Answer Key for California State Standards: Algebra I

Algebra I: Symbolic reasoning and calculations with symbols are central in algebra. Through the study of algebra, a student develops an understanding of the symbolic language of mathematics and the sciences. In addition, algebraic skills and concepts are developed and used in a wide variety of problem-solving situations.

1.0: Students identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers, including closure properties for the four basic arithmetic operations where applicable.

a. Fill in the blanks below with a single appropriate letter to identify each set of numbers with the properties or descriptions of the elements which characterize that set:

The set of:

E	Even Numbers	B	Integers
D	Rational Numbers	F	Odd Numbers
G	Irrational Numbers	J	Natural Numbers
C	Real Numbers	H	Whole Numbers

A. any number equal to a terminating decimal expression

B. {..., -3, -2, -1, 0, 1, 2, 3, ...}

C. any number which is rational or irrational

D. any number of the form $\frac{p}{q}$ where p and q are integers and q is not zero

E. any integer of the form 2k, where k is an integer

F. any integer of the form 2k + 1, where k is an integer

G. any number equal to an infinite decimal expression with no repeating block of digits

H. {0, 1, 2, 3, ...}

I. any number which can be expressed as a ratio

J. {1, 2, 3, ...}

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Answer Key for California State Standards: Algebra I

1.0: Students identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers, including closure properties for the four basic arithmetic operations where applicable.

[CONTINUED]

b.

Which of the following sets of numbers are not closed under addition?

__ The set of real numbers

__ The set of rational numbers

X The set of irrational numbers

__ The set of positive integers

Answer Key for California State Standards: Algebra I

2.0: Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.

a. Which number below is the same as $-\frac{2}{3}-(-(-\frac{3}{5})$?

A. $\frac{4}{15}$ B. $-\frac{4}{15}$ C. $-\frac{1}{2}$ D. $-\frac{19}{15}$

$$-\frac{2}{3}-(-(-\frac{3}{5})) = -\frac{2}{3} + -\frac{3}{5}$$

$$= -(\frac{2}{3}+\frac{3}{5})$$

$$= -(\frac{10}{15}+\frac{9}{15})$$

$$= -\frac{19}{15}$$

b. What number z satisfies the equation $\frac{2}{3}z = 1$?

$$\frac{2}{3}z = 1$$

$$\frac{3}{2}(\frac{2}{3}z) = \frac{3}{2}\cdot 1$$

$$(\frac{3}{2}\cdot\frac{2}{3})z = \frac{3}{2}$$

$$z = \frac{3}{2}$$

Los Angeles County Office of Education: ***Mathematics***
National Center to Improve the Tools of Education

Answer Key for California State Standards: Algebra I

2.0: Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.

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c. Calculate and simplify the expression $\sqrt[3]{2\sqrt{16}}$

$$\sqrt[3]{2\sqrt{16}} = \sqrt[3]{2 \cdot 4} = \sqrt[3]{8} = 2$$

d. If x = 4, what is $x^{-3/2}\,(x^{100} / x^{99})$?

$$x^{-3/2}\,(x^{100} / x^{99}) = x^{-3/2} \cdot x$$

$$= x^{-3/2 + 1}$$

$$= x^{-1/2}$$

When x = 4, $x^{-1/2} = \frac{1}{\sqrt{4}} = \frac{1}{2}$

2.0: Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.

[CONTINUED]

e. Write $(x^{3/2})^{4/3}\dfrac{\sqrt[4]{x}}{x}$ as x raised to a power

$$(x^{3/2})^{4/3}\ \frac{\sqrt[4]{x}}{x} = x^{3/2 \cdot 4/3} \cdot \frac{x^{1/4}}{x}$$

$$= x^2 \cdot x^{1/4 - 1}$$

$$= x^2 \cdot x^{-3/4}$$

$$= x^{2-3/4}$$

$$= x^{5/4}$$

Answer Key for California State Standards: Algebra I

3.0: Students solve equations and inequalities involving absolute values.

a $|2x - 1| = 5$ Find all values of x which make this equation true

If $|2x - 1| = 5$, then $2x - 1 = 5$ or $2x - 1 = -5$

Therefore $x = 3$ or $x = -2$

b. $3|2 - 5x| + 1 < 10$ Find all values of x which make this inequality true

$3|2 - 5x| + 1 < 10$

$3|2 - 5x| < 9$

$|2 - 5x| < 3$

$-3 < 2 - 5x < 3$

$-5 < -5x < 1$

$1 > x > -\frac{1}{5}$

Solution: All numbers greater than $-\frac{1}{5}$ and less than 1

c. $4|x - 1| = 16$ Find all values of x which make this equation true

$4|x - 1| = 16$

$|x - 1| = 4$

$x - 1 = 4$ or $x - 1 = -4$

$x = 5$ or $x = -3$

Answer Key for California State Standards: Algebra I

3.0: Students solve equations and inequalities involving absolute values.

[CONTINUED]

d. $|x-2| > 4$ Find all values for x which make this inequality true

Either $x - 2 > 4$ or $x - 2 < -4$

Either $x > 6$ or $x < -2$

Solution: All numbers less than -2 or greater than 6

4.0: Students simplify expressions before solving linear equations and inequalities in one variable, such as $3(2x-5) + 4(x-2) = 12$.

a. Simplify

1. $3(2x - 5) + 4(x - 2)$

$$\begin{aligned} 3(2x - 5) + 4(x - 2) &= 6x - 15 + 4x - 8 \\ &= 6x + 4x - 15 - 8 \\ &= 10x - 23 \end{aligned}$$

2. $\frac{1}{2}(6x + 4) - \frac{1}{3}(3 - 6x)$

$$\begin{aligned} \frac{1}{2}(6x + 4) - \frac{1}{3}(3 - 6x) &= 3x + 2 - 1 + 2x \\ &= 3x + 2x + 2 - 1 \\ &= 5x + 1 \end{aligned}$$

Answer Key for California State Standards: Algebra I

4.0: Students simplify expressions before solving linear equations and inequalities in one variable, such as $3(2x-5) + 4(x-2) = 12$.

