$
\begin{array}{r}
.3076\ldots \\
13\overline{)4.0000\ldots} \\
\underline{39} \\
100 \\
\underline{91} \\
90 \\
\underline{78}
\end{array}
\quad \text{etc.}
$$

The reason is that in applying the long division algorithm, the dividend 4 is written with an unending string of zeros to the right of the decimal point. Each time we apply the algorithm to obtain a new digit in the quotient, we obtain a remainder that is less than 13. Since the numbers 0 through 12 are the only possible remainders, 12 applications of the division process are sure to lead either to a remainder of zero (in which case we have a terminating decimal), or else to a positive remainder that has occurred previously. Since we are always bringing down zeros, a repeated remainder leads to an operation identical with one that has occurred before. As a result, this operation is followed by a pattern that will repeat indefinitely. This gives us the following point to consider.

H-3A

A Point to Consider: The number of places in a repeating pattern is, at most, 1 less than the divisor.

Have participants do worksheet H-3A on repeating decimals.

T-13

We have seen that reduced fractions of the form $\frac{a}{b}$ correspond to terminating decimals whenever b has prime factorization of the form $2^m\ 5^n$. For fractions whose decimal expansion fails to terminate, long division assures that $\frac{a}{b}$ corresponds to a repeating decimal. By regarding terminating decimals as a special kind of repeating decimal; i.e., one whose repeating pattern consists of zeros - one can assert that **all** fractions correspond to repeating decimals.

T-14

Repeating Decimals into Fractions: If you didn't already know that $.\overline{3} = \frac{1}{3}$, how could you deduce it?

Let:

$$x = .333...$$
$$10x = 3.333...$$

$$\begin{array}{rl} 10x = & 3.333... \\ -\ x = & -.333... \\ \hline 9x = & 3 \end{array}$$
$$\text{or } x = \frac{3}{9} = \frac{1}{3}$$

T-15

This technique works for longer repeating patterns as well. For example, to evaluate $x = .2373737\ ... = .2\overline{37}$, we note that

$$\begin{array}{rl} 100x = & 23.73737\ldots \\ -\ x = & -\ .23737\ldots \\ \hline 99x = & 23.5 \end{array}$$
$$x = \frac{235}{990} = \frac{47}{198}$$

An easier way to do this might be to consider having only the repeating portion to the right of the decimal, then when you subtract, you will always get a whole number for the numerator.

$$\begin{array}{rl} 1000x = & 237.3737\ldots \\ -\ 10x = & -\ 2.3737\ldots \\ \hline 990x = & 235 \end{array}$$
$$x = \frac{235}{990} = \frac{47}{198}$$

H-4

Have participants do worksheet H-4 to convert these decimals to fraction form: .888 ... ; .232323 ... ; .311311311 ... ; .763545454 ... .

T-16

Theorem

T-17

Irrational Numbers: The fact that fractions correspond to terminating or repeating decimals and vice-versa has profound consequences. Non-repeating decimals such as .101001000100001000001 .... correspond to numbers that are not rational. Conversely, "irrational" numbers such as $\sqrt{2}$ correspond to decimals that fail to repeat.

T-18

The fact that a number is rational if and only if its decimal representation has a repeating block can also be used to show that the irrational numbers are "dense" in the following sense: Given any two rational numbers, there exists an irrational number between them. For example, an irrational number between $\frac{1}{4}$ and .26 is given by .2501001000100001 ... .

H-5

Have participants do worksheet H-5 to find an irrational number between $\frac{1}{2}$ and .5001. One of many possible answers is:

0.500 01 001 0001 000001…

While a full discussion of such "irrational numbers" is not called for in the standards, the study of long division and the correspondence between decimals and fractions sets the stage for an understanding of this profoundly important part of mathematics.

T-19

Show the chart on T-19.

Administer the post-test.

