



CISC - Curriculum & Instruction Steering Committee

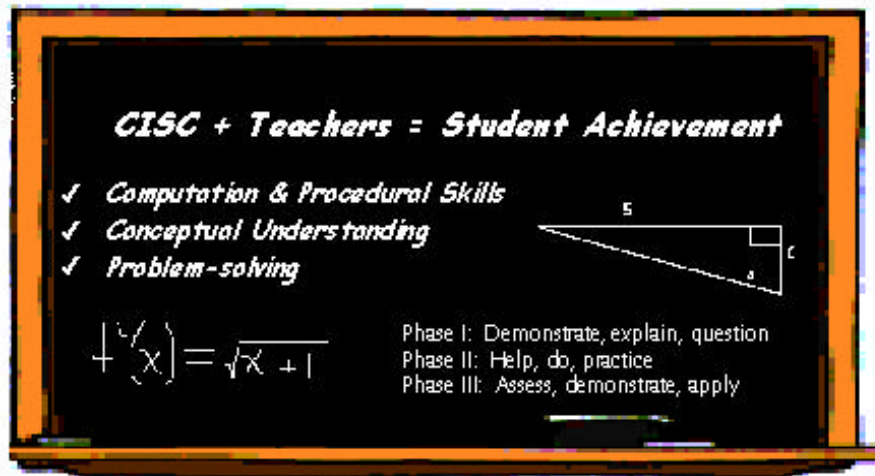
California County Superintendents Educational Services Association

Primary Content Module I

NUMBER SENSE: Exponents/Powers and Roots

The Winning EQUATION

A HIGH QUALITY MATHEMATICS PROFESSIONAL DEVELOPMENT PROGRAM FOR TEACHERS IN GRADES 4 THROUGH ALGEBRA II



STRAND: NUMBER SENSE: Exponents/Powers and Roots

MODULE TITLE: PRIMARY CONTENT MODULE I

MODULE INTENTION: The intention of this module is to inform and instruct participants in the use and properties of exponents, powers and their roots. Practice is provided to develop fluency in using exponents and facility in determining roots through estimation.

THIS ENTIRE MODULE MUST BE COVERED IN-DEPTH.

The presentation of these Primary Content Modules is a departure from past professional development models. The content here, is presented for individual teacher's depth of content in mathematics. Presentation to students would, in most cases, not address the general case or proof, but focus on presentation with numerical examples.

TIME: 2 hours

PARTICIPANT OUTCOMES:

- Define exponent, power, and root.
- Use the properties of exponents including negative whole number exponents.
- Use exponents in fractions.
- Multiply and divide expressions involving exponents with a common base.
- Use the inverse relationship between raising to a power and root extraction to find the root of perfect square integers.
- Estimate the root of any integer by determining which perfect square integer it lies between.



PRIMARY CONTENT MODULE I
NUMBER SENSE: Exponents/Powers and Roots

Facilitator's Notes

T-1

Definition: Let " a " be a number and " n " be a counting number, 1, 2, 3, Then $a^n = a \cdot a \cdot a \dots \cdot a$ (n factors of a)
" a " is called the base " n " is called the power or the exponent.

Definition: If $n = 0$, $a^0 = 1$ when $a \neq 0$ (For further elaboration, see T-12 or T-13.

T-2

Explain that 10 is a special base, because it forms the basis of the base 10 number system.

H-2

Practice

T-3

Answers to practice.

T-4

Definition: If $a \neq 0$ and $n = 1, 2, 3, \dots$ then $a^{-n} = \frac{1}{a^n}$

H-4

Ask participants to calculate:

$$10^{-2} = \frac{1}{10^2} = \frac{1}{10 \cdot 10} = \frac{1}{100} = .01$$

T-4

$$10^{-3} = \frac{1}{10^3} = \frac{1}{10 \cdot 10 \cdot 10} = \frac{1}{1000} = .001$$

$$10^{-6} = \frac{1}{10^6} = \frac{1}{1,000,000} = .000001$$

Extra Credit Problem:

$$10^2 + 10 + 10^0 + 10^{-1} + 10^{-2} + 10^{-3} =$$

$$100 + 10 + 1 + .1 + .01 + .001 = 111.111$$

Ask for volunteers to write answers on transparencies.

