1.

a. Fill in the blanks below with a single appropriate letter to identify each set of numbers with the properties or descriptions of the elements which characterize that set:

The set of:

___ Even Numbers
___ Rational Numbers
___ Irrational Numbers
___ Real Numbers
___ Integers
___ Odd Numbers
___ Natural Numbers
___ Whole Numbers

A. any number equal to a terminating decimal expression

B. {..., -3, -2, -1, 0, 1, 2, 3, ...}

C. any number which is rational or irrational

D. any number of the form $\frac{p}{q}$ where p and q are integers and q is not zero

E. any integer of the form 2k, where k is an integer

F. any integer of the form 2k + 1, where k is an integer

G. any number equal to an infinite decimal expression with no repeating block of digits

H. {0, 1, 2, 3, ...}

I. any number which can be expressed as a ratio

J. {1, 2, 3, ...}

California State Standards: Algebra I

1.

b. Which of the following sets of numbers are not closed under addition?

__ The set of real numbers   __ The set of rational numbers

__ The set of irrational numbers   __ The set of positive integers

---

2.

a. Which number below is the same as $-\frac{2}{3} - (-(-\frac{3}{5}))$?

  A. $\frac{4}{15}$ B. $-\frac{4}{15}$ C. $-\frac{1}{2}$ D. $-\frac{19}{15}$

b. What number z satisfies the equation $\frac{2}{3}z = 1$?

c. Calculate and simplify the expression $\sqrt[3]{2\sqrt{16}}$

d. If x = 4, what is $x^{-3/2} (x^{100} / x^{99})$?

e. Write $(x^{3/2})^{4/3} \dfrac{\sqrt[4]{x}}{x}$ as x raised to a power

3.

a. $|2x - 1| = 5$ Find all values of x which make this equation true

b. $3|2 - 5x| + 1 < 10$ Find all values of x which make this inequality true

c. $4|x - 1| = 16$ Find all values of x which make this equation true

d. $|x - 2| > 4$ Find all values for x which make this inequality true

4.

a. Simplify

   1. $3 (2x - 5) + 4 (x - 2)$

   2. $\frac{1}{2} (6x + 4) - \frac{1}{3} (3 - 6x)$

b. Solve for x

   1. $8 (x + 1) + 3 (2x - 2) = 44$

   2. $\frac{1}{3} (12x - 9) - 2 (x - 5) \geq 17$

5.

a. Justify each step below for the solution for x from the equation

$$\frac{2}{3}(x + 3) + 4(x - 8) = 2$$

Use the following list:

A. Commutative Property of Addition
B. Associative Property of Addition
C. Commutative Property of Multiplication
D. Associative Property of Multiplication
E. Distributive Property
F. adding the same quantity to both sides of an equation preserves equality
G. multiplying both sides of an equation by the same number preserves equality
H. 0 is the additive identity
I. 1 is the multiplicative identity

To the right of each equation below (and on the following pages) where there is an empty space, write one of the letters 'A', 'B', 'C', 'D', 'E', 'F', 'G', 'H', or 'I' to justify how that equation follows from the one above it. For example, the second equation below is justified by 'G' and the third one by 'E'.

Step	Justification
$\frac{2}{3}(x + 3) + 4(x - 8) = 2$	The given equation
$\frac{3}{2}[\frac{2}{3}(x + 3) + 4(x - 8)] = (\frac{3}{2})2$	G
$(x + 3) + 6(x - 8) = 3$	E
$(x + 3) + (6x - 48) = 3$	

5.

[CONTINUED]

Step	Justification
$[(x + 3) + 6x] - 48 = 3$	
$[x + (3 + 6x)] - 48 = 3$	
$[x + (6x + 3)] - 48 = 3$	
$[(x + 6x) + 3] - 48 = 3$	
$[(1 + 6) x + 3] - 48 = 3$	
$(7x + 3) - 48 = 3$	$1 + 6 = 7$
$7x + (3 - 48) = 3$	
$7x - 45 = 3$	$3 - 48 = -45$
$(7x - 45) + 45 = 3 + 45$	
$7x + (-45 + 45) = 48$	
$7x + 0 = 48$	$-45 + 45 = 0$
$7x = 48$	
$\frac{1}{7} (7x) = \frac{48}{7}$	
$(\frac{1}{7} 7) x = \frac{48}{7}$	

5.

[CONTINUED]

Step	Justification
$1x = \dfrac{48}{7}$	$\dfrac{1}{7} \cdot 7 = 1$
$x = \dfrac{48}{7}$	

b. The sum of three integers is 66. The second is 2 more than the first, and the third is 4 more than twice the first. What are the integers?

c. During an illness, a patient's body temperature T satisfied the inequality $|T - 98.6| \leq 2$. Find the lowest temperature the patient could have had during the illness.