[CONTINUED]

b. Solve for x

1. $8(x + 1) + 3(2x - 2) = 44$

$8(x + 1) + 3(2x - 2) = 44$

$8x + 8 + 6x - 6 = 44$

$8x + 6x + 8 - 6 = 44$

$14x + 2 = 44$

$14x = 42$

$x = \frac{42}{14}$

$x = 3$

2. $\frac{1}{3}(12x - 9) - 2(x - 5) \geq 17$

$\frac{1}{3}(12x - 9) - 2(x - 5) \geq 17$

$4x - 3 - 2x + 10 \geq 17$

$2x + 7 \geq 17$

$2x \geq 10$

$x \geq 5$

Answer Key for California State Standards: Algebra I

5.0: Students solve multi-step problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.

a. Justify each step below for the solution for x from the equation

$$\frac{2}{3}(x + 3) + 4(x - 8) = 2$$

Use the following list:

A. **Commutative Property of Addition**
B. **Associative Property of Addition**
C. **Commutative Property of Multiplication**
D. **Associative Property of Multiplication**
E. **Distributive Property**
F. **adding the same quantity to both sides of an equation preserves equality**
G. **multiplying both sides of an equation by the same number preserves equality**
H. **0 is the additive identity**
I. **1 is the multiplicative identity**

To the right of each equation below (and on the following pages) where there is an empty space, write one of the letters 'A', 'B', 'C', 'D', 'E', 'F', 'G', 'H', or 'I' to justify how that equation follows from the one above it. For example, the second equation below is justified by 'G' and the third one by 'E'.

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Answer Key for California State Standards: Algebra I

5.0: Students solve multi-step problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.

[CONTINUED]

Step	**Justification**
$\frac{2}{3}(x + 3) + 4(x - 8) = 2$	The given equation
$\frac{3}{2}\left[\frac{2}{3}(x + 3) + 4(x - 8)\right] = \left(\frac{3}{2}\right)2$	G
$(x + 3) + 6(x - 8) = 3$	E
$(x + 3) + (6x - 48) = 3$	E
$[(x + 3) + 6x] - 48 = 3$	B
$[x + (3 + 6x)] - 48 = 3$	B
$[x + (6x + 3)] - 48 = 3$	A
$[(x + 6x) + 3] - 48 = 3$	B
$[(1 + 6)x + 3] - 48 = 3$	E
$(7x + 3) - 48 = 3$	$1 + 6 = 7$
$7x + (3 - 48) = 3$	B

Answer Key for California State Standards: Algebra I

5.0: Students solve multi-step problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.

[CONTINUED]

Step	Justification
$7x - 45 = 3$	$3 - 48 = -45$
$(7x - 45) + 45 = 3 + 45$	F
$7x + (-45 + 45) = 48$	B
$7x + 0 = 48$	$-45 + 45 = 0$
$7x = 48$	H
$\frac{1}{7}(7x) = \frac{48}{7}$	G
$(\frac{1}{7} 7)x = \frac{48}{7}$	D
$1x = \frac{48}{7}$	$\frac{1}{7} \cdot 7 = 1$
$x = \frac{48}{7}$	I

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Answer Key for California State Standards: Algebra I

5.0: Students solve multi-step problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.

b. The sum of three integers is 66. The second is 2 more than the first, and the third is 4 more than twice the first.

What are the integers?

Let x be the first number. The the second number is $x + 2$ and the third number is $2x + 4$.

$x + (x + 2) + (2x + 4) = 66$
$4x + 6 = 66$
$4x = 60$
$x = 15$
$x + 2 = 17$
$2x + 4 = 34$
The three numbers are 15, 17, and 34

c. During an illness, a patient's body temperature T satisfied the inequality $|T - 98.6| \leq 2$. Find the lowest temperature the patient could have had during the illness.

$|T - 98.6| \leq 2$

$-2 \leq T - 98.6 \leq 2$

$-2 + 98.6 \leq T \leq 2 + 98.6$

$96.6 \leq T \leq 100.6$

The lowest temperature the patient could have had is 96.6

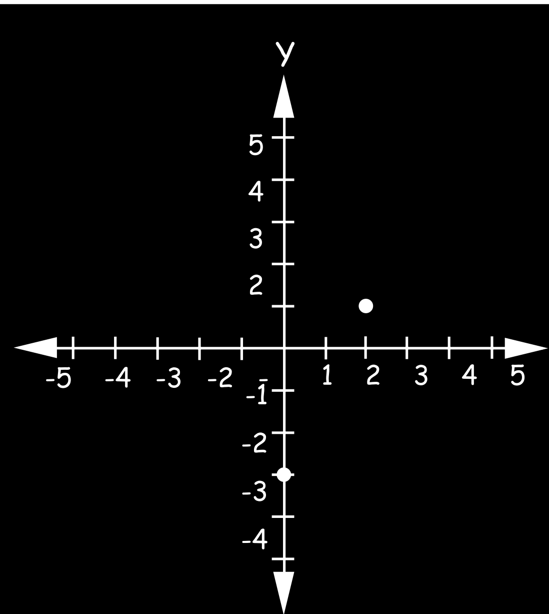
Answer Key for California State Standards: Algebra I

6.0: Students graph a linear equation and compute the *x*- and *y*-intercepts (e.g., graph $2x + 6y = 4$). They are also able to sketch the region defined by linear equality (e.g., they sketch the region defined by $2x + 6y < 4$).

a. Graph the equation: $2x - y = 3$

$$2x - y = 3$$
$$y = 2x - 3$$

x	y
0	-3
2	1



Two points on the line are (0,-3) and (2,1). Note: Two points determine a line. Calculating the coordinates of any two points on the line determines the same graph.

b. What is the x intercept?

In the equation $2x - y = 3$ (or $y = 2x - 3$), substitute $y = 0$.

$$2x - 0 = 3$$
$$2x = 3$$
$$x = \frac{3}{2}$$

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Answer Key for California State Standards: Algebra I

