Laws of Exponents

Addition of Exponents

If: $a \neq 0$,

$$a^m \cdot a^n = a^{m+n}$$

Example:

$$2^3 \cdot 2^2 = (2 \cdot 2 \cdot 2) \cdot (2 \cdot 2)$$

$$= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

$$= 2^5$$

$$= 2^{3+2}$$

$$3^1 \cdot 3^5 = ?$$

Another Example For Negative Exponents

$$2^{-3} \cdot 2^{-2} = \frac{1}{2^3} \cdot \frac{1}{2^2} =$$

$$= \frac{1}{2^3 \cdot 2^2}$$

$$= \frac{1}{2^{3+2}}$$

$$= \frac{1}{2^5}$$

$$= 2^{-5}$$

$$= 2^{-3+-2}$$

$$3^{-1} \cdot 3^{-4} = ?$$

Still More Examples

$$2^3 \cdot 2^{-2} = 2^{3+(-2)}$$

$$= 2^1$$

$$= 2$$

Why?

$$2^3 \cdot 2^{-2} = 2^3 \cdot \frac{1}{2^2}$$

$$= \frac{2^3}{2^2}$$

$$= \frac{2 \cdot 2 \cdot 2}{2 \cdot 2}$$

$$= \frac{2 \cdot 2}{2 \cdot 2} \cdot 2$$

$$= 1 \cdot 2 = 2 = 2^1$$

More Exponents

If $a \neq 0$ $\quad \dfrac{a^m}{a^n} = a^{m-n}$

Example:

$$\frac{2^7}{2^3} = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2}$$

$$= \frac{2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2} \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

$$= 1 \cdot 2^4$$

$$= 2^4 \ (=16)$$

So, $\quad \dfrac{2^7}{2^3} = 2^{7-3} = 2^4$

More Exponents

Example:

$$\frac{3^2}{3^5} = \frac{3 \cdot 3}{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}$$

$$= \frac{3 \cdot 3}{3 \cdot 3} \cdot \frac{1}{3 \cdot 3 \cdot 3}$$

$$= 1 \cdot \frac{1}{3^3}$$

$$= 3^{-3} \left(= \frac{1}{3^3} = \frac{1}{27} \right)$$

So, $\frac{3^2}{3^5} = 3^{2-5} = 3^{-3}$

Example:

$$\frac{4^5}{4^5} = \frac{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}$$

$$= 1$$

So, $\frac{4^5}{4^5} = 4^{5-5}$

$$= 4^0 = 1$$

If a and b are not zero,

$$(ab)^n = a^n b^n$$

Example:

$$(3 \cdot 5)^4 = (3 \cdot 5)(3 \cdot 5)(3 \cdot 5)(3 \cdot 5)$$
$$= 3 \cdot 3 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 \cdot 5$$
$$= 3^4 \cdot 5^4 \quad (= 81 \text{ x } 625 = 50{,}625)$$

Example:

$$(2 \cdot 5)^3 = (2 \cdot 5)(2 \cdot 5)(2 \cdot 5)$$
$$= 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5$$
$$= 2^3 \cdot 5^3 \quad (= 1{,}000)$$

Note that $(2 \cdot 5)^3 = 10^3 = 1{,}000$

So we have just "factored" 1,000

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Example:

$$\left(\frac{3}{5}\right)^4 = \frac{3}{5} \cdot \frac{3}{5} \cdot \frac{3}{5} \cdot \frac{3}{5}$$

$$= \frac{3^4}{5^4}$$

If $a \neq 0$ $\quad (a^{m})^{n} = a^{mn}$

Example:

$$(2^{3})^{4} = 2^{3} \cdot 2^{3} \cdot 2^{3} \cdot 2^{3}$$

$$= 2^{3+3+3+3} = 2^{3 \cdot 4}$$

$$= 2^{12} \quad (= 4096)$$

Example:

$$\left[\left(\frac{1}{3}\right)^{2}\right]^{3} = \left(\frac{1}{3}\right)^{3} \cdot \frac{1}{3}$$

$$= \left(\frac{1}{3}\right) \cdot \frac{1}{3}\left(\frac{1}{3}\right) \cdot \frac{1}{3}\left(\frac{1}{3}\right) \cdot \frac{1}{3}$$

$$= \left(\frac{1}{3}\right)^{6} \quad \left(= \frac{1}{729}\right)$$

So, $\left[\left(\frac{1}{3}\right)^{2}\right]^{3} = \left(\frac{1}{3}\right)^{2x3}$

$$= \left(\frac{1}{3}\right)^{6}$$

Practice

Simplify each expression (you may leave answers in exponential form.)