T-5/H-5

Practice with negative exponents.

T-6

Answers to practice.

T-7

Explain that 10^n is, in standard notation, "1" followed by n zeros.

**T-8**

Explain the 10^{-n} is, in standard notation, a decimal followed by $n - 1$ zeros which ends with a "1".

T-9

Laws of Exponents

Addition of Exponents

$$\text{If } a \neq 0 \quad a^m \cdot a^n = a^{m+n}$$

T-10

Follow the example presented on T-10. Emphasize that the first step is to use the definition of negative exponents.

For example: $2^{-3} \cdot 2^{-2} = \frac{1}{2^3} \cdot \frac{1}{2^2}$. Then use addition of exponents: $2^{-3} \cdot 2^{-2} = 2^{-3+(-2)} = 2^{-5} = \frac{1}{2^5}$

T-11

Show examples of work with multiplication of bases which are brought to positive and negative powers.

T-12

If $a \neq 0$, $\frac{a^m}{a^n} = a^{m-n}$. Show example $\frac{2^7}{2^3}$ and work through with participants.

T-13

Show example $\frac{3^2}{3^5}$. Emphasize that since exponent of denominator is larger than the exponent of the numerator, the result is a fraction. For the next example, $\frac{4^5}{4^5}$, emphasize that since base is the same in the numerator and denominator, as is the exponent of the base, the result is 1. Remind participants that $4^{5-5} = 4^0 = 1$ which was one of the exponential laws that has been previously discussed as defined in T-1.

T-14

If a and b are not zero $(ab)^n = a^n b^n$. Work through example $(3 \cdot 5)^4$. Emphasize that using reordering of factors by commutative and associative properties, makes the answers more easily found.

T-15

If a and b are not zero, then $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$. Work through example

$\left(\frac{3}{5}\right)^4$ with participants.

**T-16**

If $a \neq 0$ $(a^m)^n = a^{mn}$. Work through example $(2^3)^4$ with participants.

With the next example $\frac{1}{3}^2 \cdot 3^3$ emphasize the exponent rule.

H-16

Practice Problems.

T-17

Answers to practice problems.

T-18**Definitions**

Square Root - a number that when multiplied by itself produces a given number.

Radical Sign - $\sqrt{\quad}$ is by convention, the symbol for taking positive root. Radical Sign $\sqrt{36}$ Radicand

T-19

Squaring and taking a square root are inverse operations.

T-20

Chart shows square numbers and square roots.

T-21

A perspective: When taking a square root, one question you could ask yourself is: "What number times itself is the radicand?"

T-22/H-22

Practice taking the square root.

T-23

Answers to practice.

T-24

Finding estimation of square roots when the radicand is not a perfect square. Work through example, $\sqrt{45}$, with participants.

T-25/H-25

Practice. The answer is at the bottom of the page.

T-26

Check your understanding

T-27

Check your understanding, part 2.

T-28

Opening or Closing Activity:

(Note: To use as an opening activity, pull out T-20 through T-23 and go through the slides before T-1.)

Your rich uncle needs your help in his office for a month. He made an unusual offer to pay you 1¢ the first day, 2¢ the second day, 4¢ the third day, each day doubling the previous days pay. Of course, you were

**T-29**

insulted and were about to refuse employment for the month when he said, "Ok, ok, I really need your help. So how about \$1,000.00 a day with a \$1,000.00 a day raise." You gladly accepted and your rich eccentric uncle laughed all the way back to his office. What did you do that was so funny to him? Allow participants to work this out before continuing.

Let's look at this problem mathematically.

Problem #1 Doubling 1¢ per day.