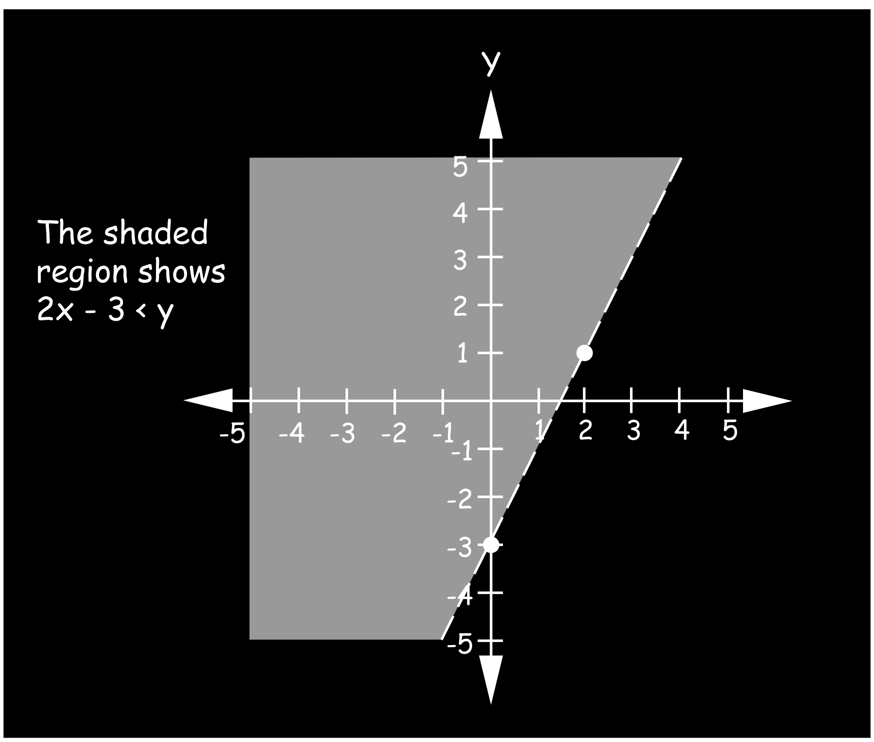
6.0: Students graph a linear equation and compute the *x*- and *y*-intercepts (e.g., graph $2x + 6y = 4$). They are also able to sketch the region defined by linear equality (e.g., they sketch the region defined by $2x + 6y < 4$).

[CONTINUED]

c. What is the y intercept?

In the equation $2x - y = 3$, substitute $x = 0$ to get $y = -3$. Alternatively, since b is the y intercept for $y = mx + b$, it follows that -3 is the y intercept for $y = 2x - 3$.

d. On your graph, mark the region showing $2x - 3 < y$



Answer Key for California State Standards: Algebra I

7.0: Students verify that a point lies on a line, given an equation of the line. Students are able to derive linear equations by using the point-slope formula.

a. Write an equation involving only numbers that shows that the point $(1\frac{1}{2}, 2)$ lies on the graph of the equation $2y = 6x - 5$.

$2 \cdot 2 = 6(1\frac{1}{2}) - 5$ or $2 \cdot 2 = 6(\frac{3}{2}) - 5$

b. A line has a slope of $\frac{1}{2}$ and passes through the point (5, 8).
What is the equation for the line?

The equation of the line must be of the form $y = mx + b$. The slope $m = \frac{1}{2}$ is given. Therefore, $y = \frac{1}{2}x + b$. To find the y intercept b, substitute the coordinates of the point (5,8) for x and y in the equation $y = \frac{1}{2}x + b$. This gives:

$$8 = \frac{1}{2} \cdot 5 + b$$
$$b = 8 - \frac{5}{2}$$
$$b = \frac{16 - 5}{2}$$
$$b = \frac{11}{2}$$
$$\text{Therefore } y = \frac{1}{2}x + \frac{11}{2}$$

This result may also be obtained by using the point-slope formula for a nonvertical line: $y - y_o = m(x - x_o)$ and substituting $m = \frac{1}{2}$, $x_o = 5$, and $y_o = 8$.

Answer Key for California State Standards: Algebra I

8.0: Students understand the concepts of parallel lines and perpendicular lines and how those slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.

a. A line is parallel to the line for the equation:
$\frac{1}{2}y = \frac{1}{2}x - 9$. What is the slope of the parallel line?

$\frac{1}{2}y = \frac{1}{2}x - 9$ may be rewritten as $y = x - 18$, which has slope 1. Any line parallel to this one must have the same slope, 1.

b. What is the slope of a line perpendicular to the line for the equation $3y = 7 - 6x$?

$3y = 7 - 6x$ may be rewritten as $y = -2x + \frac{7}{3}$. The slope m of any line perpendicular to this one must satisfy $m(-2) = -1$. Therefore $m = \frac{1}{2}$.

c. What is the equation of a line passing through the point (7, 4) and perpendicular to the line having the equation $3x - 4y - 12 = 0$?

The equation $3x - 4y - 12 = 0$ may be rewritten as $y = \frac{3}{4}x - 3$. The slope of this line is $\frac{3}{4}$. The slope m of any line perpendicular to this one must satisfy $m(\frac{3}{4}) = -1$. Therefore $m = -\frac{4}{3}$. So the equation of any perpendicular line must be of the form $y = -\frac{4}{3}x + b$. Since the graph of the line contains the point (7,4), it is also true that

$$4 = -\frac{4}{3}(7) + b$$

$$4 = -\frac{28}{3} + b$$

$$b = \frac{12}{3} + \frac{28}{3} = \frac{40}{3}$$

So the answer is $y = -\frac{4}{3}x + \frac{40}{3}$. This answer may also be obtained by using the point-slope formula $y - y_o = m(x - x_o)$ with $m = -\frac{4}{3}$, $x_o = 7$, and $y_o = 4$.

Answer Key for California State Standards: Algebra I

9.0: Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.

a. Solve for the numbers x and y from the equations $2x - y = 1$ and $3x - 2y = -1$

$2x - y = 1$ $3x - 2y = -1$	Multiply the first equation by -2 and then add it to the second equation
$-4x + 2y = -2$ $3x - 2y = -1$	
$-x + 0y = -3$ $x = 3$	Solve for x
$2 \cdot 3 - y = 1$	Substitute $x = 3$ into the first equation and solve for y
$6 - y = 1$ $y = 5$	
$x = 3, y = 5$	Solution

There are other ways to solve this problem. One can use one of the equations to solve for one of the variables in terms of the other and substitute that expression into the other equation. For example, from $2x - y = 1$, solving for y gives $y = 2x - 1$. Substituting this expression for y in the second equation gives

$$3x - 2(2x - 1) = -1$$
$$3x - 4x + 2 = -1$$
$$-x + 2 = -1$$
$$-x = -3$$
$$x = 3$$

Now substituting $x = 3$ into either equation yields $y = 5$. So the solution again is $x = 3$ and $y = 5$.

Answer Key for California State Standards: Algebra I

9.0: Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.