Fractions and Decimals

Pre- Post-Test

Convert each fraction to a decimal:

1. $\frac{1}{4} =$

2. $\frac{3}{5} =$

3. $\frac{7}{16} =$

4. $\frac{4}{11} =$

Convert each decimal to a fraction:

5. .75 =

6. .3 =

7. .5625

8. .6666... =

9. .232323… =

Fractions and Decimals

Pre- Post-Test Answer Key

1. 0.25
2. 0.6
3. 0.4375
4. $0.36\overline{36}$
5. $\frac{3}{4}$
6. $\frac{3}{10}$
7. $\frac{9}{16}$
8. $\frac{2}{3}$
9. $\frac{23}{99}$

Decimals to Fractions

Recall:

$$428 = 4 \bullet 10^2 + 2 \bullet 10^1 + 8 \bullet 10^0$$

We can use this same process to change .428 to a fraction

$$.428 = 4 \bullet 10^{-1} + 2 \bullet 10^{-2} + 8 \bullet 10^{-3}$$

$$.428 = 4 \bullet \frac{1}{10} + 2 \bullet \frac{1}{100} + 8 \bullet \frac{1}{1000}$$

$$= \frac{4}{10} + \frac{2}{100} + \frac{8}{1000}$$

$$= \frac{4}{10}\left(\frac{100}{100}\right) + \frac{2}{100}\left(\frac{10}{10}\right) + \frac{8}{1000}$$

$$= \frac{400}{1000} + \frac{20}{1000} + \frac{8}{1000}$$

$$= \frac{428}{1000}$$

This can be reduced using GCD(428, 1000) = 4

$$.428 = \frac{428}{1000} = \frac{107}{250}$$

Fractions and Decimals

Worksheet

Convert these decimals to fraction form:

.6 =

.415 =

.503 =

Fractions to Decimals

The fraction $\frac{a}{b}$ is

"the answer to the division problem $a \div b$."

So $\frac{3}{4}$ can be converted to its decimal equivalent by dividing 3 by 4.

$$\begin{array}{r} .75 \\ 4\overline{)3.00} \\ \underline{2\,8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

Fractions and Decimals

Worksheet

Convert these fractions to decimal form:

$$\frac{3}{8} =$$

$$\frac{1}{50} =$$

$$\frac{12}{75} =$$

Fractions to Decimals

Why does dividing numerator by denominator give the correct decimal?

When converting a fraction into a decimal by dividing numerator by denominator, we essentially find an equivalent fraction whose denominator is a power of 10, i.e., a fraction of the form:

$$\frac{N}{10^n}$$

In the case of $\frac{3}{4}$, we choose n = 2. Thus,

$$\frac{3}{4} = \frac{N}{10^2} = \frac{N}{100}$$

Fractions to Decimals

Why does dividing numerator by denominator give the correct decimal?

To find an equivalent fraction for $\frac{3}{4}$, set

$$\frac{3}{4} = \frac{N}{100}$$

Use cross multiplication:

$$4N = 3 \bullet 100 = 300$$

$$N = \frac{300}{4} = \frac{300 \div 4}{4 \div 4} = \frac{300 \div 4}{1} = 300 \div 4$$

Then use long division to obtain N = 75. That is,

$$\frac{3}{4} = \frac{75}{100}$$

This is why long division yields

$$\frac{3}{4} = \frac{75}{100} = 0.75$$

But, how do we know which power of 10 to use in the denominator, e.g. 10^2 for $\frac{3}{4}$?

Fractions to Decimals

Not all fractions may have an equivalent fraction whose denominator is a power of 10. Fractions like:

$$\frac{3}{4} = \frac{75}{10^2} = \frac{75}{100} = .75$$

$$\frac{2}{5} = \frac{4}{10^1} = \frac{4}{10} = .4$$

$$\frac{1}{16} = \frac{625}{10^4} = \frac{625}{10000} = .0625$$

have corresponding terminating decimal forms.

But other fractions can only be converted into decimals with an infinite number of decimal places.

$$\frac{1}{3} = .3333... = 0.\overline{3}$$

$$\frac{3}{11} = .272727... = 0.\overline{27}$$

etc.

Fractions to Decimals

Question: How can we tell if a fraction can be converted into a terminating decimal?

Answer: A fraction $\frac{a}{b}$ that is written in the lowest terms can be converted into a terminating decimal if and only if $b = 2^m \bullet 5^n$ for m, n = 0, 1, 2,… In other words, if and only if the prime factors of b are 2 and/or 5.

Rationale: As shown above, a terminating decimal is a fraction whose denominator is a power of 10. That is,

$$\frac{a}{b} = \frac{N}{10^n}$$

Recall that $(cd)^n = c^n d^n$ from Module I. Therefore,

$$10^n = (2 \bullet 5)^n = 2^n \bullet 5^n$$

This means that the only prime factors of 10^n are 2 and 5.

Fractions to Decimals

In order for $\frac{a}{b} = \frac{N}{10^n}$ we must multiply *a* and *b* by some whole number *k* so that $ak = N$ and $bk = 10^n$.