Day 1	\$.01	=	2^0 ¢
Day 2	\$.02	=	2^1 ¢
Day 3	\$.03	=	2^2 ¢
Day 4	\$.04	=	2^3 ¢
Day 30	\$5,368.709.12	=	2^{29} ¢

So on Day 30 your uncle would owe you 2^{29} or \$5,368.709.12

T-30

Problem #2 \$1,000.00 a day raise

Day 1	\$1,000.00
Day 2	\$2,000.00
Day 3	\$3,000.00
Day 4	\$4,000.00
Day 30	\$30,000.00

T-31

Notice that when we pair Day 1 with Day 30, Day 2 with Day 29, etc., the totals are always \$30,000.00. Your total income for the 30 days will be $15(30,000) = \$465,000.00$

On T-21 you exceeded this total on daily salary on the 27th day, without even figuring the total for the month! The total for all 30 days will be \$10,737,418.23

**Check Your
Understanding
Answer Keys**

Answers to "Check your Understanding" are for facilitators only. Facilitators should fill in the boxes with the correct answers.

Exponents

Definition: Let " a " be a number and " n " be a counting number; 1, 2, 3, ...

then $a^n = a \cdot a \dots \cdot a$

n factors of a

" n " is called the exponent or power, and

" a " is called the base.

Examples:

$$2^3 = 2 \cdot 2 \cdot 2 = 8$$

$$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$$

$$2^1 = 2$$

Definition: If $a \neq 0$, $a^0 = 1$

Example: $2^0 = 1$

Important Special Case

$$a = 10$$

$$10^0 = 1$$

$$10^1 = 10$$

$$10^2 = 10 \cdot 10 = 100$$

$$10^3 = 10 \cdot 10 \cdot 10 = 1,000$$

Question: $10^? = 1,000,000$

Practice

Expand the following:

a) 5^3

b) 10^8

c) $\frac{1}{2}^3$

d) $(.4)^2$

e) $(2.9376)^0$

Practice Answers

$$\text{a) } 5^3 = 5 \cdot 5 \cdot 5 = 125$$

$$\text{b) } 10^8 = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 100,000,000$$

$$\text{c) } \frac{1}{2}^3 = \frac{1}{2} \frac{1}{2} \frac{1}{2} = \frac{1}{8}$$

$$\text{d) } (.4)^2 = (.4)(.4) = .16$$

$$\text{e) } (2.9376)^0 = 1$$

Negative Exponents

Definition:

If $a \neq 0$ and $n = 1, 2, 3, \dots$,

$$\text{then } a^{-n} = \frac{1}{a^n}$$

Examples:

$$2^{-3} = \frac{1}{2^3}$$

$$= \frac{1}{2 \cdot 2 \cdot 2}$$

$$= \frac{1}{8}$$

$$2^{-1} = \frac{1}{2^1}$$

$$= \frac{1}{2}$$

Negative Power of 10

Recall: $10^0 = 1$

What Is: $10^{-1} = ?$

$$10^{-1} = \frac{1}{10} = .1$$

Find: $10^{-2} =$

$$10^{-3} =$$

$$10^{-6} =$$

Extra Credit

$$10^2 + 10^1 + 10^0 + 10^{-1} + 10^{-2} + 10^{-3} =$$

How many different digits do you need?

Practice

Simplify the following:

a) 3^{-2}

b) 5^{-1}

c) 10^{-8}

d) 2^{-5}

e) $(28.329186)^0$

Practice Answers

$$\text{a) } 3^{-2} = \frac{1}{3 \cdot 3} = \frac{1}{9}$$

$$\text{b) } 5^{-1} = \frac{1}{5}$$

$$\text{c) } 10^{-8} = \frac{1}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10} =$$

$$\frac{1}{100,000,000}$$

$$\text{d) } 2^{-5} = \frac{1}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = \frac{1}{32}$$

$$\text{e) } (28.329186)^0 = 1$$