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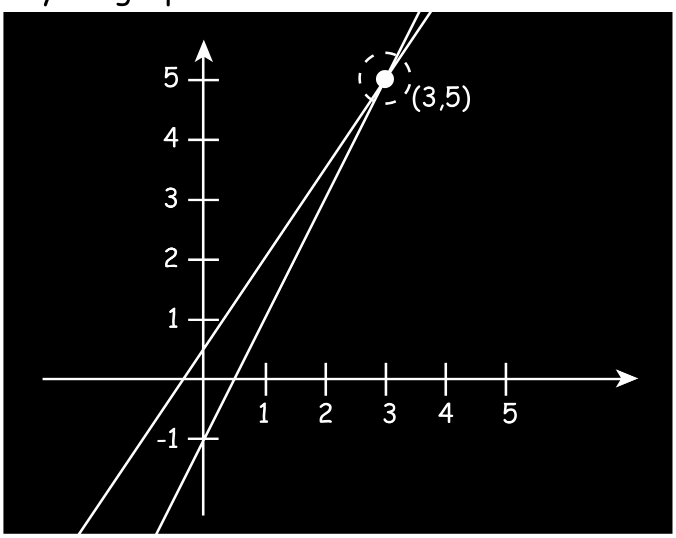
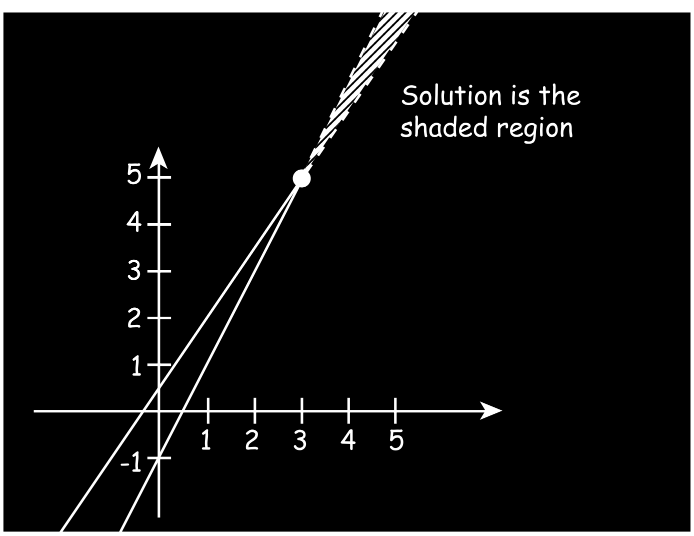
b. Graph the equations $2x - y = 1$ and $3x - 2y = -1$ and circle the portion of the graph which corresponds to the solution to the above problem on your graph.



c. Graph the solution to the linear inequalities

$2x - y > 1$ and $3x - 2y < -1$



Answer Key for California State Standards: Algebra I

10.0: Students add, subtract, multiply, and divide monomials and polynomials. Students solve multistep problems, including word problems, by using these techniques.

a. Simplify

1. $3x^2 \ x^4 \ x^5$

$$3x^2 x^4 x^5 = 3x^{2+4+5} = 3x^{11}$$

2. $\dfrac{4x^3}{2x}$

$$\frac{4x^3}{2x} = \frac{4}{2} \cdot \frac{x^3}{x} = 2x^{3-1} = 2x^2$$

3. $6x^2 + 9x^2$

$$6x^2 + 9x^2 = (6 + 9)x^2 = 15x^2$$

b. Let $P = 2x^2 + 3x - 1$ and $Q = -3x^2 + 4x - 1$

1. Calculate P + Q and collect like terms.

$$\begin{aligned} P + Q &= 2x^2 + 3x - 1 + (-3x^2 + 4x - 1) \\ &= (2x^2 - 3x^2) + (3x + 4x) - 1 - 1 \\ &= -x^2 + 7x - 2 \end{aligned}$$

2. Calculate P - Q and collect like terms.

$$\begin{aligned} P - Q &= 2x^2 + 3x - 1 - (-3x^2 + 4x -1) \\ &= 2x^2 + 3x - 1 + 3x^2 - 4x + 1 \\ &= (2x^2 + 3x^2) + (3x - 4x) - 1 + 1 \\ &= 5x^2 - x \end{aligned}$$

c. Calculate the product $(x^2 - 1) (2x^2 - x - 3)$ and collect like terms.

$$\begin{aligned} &(x^2 - 1)(2x^2 - x - 3) \\ &= (x^2 - 1)2x^2 + (x^2 - 1)(-x) + (x^2 - 1)(-3) \\ &= 2x^4 - 2x^2 - x^3 + x - 3x^2 + 3 \\ &= 2x^4 - x^3 - 5x^2 + x + 3 \end{aligned}$$

Answer Key for California State Standards: Algebra I

10.0: Students add, subtract, multiply, and divide monomials and polynomials. Students solve multistep problems, including word problems, by using these techniques.

[CONTINUED]

d. The area of a rectangle is 16. The length of the rectangle is $\frac{x^5}{x+1}$ and the width is $\frac{x+1}{x^3}$. What is x?

A = length times width

$$16 = \frac{x^5}{x+1} \cdot \frac{x+1}{x^3}$$

$$16 = \frac{x^5}{x^3}$$

$$16 = x^2$$

$$x = 4$$

Answer Key for California State Standards: Algebra I

11.0: Students apply basic factoring techniques to second- and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.

Factor the following expressions:

a. $x^2 + 5x + 4$

$x^2 + 5x + 4 = (x + 4)(x + 1)$

b. $x^3 + 6x^2 + 9x$

$x^3 + 6x^2 + 9x = x(x^2 + 6x + 9) = x(x + 3)^2$

c. $(a + b)\,x + (a + b)\,y$

$(a + b)x + (a + b)y = (a + b)(x + y)$

d. $3x^2 + 7x + 2$

$3x^2 + 7x + 2 = (3x + 1)(x + 2)$

e. $p^2 - q^2$

$p^2 - q^2 = (p - q)(p + q)$

f. $199^2 - 99^2$ (calculate by performing only one multiplication)

$199^2 - 99^2 = (199 - 99)(199 + 99) = 100 \cdot 298 = 29{,}800$

Answer Key for California State Standards: Algebra I

12.0: Students simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.

Reduce to the lowest terms: $\dfrac{x^5 - x^3}{x^2 - 3x + 2}$

$$\frac{x^5 - x^3}{x^2 - 3x + 2}$$

$$= \frac{x^3(x^2 - 1)}{(x - 2)(x - 1)}$$

$$= \frac{x^3(x + 1)(x - 1)}{(x - 2)(x - 1)}$$

$$= \frac{x^3(x + 1)}{x - 2}$$

Answer Key for California State Standards: Algebra I

13.0: Students add, subtract, multiply, and divide rational expressions and functions. Students solve both computationally and conceptually challenging problems by using these techniques.