a) $2^3 \cdot 2^8$

b) $\left(\frac{2}{5}\right)^4 \cdot 5$

c) $\frac{5^6}{5^4}$

d) $2^3 \cdot 3^2$

e) $\frac{x^{20}}{x^{20}}$

f) $7^{-2} \cdot 7^{-4}$

g) $(2^4)^7$

h) 3^0

i) $(x^2y)^4$

Challenge:

j) $\left(\frac{1}{2}\right)^{-3}$

Answers to Practice Problems

a) $2^3 \cdot 2^8 = 2^{3+8} = 2^{11}$

b) $\left(\frac{2}{5}\right)^4 \cdot 5 = \frac{2^4}{5^4} \cdot 5^1 = \frac{2^4}{5^3}$

c) $\frac{5^6}{5^4} = 5^{6-4} = 5^2 = 25$

d) $2^3 \cdot 3^2 = 2^3 \cdot 3^2$ (bases are different)

e) $\frac{x^{20}}{x^{20}} = x^{20-20} = x^0 = 1$

f) $7^{-2} \cdot 7^{-4} = 7^{-2+-4} = 7^{-6} = \frac{1}{7^6}$

g) $\left(2^4\right)^7 = 2^{28}$

h) $3^0 = 1$

i) $\left(x^2 \cdot y\right)^4 = x^8 y^4$

j) $\left(\frac{1}{2}\right)^{-3} = (2^{-1})^{-3} = 2^{-1 \cdot -3} = 2^3 = 8$

Definitions

- Square Root - a number that when multiplied by itself produces a given number.

  Example: 3 is a square root of 9.
  -3 is also a square root of 9.

- Radical Sign $\sqrt{\ }$ The symbol for the positive square root.

  By convention, the radical sign, $\sqrt{a}$ designates only the positive square root.

Radical Sign → $\sqrt{36}$ ← Radicand

Squaring and Taking a Square Root are Inverse Operations

Like Addition is to Subtraction

Like Multiplication is to Division

$5^2 = 25$ and $\sqrt{25} = 5$

$8^2 = 64$ and $\sqrt{64} = 8$

Square Numbers and Square Roots

Square Numbers		Pos. Square Roots
1^2	1	1
2^2	4	2
3^2	9	3
4^2	16	4
5^2	25	5
6^2	36	6
7^2	49	7
8^2	64	8
9^2	81	9
10^2	100	10

Perspective

When taking a Square Root, one question you could ask yourself is:

"What number times itself is the radicand?”

$$\sqrt{36} = ?$$

"What number times itself is 36?"

Practice Taking the Square Root

1. $\sqrt{49}$
2. $\sqrt{121}$
3. $\sqrt{100}$
4. $\sqrt{1}$
5. $\sqrt{225}$
6. $\sqrt{81}$
7. $\sqrt{169}$
8. $\sqrt{36}$
9. $\sqrt{289}$
10. $\sqrt{144}$

Answers to Practice

1. $\sqrt{49} = 7$
2. $\sqrt{121} = 11$
3. $\sqrt{100} = 10$
4. $\sqrt{1} = 1$
5. $\sqrt{225} = 15$
6. $\sqrt{81} = 9$
7. $\sqrt{169} = 13$
8. $\sqrt{36} = 6$
9. $\sqrt{289} = 17$
10. $\sqrt{144} = 12$

Question

Between which whole numbers is the Square Root of 45?

$$\sqrt{45}$$

45 is not a square number, but it is between two square numbers.

$$36 < 45 < 49$$

and $$\sqrt{36} < \sqrt{45} < \sqrt{49}$$

and $$6 < \sqrt{45} < 7$$

$\sqrt{45}$ is a little less than 7.

$$\left(\sqrt{45} \approx 6.708\right)$$

Practice

Estimate the value of

$$\sqrt{67}$$

Practice

Estimate the value of

$$\sqrt{67}$$

Since $64 < 67 < 81$

and $\sqrt{64} < \sqrt{67} < \sqrt{81}$

and $8 < \sqrt{67} < 9$

$\sqrt{67}$ is a little more than 8.

$$\sqrt{67} \approx 8.185$$

Check Your Understanding
Fill in the missing parts

2^7			
	5 to the 4th power		
		(ab)(ab)(ab)	
			121
$(a^3b^4)^5$			
			$3^8x^8y^8$
$\frac{8x^3y^5}{4xy^2}$			
$(5)^{-2}$			

Check your Understanding
Part 2

1. Explain how 9 and 81 are related.

   _____

   _____

2. Without a calculator, identify the two integers between which the square root lies, and explain why.

   a. $\sqrt{30}$ b. $\sqrt{72}$ c. $\sqrt{12}$

Opening Activity

Your rich uncle needs your help in his office for a 30 day month. He made an unusual offer to pay you 1¢ the first day, 2¢ the second day, 4¢ the third day, each day doubling the previous days pay. Of course, you were insulted and were about to refuse employment for the month when he said, "Ok, ok, I really need your help. So how about $1,000.00 a day with $1,000.00 raise each day." You gladly accepted and your rich eccentric uncle laughed all the way back to his office. What did you do that was so funny to him?