That is, $\frac{ak}{bk} = \frac{N}{10^n}$

Since $bk = 10^n = 2^n \bullet 5^n$, we conclude that *b* (as well as *k*) has only prime factors of 2 and/or 5. For example,

$$\frac{1}{8} = \frac{1 \bullet 125}{8 \bullet 125} = \frac{125}{1000}$$

or $$\frac{1}{8} = \frac{1}{2^3} = \frac{1 \bullet 5^3}{2^3 \bullet 5^3} = \frac{125}{(2 \bullet 5)^3} = \frac{125}{1000}$$

$$\frac{3}{40} = \frac{3 \bullet 25}{40 \bullet 25} = \frac{75}{1000}$$

or $$\frac{3}{40} = \frac{3}{2^3 \bullet 5} = \frac{3 \bullet 5^2}{(2^3 \bullet 5) \bullet 5^2} = \frac{75}{(2 \bullet 5)^3} = \frac{75}{1000}$$

$\frac{3}{4} = \frac{75}{100}$ Revisited

In the argument that $\frac{3}{4} = .75$, using long division,

we wrote $\frac{3}{4} = \frac{N}{10^2}$ Why 10^2?

Answer: $\frac{3}{4} = \frac{3}{2^2} = \frac{3 \bullet 5^2}{2^2 \bullet 5^2} = \frac{3 \bullet 25}{10^2} = \frac{75}{100}$

Since $4 = 2^2$, we needed a factor of 5 to the same power.

<u>Note</u>: Any higher power of 10 would also work,

$$\frac{3}{4} = \frac{750}{1000} = \frac{750}{10^3}$$

$$\frac{3}{4} = \frac{3}{2^2} = \frac{3 \bullet 2 \bullet 5^3}{2^2 \bullet 2 \bullet 5^3} = \frac{3 \bullet 2 \bullet 5^3}{2^3 \bullet 5^3} = \frac{750}{10^3} = \frac{750}{1000}$$

This may be reduced to $\frac{75}{100}$.

Fractions Into Repeating Decimals

Long division converts some fractions to repeating decimals.

$\frac{1}{3} = .3333\ ... = .\overline{3}$ by doing long division for $1 \div 3$

$$
\begin{array}{r}
.3333... \\
3\overline{)1.0000...} \\
\underline{9} \\
10 \\
\underline{9} \\
10 \text{ etc.}
\end{array}
$$

$\frac{4}{13}$ and $\frac{5}{12}$ also have infinite decimals because the denominators have prime factors other than 2 and 5.

Example:

Converting $\frac{3}{11}$ to a decimal corresponds to

$$
\begin{array}{r}
.2727\ldots \\
11\overline{)3.0000\ldots} \\
\underline{22} \\
80 \\
\underline{77} \\
30 \\
\underline{22} \\
8 \quad \text{etc.}
\end{array}
$$

$$\frac{3}{11} = .272727\ldots = .\overline{27}$$

Assignment

Use long division to find the decimal for $\frac{1}{7}$. Be prepared to explain why the decimal has a repeating block to the other teachers here today.

What feature of the standard long division algorithm is crucial to your argument?

$$\frac{1}{7} = ?$$

$$7\overline{)1.000000}$$

What is the maximum number of digits in a repeating block in relation to the divisor?

A calculator will not show $\frac{4}{13}$ as a repeating decimal.

Long division determines the repeating pattern.

$$
\begin{array}{r}
.3076... \\
13\overline{)4.0000...} \\
\underline{3\,9} \\
100 \\
\underline{91} \\
90 \\
\underline{78}
\end{array}
\qquad \text{etc.}
$$

Fractions and Decimals

Worksheet

Continue the long division problem. At which point does it repeat?

$$
\begin{array}{r}
.3076 \\
13\overline{\smash{)}4.0000...} \\
\underline{39} \\
100 \\
\underline{91} \\
90 \\
\underline{78}
\end{array}
$$

Is $\frac{5}{17}$ a repeating or terminating decimal?

Show how you determined your answer.

Terminating Decimals

Terminating decimals can be regarded as a special kind of repeating decimal

with a repeating pattern of <u>zeros</u>.

Converting $\frac{3}{4}$ to a decimal using long division

$$\begin{array}{r} .7500... \\ 4\overline{)3.0000...} \\ \underline{28} \\ 20 \\ \underline{20} \\ 00 \\ \underline{00} \\ 0... \end{array}$$

means $\frac{3}{4} = .75$

So, **all** fractions correspond to repeating decimals.