Express each of the following as a quotient of two polynomials reduced to lowest terms

a. $\frac{3}{x+1} - \frac{4}{x-2}$

$$\frac{3}{x+1} - \frac{4}{x-2} = \frac{3(x-2)}{(x+1)(x-2)} - \frac{4(x+1)}{(x+1)(x-2)} =$$

$$\frac{3x-6-4x-4}{(x+1)(x-2)} = \frac{-(x+10)}{(x+1)(x-2)} \text{ or } \frac{-x-10}{x^2-x-2} \text{ or } -\frac{x+10}{x^2-x-2} \text{ etc.}$$

b. $\frac{x}{2x-1} + \frac{x-1}{2x+1} + \frac{2x}{4x^2-1}$

$$\frac{x}{2x-1} + \frac{x-1}{2x+1} + \frac{2x}{4x^2-1} = \frac{x(2x+1)}{(2x-1)(2x+1)} + \frac{(x-1)(2x-1)}{(2x+1)(2x-1)} + \frac{2x}{4x^2-1}$$

$$= \frac{2x^2+x}{4x^2-1} + \frac{2x^2-3x+1}{4x^2-1} + \frac{2x}{4x^2-1}$$

$$= \frac{(2x^2+x)+(2x^2-3x+1)+2x}{4x^2-1} = \frac{4x^2+1}{4x^2-1}$$

13.0: Students add, subtract, multiply, and divide rational expressions and functions. Students solve both computationally and conceptually challenging problems by using these techniques.

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Express each of the following as a quotient of two polynomials reduced to lowest terms

c. $\dfrac{a^2 - 4}{a^3 + a} \times \dfrac{4a}{a - 2}$

$$\frac{a^2 - 4}{a^3 + a} \cdot \frac{4a}{a - 2} = \frac{(a - 2)(a + 2)}{a(a^2 + 1)} \cdot \frac{4a}{a - 2} = \frac{4(a + 2)}{a^2 + 1}$$

d. $\dfrac{t^2 + 2t + 1}{t + 2} \div \dfrac{t + 1}{t^2 + 5t + 6}$

$$\frac{t^2 + 2t + 1}{t + 2} \div \frac{t + 1}{t^2 + 5t + 6} = \frac{t^2 + 2t + 1}{t + 2} \cdot \frac{t^2 + 5t + 6}{t + 1} =$$

$$\frac{(t + 1)^2}{t + 2} \cdot \frac{(t + 2)(t + 3)}{t + 1} = (t + 1)(t + 3) = t^2 + 4t + 3$$

provided $t \neq -1, -2, \text{ or } -3$

Answer Key for California State Standards: Algebra I

14.0: Students solve a quadratic equation by factoring or completing the square.

a. Solve by factoring: $2x^2 - x - 15 = 0$

$2x^2 - x - 15 = 0$

$(2x + 5)(x - 3) = 0$

Either $2x + 5 = 0$ or $x - 3 = 0$

Either $x = -\frac{5}{2}$ or $x = 3$

b. 1. Complete the square of the polynomial $x^2 + 6x + 5$ by finding numbers h and k such that $x^2 + 6x + 5 = (x + h)^2 + k$.

h = 3
k = -4

$$\begin{aligned} x^2 + 6x + 5 &= x^2 + 6x + \left(\tfrac{6}{2}\right)^2 - \left(\tfrac{6}{2}\right)^2 + 5 \\ &= (x^2 + 6x + 9) - 9 + 5 \\ &= (x + 3)^2 - 4 \end{aligned}$$

2. Solve for x if $2(x - 3)^2 - 5 = 0$

$2(x - 3)^2 - 5 = 0$

$2(x - 3)^2 = 5$

$(x - 3)^2 = \frac{5}{2}$

$x - 3 = \pm\sqrt{\frac{5}{2}}$

$x = 3 \pm \sqrt{\frac{5}{2}}$

The two roots are $3 + \sqrt{\frac{5}{2}}$ and $3 - \sqrt{\frac{5}{2}}$

Answer Key for California State Standards: Algebra I

15.0: Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.

a. What percent of $225 is $180?

$\frac{180}{225} = \frac{n}{100}$ $\qquad$ $\frac{4}{5} = \frac{n}{100}$

$5n = 4 \cdot 100$ $\qquad$ $n = \frac{400}{5}$

$n = 80$ $\qquad$ $180 is 80% of $225

b. The Smith family is traveling to a vacation destination in two cars.
Mrs. Smith leaves home at noon with the children, traveling 40 miles per hour.
Mr. Smith leaves 1 hour later and travels at
55 miles per hour.

At what time does Mr. Smith overtake Mrs. Smith?

A car travelling for t hours at a rate of r miles per hour travels a distance of d miles where $d = rt$.

Let d_1 = distance traveled by Mrs Smith t hours past 1:00 p.m.
Let d_2 = distance traveled by Mr Smith t hours past 1:00 p.m.

Then $d_1 = 40(t + 1)$ and $d_2 = 55t$

To find the number of hours past 1:00 p.m. at which time they meet, set $d_1 = d_2$

$d_1 = d_2$ $\qquad$ $40(t + 1) = 55t$

$40t + 40 = 55t$ $\qquad$ $15t = 40$

$t = \frac{40}{15}$ hours $\qquad$ $t = 2\frac{2}{3}$ hours

$t = 2$ hours and 40 minutes

Mrs Smith and Mr Smith will meet at 3:40 p.m.

Answer Key for California State Standards: Algebra I

15.0: Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.

[CONTINUED]

c. A chemist has one solution of hydrochloric acid and water that is 25% acid and a second that is 75% acid. How many liters of each should be mixed together to get 250 liters of a solution that is 40% acid?

Let x = the no. of liters needed of the 25% acid solution.
Let y = the no. of liters needed of the 75% acid solution.

The mixture must satisfy two equations: $x + y = 250$, and $(.25)x + (.75)y = (.40)250$. The second equation may be rewritten as $\frac{1}{4}x + \frac{3}{4}y = \frac{2}{5}(250)$ or $\frac{1}{4}x + \frac{3}{4}y = 100$. Multiplying both sides by 4 gives $x + 3y = 400$. Subtracting the first equation from this last one gives:

$$\begin{array}{r} x + 3y = 400 \\ \underline{x + y \;\; = 250} \\ 2y \;\;\; = 150 \end{array}$$

So $y = 75$. Substituting $y = 75$ into either equation gives $x = 175$. The answer is that 175 liters of the 25% acid solution must be mixed with 75 liters of 75% acid solution to produce 250 liters of a 40% acid solution.

d. Molly can deliver the papers on her route in 2 hours. Tom can deliver the same route in 3 hours. How long would it take them to deliver the papers if they worked together?