6.

a. Graph the equation: $2x - y = 3$

b. What is the x intercept?

c. What is the y intercept?

d. On your graph, mark the region showing $2x - 3 < y$
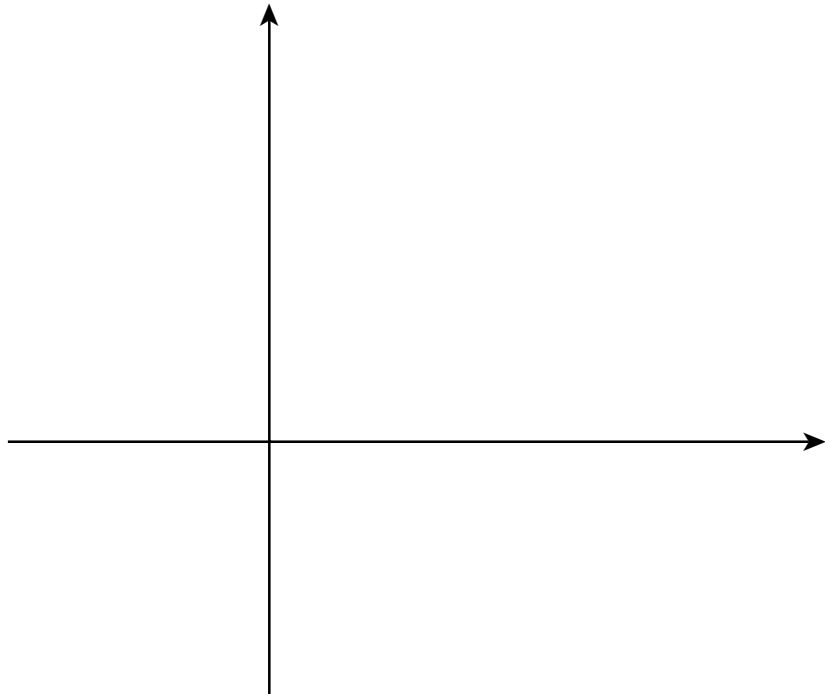
California State Standards: Algebra I

7.

a. Write an equation involving only numbers that shows that the point $(1\frac{1}{2}, 2)$ lies on the graph of the equation $2y = 6x - 5$.

b. A line has a slope of $\frac{1}{2}$ and passes through the point $(5, 8)$. What is the equation for the line?

8.

a. A line is parallel to the line for the equation: $\frac{1}{2}y = \frac{1}{2}x - 9$. What is the slope of the parallel line?

b. What is the slope of a line perpendicular to the line for the equation $3y = 7 - 6x$?

c. What is the equation of a line passing through the point $(7, 4)$ and perpendicular to the line having the equation $3x - 4y - 12 = 0$?
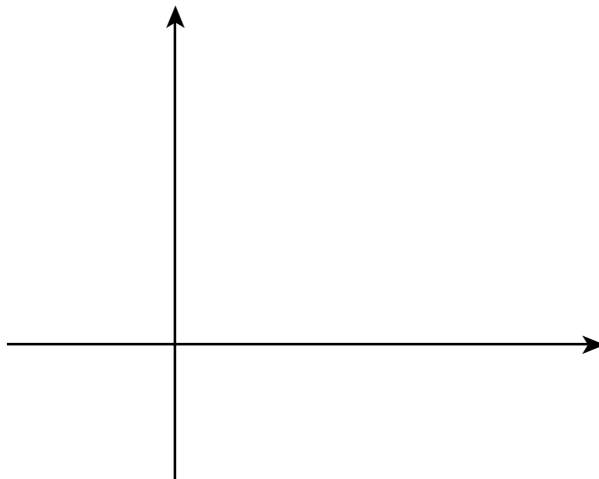
9.

a. Solve for the numbers x and y from the equations $2x - y = 1$ and $3x - 2y = -1$

b. Graph the equations $2x - y = 1$ and $3x - 2y = -1$ and circle the portion of the graph which corresponds to the solution to the above problem on your graph.
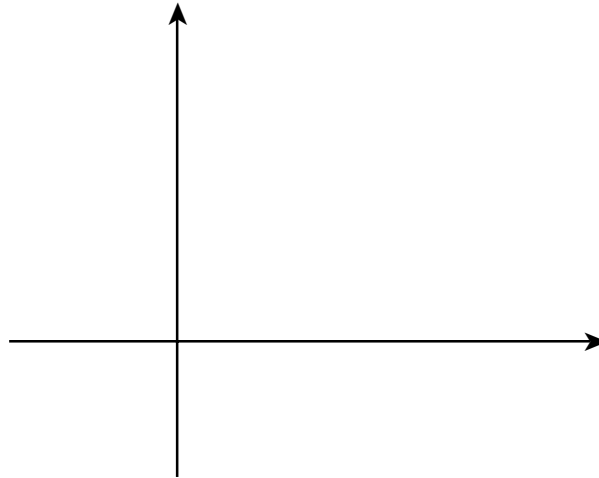
c. Graph the solution to the linear inequalities
$2x - y > 1$ and $3x - 2y < -1$