Repeating Decimals to Fractions

If you didn't already know that .333… = $\frac{1}{3}$, how could you deduce it?

Let x = .333… = $.\bar{3}$

Then 10x = 3.333…

Subtract x from both sides.

$$\begin{array}{rcl} 10x & = & 3.333\ldots. \\ \underline{-\ \ x} & = & \underline{-\ .333\ldots.} \\ 9x & = & 3 \end{array}$$

$$x = \frac{3}{9} = \frac{1}{3}$$

For more complicated decimals, this also works:

Evaluate $x = .2373737\ldots = .2\overline{37}$

$$
\begin{array}{rl}
100x & = 23.73737\ldots \\
\underline{-\quad x} & \underline{= -.23737\ldots} \\
99x & = 23.5 \\
x & = \dfrac{235}{990} = \dfrac{47}{198}
\end{array}
$$

An easier way is to consider having only the repeating portion to the right of the decimal.

$$
\begin{array}{rl}
1000x & = 237.3737\ldots \\
\underline{-10x} & \underline{= -2.3737\ldots} \\
990x & = 235 \\
x & = \dfrac{235}{990} = \dfrac{47}{198}
\end{array}
$$

Fractions and Decimals

Worksheet

Convert these decimals to fraction form:

.888 ... =

.232323 ... =

.311311311 ... =

.763545454 ... =

Theorem

Combining the algebraic method for converting repeating decimals to fractions with the long division argument gives this important result:

Theorem – Any fraction is equal to a decimal with a repeating block, and any decimal with a repeating block is equal to a fraction.

What about infinite decimals without repeating blocks?

Irrational Numbers

Not all decimals have repeating blocks. Look carefully at

$$.101001000100001\ldots$$

This decimal has no repeating block. It is an example of an

Irrational Number

An irrational number is an infinite decimal which has no repeating block. Another example is

$$\sqrt{2}$$

but it requires a careful argument to show this.

Real Numbers

The set of real numbers is the set of all rational numbers together with the set of all irrational numbers.

Between any two different real numbers is a rational number and an irrational number.

Example: One irrational number between

$$\frac{1}{4} \text{ and } .26$$

is: .2501001000100001…

A rational number between $\frac{1}{4} = .25$ and .26 is

$.255$ or $\frac{255}{1000}$.

Fractions and Decimals

Worksheet

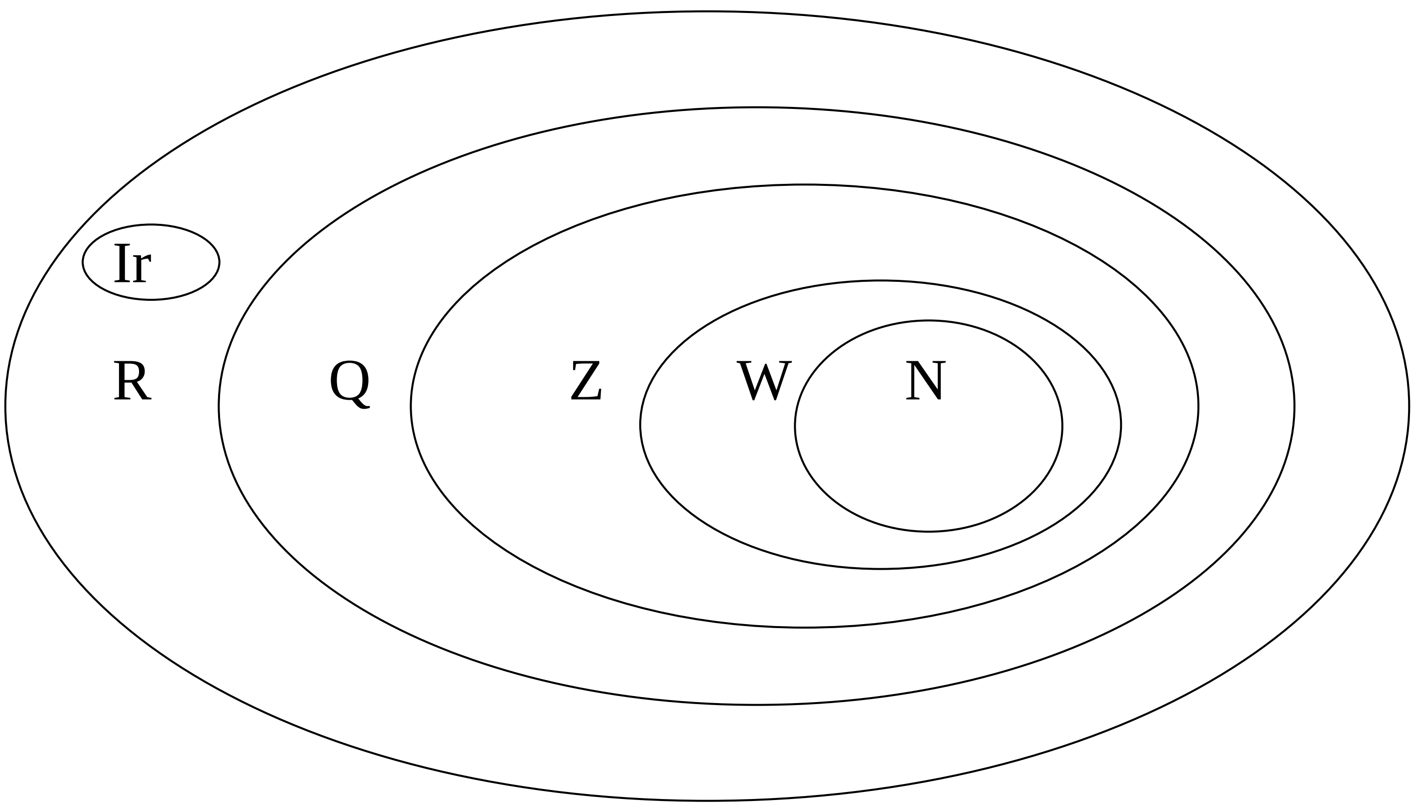
Find an irrational number between $\frac{1}{2}$ and .5001.