If it takes Molly 2 hours to complete the job, she does $\frac{1}{2}t$ jobs in t hours. If it takes Tom 3 hours to complete the job, he can do $\frac{1}{3}t$ jobs in t hours. To find the number of hours it takes to complete the job with both of them working, solve the equation:

$$\frac{1}{2}t + \frac{1}{3}t = 1$$

$$\left(\frac{1}{2} + \frac{1}{3}\right)t = 1$$

$$\frac{5}{6}t = 1$$

$$t = \frac{6}{5} \text{ or 1 hour and 12 minutes}$$

Answer Key for California State Standards: Algebra I

16.0: Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.
17.0: Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.
18.0: Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion.

a. Which value of x would cause the relation below NOT to be a function?

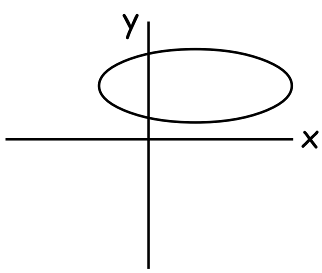
$$\{ (1, 3), (x, 7), (6, 8) \}$$

A. 1
B. 3
C. 7
D. 8

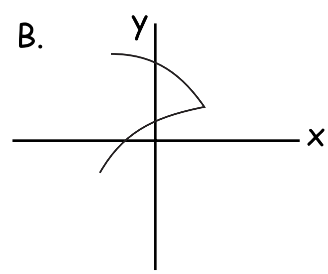
A function is a relation satisfying the condition that for each element in the domain, there is one and only one element in the range.

b. Which of the following graphs of relations is also the graph of a function?

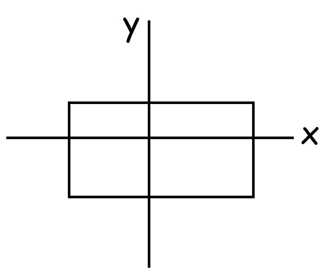
A.



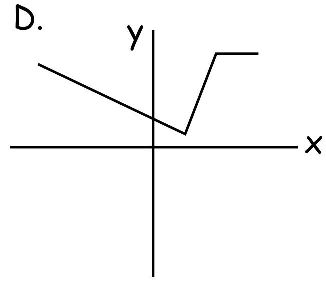
B.



C.



D.



D. is the only graph satisfying the vertical line test

Answer Key for California State Standards: Algebra I

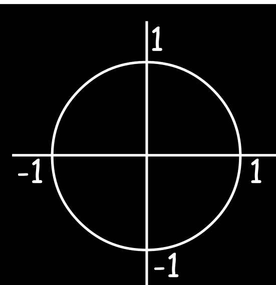
16.0: Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.
17.0: Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.
18.0: Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion.

[CONTINUED]

c. Determine the range and the domain of the relation
$\{ (x, y) : x^2 + y^2 = 1 \}$

Domain = [-1, 1]

Range = [-1, 1]



The graph of this relation is the unit circle. The x values range from -1 to 1 and the y values range from -1 to 1.

d. Find the natural domain of the function $f(x) = x^2 / \sqrt{1 + x}$

The natural domain is the largest subset of the set of real numbers for which the formula is meaningful. In order to avoid dividing by zero or taking the square root of a negative number, we must require $1 + x > 0$ or $x > -1$. The answer is $\{x : x > -1\}$

Answer Key for California State Standards: Algebra I

16.0: Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.
17.0: Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.
18.0: Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion.

e. Find the range of the function $g(x) = 5x^2 + 13$

The range of $g(x)$ is the set of all values the function $g(x)$ can take. The range of $g(x)$ is the set of all numbers greater than or equal to 13, i.e. the range is $\{y : y \geq 13\}$. This is because $5x^2 \geq 0$ for any value of x and there is a value of x such that $5x^2$ is equal to any given non-negative number.

f. Find the range of the relation
$\{ (1, 2), (1, 4), (3, 4), (5, 6), (7, 8) \}$

The range of a relation is the set of all second coordinates, in this case, $\{2, 4, 6, 8\}$. Notice that 4 is listed only once.

Answer Key for California State Standards: Algebra I

19.0: Students know the quadratic formula and are familiar with its proof by completing the square.

a. Given a quadratic equation of the form: $ax^2 + bx + c = 0,\ a \neq 0$
What is the formula for finding the solutions to the equation?

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

b. The equations below are part of a derivation of the quadratic formula by completing the square:

$$ax^2 + bx + c = 0$$

$$a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Which of the following is the best next step for the derivation of the quadratic formula?

A. $ax^2 + bx = -c$

B. $x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$

C. $\left(x^2 + \frac{b}{a}x\right)^2 = \left(\frac{c}{a}\right)$

D. $\sqrt{x^2 + \frac{b}{a}x} = \sqrt{-\frac{c}{a}}$

The derivation of the quadratic formula by completing the square is standard material in any good Algebra I textbook.

Answer Key for California State Standards: Algebra I

20.0: Students use the quadratic formula to find the roots of a second-degree polynomial and to solve quadratic equations.

Find all values of x which satisfy the equation $4x^2 - 4x - 1 = 0$

$$4x^2 - 4x - 1 = 0$$

$$x = \frac{- -4 \pm \sqrt{(-4)^2 - 4 \cdot 4(-1)}}{2.4}$$

$$x = \frac{4 \pm \sqrt{16 + 16}}{8}$$

$$x = \frac{4 \pm \sqrt{32}}{8}$$

$$x = \frac{4 \pm 4\sqrt{2}}{8}$$

$$x = \frac{1 \pm \sqrt{2}}{2}$$

The roots are $\frac{1 + \sqrt{2}}{2}$ and $\frac{1 - \sqrt{2}}{2}$

Answer Key for California State Standards: Algebra I

21.0: Students graph quadratic functions and know that their roots are the x-intercepts.