10.

a. Simplify

1. $3x^2 \ x^4 \ x^5$

2. $\dfrac{4x^3}{2x}$

3. $6x^2 + 9x^2$

b. Let $P = 2x^2 + 3x - 1$ and $Q = -3x^2 + 4x - 1$

1. Calculate $P + Q$ and collect like terms.

2. Calculate $P - Q$ and collect like terms.

c. Calculate the product $(x^2 - 1)(2x^2 - x - 3)$ and collect like terms.

d. The area of a rectangle is 16. The length of the rectangle is $\dfrac{x^5}{x+1}$ and the width is $\dfrac{x+1}{x^3}$. What is x?

11. Factor the following expressions:

    a. $x^2 + 5x + 4$

    b. $x^3 + 6x^2 + 9x$

    c. $(a + b)x + (a + b)y$

    d. $3x^2 + 7x + 2$

    e. $p^2 - q^2$

    f. $199^2 - 99^2$ (calculate by performing only one multiplication)

12. Reduce to the lowest terms: $\dfrac{x^5 - x^3}{x^2 - 3x + 2}$

13. Express each of the following as a quotient of two polynomials reduced to lowest terms

a. $\dfrac{3}{x+1} - \dfrac{4}{x-2}$

b. $\dfrac{x}{2x-1} + \dfrac{x-1}{2x+1} + \dfrac{2x}{4x^2-1}$

c. $\dfrac{a^2-4}{a^3+a} \times \dfrac{4a}{a-2}$

d. $\dfrac{t^2+2t+1}{t+2} \div \dfrac{t+1}{t^2+5t+6}$

14. a. Solve by factoring: $2x^2 - x - 15 = 0$

14.

[CONTINUED]

b. 1. Complete the square of the polynomial $x^2 + 6x + 5$ by finding numbers h and k such that $x^2 + 6x + 5 = (x + h)^2 + k$.

h = _____

k = _____

2. Solve for x if $2(x - 3)^2 - 5 = 0$

15.

a. What percent of $225 is $180?

b. The Smith family is traveling to a vacation destination in two cars. Mrs. Smith leaves home at noon with the children, traveling 40 miles per hour. Mr. Smith leaves 1 hour later and travels at 55 miles per hour.

At what time does Mr. Smith overtake Mrs. Smith?

15.

[CONTINUED]

c. A chemist has one solution of hydrochloric acid and water that is 25% acid and a second that is 75% acid. How many liters of each should be mixed together to get 250 liters of a solution that is 40% acid?

d. Molly can deliver the papers on her route in 2 hours. Tom can deliver the same route in 3 hours. How long would it take them to deliver the papers if they worked together?

16.

17.

18.

a. Which value of x would cause the relation below NOT to be a function?

$$\{ (1, 3), (x, 7), (6, 8) \}$$

A. 1
B. 3
C. 7
D. 8

16.

17.

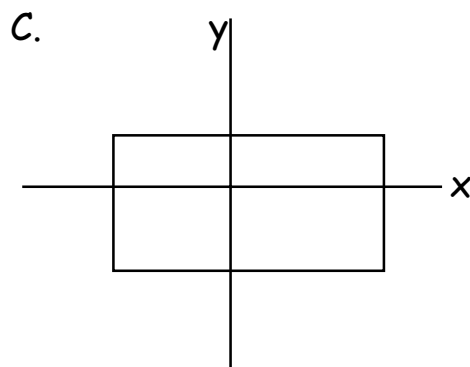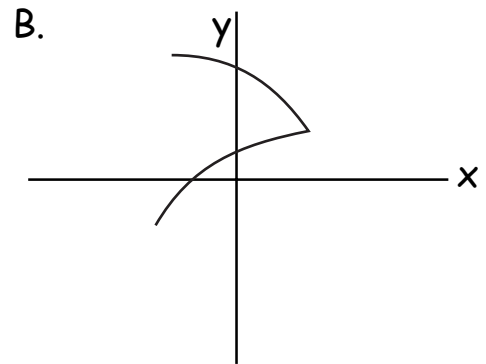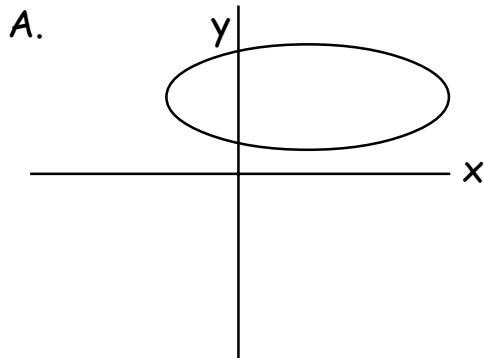**18.**

b. Which of the following graphs of relations is also the graph of a function?