Sets of numbers

$$N \qquad W \qquad Z \qquad Q \qquad R$$

$$(Q \qquad Ir) = R$$

N: natural numbers (counting numbers)
W: whole numbers (natural numbers and 0)
Z: integers
Q: rational numbers
R: real numbers
Ir: irrational numbers



Fractions and Decimals

Pre- Post-Test Answer Key

1. $\frac{1}{4} = 0.25$

2. $\frac{3}{5} = 0.6$

3. $\frac{7}{16} = 0.4375$

4. $\frac{4}{11} = 0.\overline{36}$

5. $.75 = \frac{3}{4}$

6. $.3 = \frac{3}{10}$

7. $.5625 = \frac{9}{16}$

8. $.6666 = \frac{2}{3}$

9. $.232323... = \frac{23}{99}$

Fractions and Decimals

Worksheet Answer Key

$$.6 = \frac{6}{10} = \frac{3}{5}$$

$$.415 = \frac{415}{1000} = \frac{3}{5}$$

$$.503 = \frac{503}{1000}$$

Fractions and Decimals

Worksheet Answer Key

$$\frac{3}{8} = 0.375$$

$$\frac{1}{50} = 0.02$$

$$\frac{12}{75} = 0.16$$

Assignment

Answer Key

$$\frac{1}{7} = ?$$

$$\begin{array}{r}
0.142857 \\
7\overline{)1.000000} \\
\underline{7} \\
30 \\
\underline{28} \\
20 \\
\underline{14} \\
60 \\
\underline{56} \\
40 \\
\underline{35} \\
50 \\
\underline{49} \\
1
\end{array}$$

Fractions and Decimals
Worksheet Answer Key

1.

```
       .3076923
13) 4.0000000
    39
     100
      91
       90
       78
       120
       117
         30
         26
          4
```

2.

```
     .2941176470588235
17) 5.0000000000000000
    3 4
    1 60
    1 53
       70
       68
        20
        17
         30
         17
         130
         119
          110
          102
            80
            68
            120
            119
              100
               85
               150
               136
                140
                136
                  40
                  34
                   60
                   51
                    90
                    85
                     5
```


Fractions and Decimals

Worksheet Answer Key

$$.888... = \frac{8}{9}$$

$$.232323... = \frac{23}{99}$$

$$.311311311... = \frac{311}{999}$$

$$.763545454... = \frac{75591 \div 9}{99000 \div 9} = \frac{8399}{11000}$$

Fractions and Decimals

Worksheet Answer Key

$$\frac{1}{2} = .5000$$

For instance,

.50001010010001