You may assume that the following equation is correct for all values of x:

$$-3x^2 + 12x - \frac{21}{2} = -3\,(x - 2)^2 + \frac{3}{2}$$

a. For which values of x, if any, does the graph of the equation $y = -3x^2 + 12x - \frac{21}{2}$ cross the x axis?

$$-3(x - 2)^2 + \frac{3}{2} = 0$$

$$-3(x - 2)^2 = -\frac{3}{2}$$

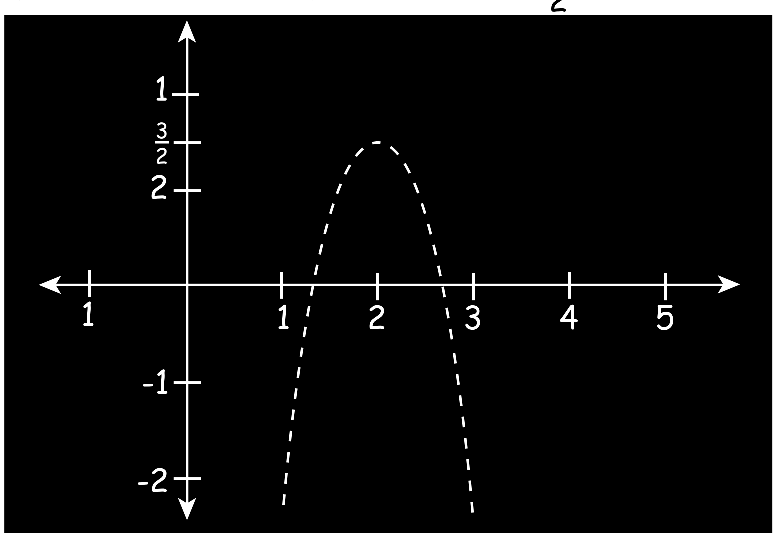
$$(x - 2)^2 = \frac{1}{2}$$

$$x - 2 = \pm\sqrt{\frac{1}{2}}$$

$$x = 2 \pm \sqrt{\frac{1}{2}} = 2 \pm \frac{1}{\sqrt{2}}$$

The graph of $y = -3x^2 + 12x - \frac{21}{2}$ crosses the x-axis at $x = 2 + \frac{1}{\sqrt{2}}$ and $x = 2 - \frac{1}{\sqrt{2}}$

b. Sketch the graph of the equation $y = -3x^2 + 12x - \frac{21}{2}$



Answer Key for California State Standards: Algebra I

22.0: Students use the quadratic formula or factoring techniques or both to determine whether the graph of quadratic function will intersect the x-axis in zero, one or two points.

Use the quadratic formula or the method of factoring to determine whether the graphs of the following functions intersect the x axis in zero, one, or two points. (Do not graph the functions.)

a. $y = x^2 + x + 1$
b. $y = 4x^2 + 12x + 5$
c. $y = 9x^2 - 12x + 4$

Each of these problems may be solved using the discriminant, $D = b^2 - 4ac$, which appears under the radical sign in the quadratic formula. If $D > 0$, the graph has exactly two x-intercepts. If $D = 0$, the graph has exactly one x-intercept. If $D < 0$, the graph does not intersect the x-axis.

a. $D = b^2 - 4ac = 1^2 - 4 \cdot 1 \cdot 1 < 0$

Therefore the graph of $y = x^2 + x + 1$ does not intersect the x-axis (equivalently $x^2 + x + 1 = 0$ has no real solutions).

Answer: 0

b. $D = b^2 - 4ac = 12^2 - 4 \cdot 4 \cdot 5 > 0$. Therefore $y = 4x^2 + 12x + 5$ has two x-intercepts. This may also be seen by factoring:

$$4x^2 + 12x + 5 = (2x + 5)(2x + 1)$$

So the intercepts are $-\frac{5}{2}$ and $-\frac{1}{2}$.

Answer: 2

c. $D = b^2 - 4ac = (-12)^2 - 4 \cdot 9 \cdot 4 = 12^2 - (3 \cdot 4)(3 \cdot 4) = 12^2 - 12^2 = 0$. Therefore $y = 9x^2 - 12x + 4$ has exactly one x-intercept. This may also be seen by factoring:

$$9x^2 - 12x + 4 = (3x - 2)^2$$

Setting this expression equal to zero gives exactly one solution, $x = \frac{3}{2}$.

Answer: 1

Answer Key for California State Standards: Algebra I

23.0: Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity.

a. If an object is thrown vertically with an initial velocity of v_o from an initial height of h_o feet, then neglecting air friction its height h(t) in feet above the ground t seconds after the ball was thrown is given by the formula

$$h(t) = -16t^2 + v_o t + h_o$$

If a ball is thrown upward from the top of a 144 foot tower at 96 feet per second, how long will it take for the ball to reach the ground if there is no air friction and the path of the ball is unimpeded?

Let h(t) be the height above the ground at time t measured in seconds. Then

$$h(t) - 16t^2 + 96t + 144$$

In order to find t such that h(t) is zero, set h(t) = 0 and solve for t.

$$-16t^2 + 96t + 144 = 0$$

$$t^2 - 6t - 9 = 0$$

$$t = \frac{6 \pm \sqrt{36 - 4(-9)}}{2}$$

$$t = \frac{6 \pm \sqrt{36 \cdot 2}}{2}$$

$$t = \frac{6 \pm 6\sqrt{2}}{2} = 3 \pm 3\sqrt{2}$$

Since the object was thrown at t = 0 and time moves forward, the correct solution is $t = 3 + 3\sqrt{2}$ seconds.

23.0: Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity.

[CONTINUED]

b. The boiling point of water depends on air pressure and air pressure decreases with altitude. Suppose that the height H above the ground in meters can be deduced from the temperature T at which water boils in degress Celsius by the following formula:

$$H = 1000(100 - T) + 580(100 - T)^2$$

1. If water on the top of a mountain boils at 99.5 degrees Celsius, how high is the mountain?

$H = 1{,}000(100 - 99.5) + 580(100 - 99.5)^2 = 1{,}000(\frac{1}{2}) + 580(\frac{1}{4})$

$H = 500 + 145 = 645$ meters

2. What is the approximate boiling point of water at sea-level (H=0 meters) according to this equation? Round your answers to the nearest 10 degrees.

The temperature at which water boils at sea level according to the formula may be deduced by setting H = 0 and solving for T:

$1{,}000(100 - T) + 580(100 - T)^2 = 0$

$(100 - T) + .58(100 - T)^2 = 0$

$(100 - T)(1 + .58(100 - T)) = 0$

$(100 - T)(1 + 58 - .58T) = 0$

$(100 - T)(59 - .58T) = 0$

So $T = 100$ or $T = \frac{59}{.58} = \frac{58}{.58} + \frac{1}{.58} \approx 102$

The equation predicts the boiling point is approximately 100°C

Answer Key for California State Standards: Algebra I

24.0: Students use and know simple aspects of a logical argument:

24.1: Students explain the difference between inductive and deductive reasoning and provide examples of each.
24.2: Students identify the hypothesis and conclusion in a logical deducation.
24.3: Students use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion.

a. Verify to your own satisfaction, by direct calculation, the correctness of the following equations (do not submit your calculations on this exam):

$$3 = \frac{3}{2}(3^1 - 1)$$

$$3 + 3^2 = \frac{3}{2}(3^2 - 1)$$

$$3 + 3^2 + 3^3 = \frac{3}{2}(3^3 - 1)$$

$$3 + 3^2 + 3^3 + 3^4 = \frac{3}{2}(3^4 - 1)$$

1. Using inductive reasoning, propose a formula that gives the sum for $3 + 3^2 + 3^3 + \ldots + 3^n$ for any counting number n.

$$3 + 3^2 + 3^3 + \ldots + 3^n = \frac{3}{2}(3^n - 1)$$

2. Does the sequence of formulas above prove that your answer to part 1 is correct? Explain your answer.

No, inductive reasoning is really a form of guessing based on previous observations.
[Note to the reader: In this case the formula given in part 1 is correct for any value of n. Inductive reasoning worked in this case, but it doesn't always give correct answers.]

Answer Key for California State Standards: Algebra I

24.0: Students use and know simple aspects of a logical argument:

24.1: Students explain the difference between inductive and deductive reasoning and provide examples of each.
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24.3: Students use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion.

[CONTINUED]

b. Consider the following mathematical statement:

If y is a positive integer, then $1 + 1141y^2$ is not a perfect square.

1. Write the hypothesis of this statement.

   y is a positive integer

2. Write the conclusion of this statement.

   $1 + 1141y^2$ is not a perfect square

3. Use whole number arithmetic to prove that the conclusion is correct when $y = 1$.

   If $y = 1$, $1 + 1141y^2 = 1142$. To show that 1142 is not a perfect square, it suffices to show that 1142 falls between the squares of two consecutive integers.

   $33^2 = 1089 < 1142 < 1156 = 34^2$

   Therefore the conclusion, $1 + 1141y^2$ is not a perfect square, is correct when $y = 1$.

Answer Key for California State Standards: Algebra I

24.0: Students use and know simple aspects of a logical argument:

24.1: Students explain the difference between inductive and deductive reasoning and provide examples of each.
24.2: Students identify the hypothesis and conclusion in a logical deducation.
24.3: Students use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion.

[CONTINUED]

4. It has been shown by mathematicians that the conclusion is correct for each positive integer y up to and including 30,693,385,322,765,657,197,397,207. However, if this number is increased by 1 so that

$$y = 30{,}693{,}385{,}322{,}765{,}657{,}197{,}397{,}208$$

then the positive square root of $1 + 1141y^2$ is

$$1{,}036{,}782{,}394{,}157{,}223{,}963{,}237{,}125{,}215$$

Is the statement, "If y is a positive integer, then $1 + 1141y^2$ is not a perfect square" correct? Explain your answer.

No, the statement is incorrect because the conclusion is false for at least one positive integer value of y.
[Note that inductive reasoning for this problem would most likely lead to a faulty conclusion.]

Answer Key for California State Standards: Algebra I

25.0: Students use properties of the number system to judge the validity of results, to justify each step of a procedure, and to prove or disprove statements:

25.1: Students use properties of numbers to construct simple, valid arguments (direct and indirect) for, or formulate counterexamples to, claimed assertions.
25.2: Students judge the validity of an argument according to whether the properties of the real number system and the order of operations have been applied correctly at each step.
25.3: Given a specific algebraic statement involving linear, quadratic, or absolute value expressions or equations or inequalities, students determine whether the statement is true sometimes, always, or never.

a. Prove, using basic properties of algebra, or disprove by finding a counterexample, each of the following statements:

1. The set of even numbers is closed under addition.

A number m is even if and only if m = 2k for some integer k. Let m and n be even and let m = 2k and m = 2j for integers k and j. Then:

$$m + n = 2k + 2j = 2(k + j)$$

Therefore m + n has a factor of 2 so it is even. This proves that the sum of any two even numbers is even and therefore the set of even numbers is closed under addition.

2. The sum of any two odd numbers is even.

A number m is odd if and only if m = 2k + 1 for some integer k. Let m and n be odd and let m = 2k +1 and n = 2j + 1 for integers k and j. Then:

$m + n = (2k + 1) + (2j + 1) = 2k + 2j + 2 = 2(k + j + 1)$

Therefore m + n has a factor of 2 so it is even. This proves that the sum of any two odd numbers is odd.

Answer Key for California State Standards: Algebra I

24.0: Students use and know simple aspects of a logical argument:

24.1: Students explain the difference between inductive and deductive reasoning and provide examples of each.
24.2: Students identify the hypothesis and conclusion in a logical deducation.
24.3: Students use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion.

[CONTINUED]

a.

3. For any positive real number x, $\sqrt{x} < x$

This statement is false. $x = \frac{1}{4}$ gives a counterexample because:

$\frac{1}{2} = \sqrt{\frac{1}{4}} > \frac{1}{4}$

b. Find all possible pairs of numbers a and b which satisfy the equation $a^2 + b^2 = (a + b)^2$. Explain your reasoning.

Suppose $a^2 + b^2 = (a + b)^2$
Then $a^2 + b^2 = a^2 + 2ab + b^2$
Therefore $0 = 2ab$
Therefore $ab = 0$
Therefore $a = 0$ or $b = 0$.
If $a = 0$ or $b = 0$ then $a^2 + b^2 = (a + b)^2$.
We conclude that $a^2 + b^2 = (a + b)^2$ if and only if $a = 0$ or $b = 0$

24.0: Students use and know simple aspects of a logical argument:

24.1: Students explain the difference between inductive and deductive reasoning and provide examples of each.
24.2: Students identify the hypothesis and conclusion in a logical deducation.
24.3: Students use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion.

[CONTINUED]

c. Identify the step below in which a fallacy occurs:

Step 1: Let $a = b = 1$
Step 2: $a^2 = ab$
Step 3: $a^2 - b^2 = ab - b^2$
Step 4: $(a - b)(a + b) = b(a - b)$
Step 5: $a + b = b$
Step 6: $2 = 1$

Answer: Step 5
Explain why the step you have chosen as the fallacy is incorrect.

Step 5 results from dividing both sides of the previous equation by zero because $a - b = 0$.

d. Is the following equation true for some values of x, no values of x or all values of x?

$$16\left(x - \frac{1}{4}\right)^2 - 1 = 16x^2 - 8x$$

The equation is true for all values of x because for any value of x:

$$16\left(x - \frac{1}{4}\right)^2 - 1 = 16\left(x^2 - \frac{1}{2}x + \frac{1}{16}\right) - 1$$

$$= 16x^2 - 8x + 1 - 1$$

$$= 16x^2 - 8x$$