A.

B.

C.

D.
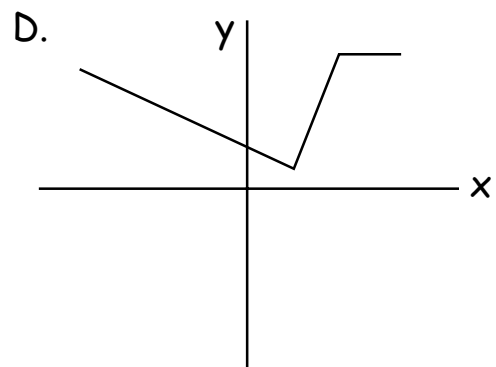
c. Determine the range and the domain of the relation
$\{ (x, y) : x^2 + y^2 = 1 \}$

Domain = _____

Range = _____

16.

17.

18.

d. Find the natural domain of the function $f (x) = x^2 / \sqrt{1 + x}$

e. Find the range of the function $g (x) = 5x^2 + 13$

f. Find the range of the relation
        $\{ (1, 2), (1, 4), (3, 4), (5, 6), (7, 8) \}$

19.

a. Given a quadratic equation of the form: $ax^2 + bx + c = 0$, $a \neq 0$
What is the formula for finding the solutions to the equation?

b. The equations below are part of a derivation of the quadratic formula by completing the square:

$$ax^2 + bx + c = 0$$

$$a \left(x^2 + \frac{b}{a} x + \frac{c}{a} \right) = 0$$

$$x^2 + \frac{b}{a} x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a} x = -\frac{c}{a}$$

Which of the following is the best next step for the derivation of the quadratic formula?

A. $ax^2 + bx = -c$

B. $x^2 + \frac{b}{a} x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$

C. $\left(x^2 + \frac{b}{a} x \right)^2 = \left(\frac{c}{a} \right)$

D. $\sqrt{x^2 + \frac{b}{a} x} = \sqrt{-\frac{c}{a}}$

20. Find all values of x which satisfy the equation $4x^2 - 4x - 1 = 0$
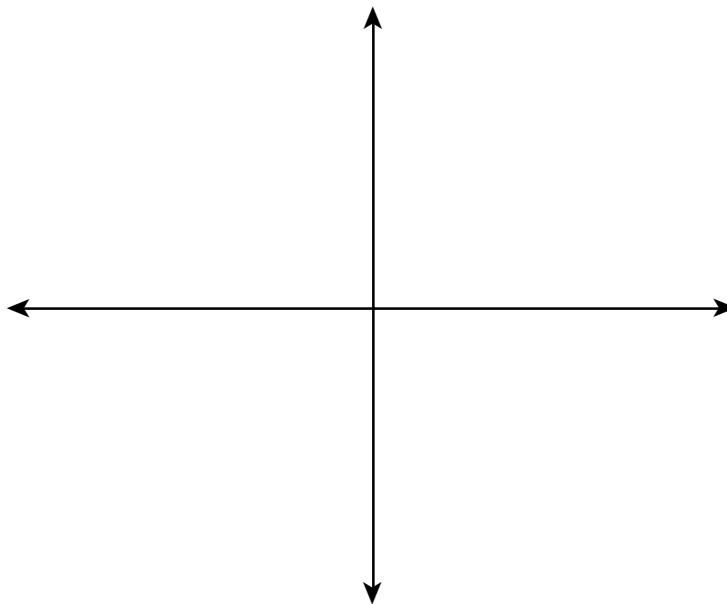
21. You may assume that the following equation is correct for all values of x:

$$-3x^2 + 12x - \frac{21}{2} = -3(x - 2)^2 + \frac{3}{2}$$

a. For which values of x, if any, does the graph of the equation $y = -3x^2 + 12x - \frac{21}{2}$ cross the x axis?

b. Sketch the graph of the equation $y = -3x^2 + 12x - \frac{21}{2}$

22. Use the quadratic formula or the method of factoring to determine whether the graphs of the following functions intersect the x axis in zero, one, or two points. (Do not graph the functions.)

   a. $y = x^2 + x + 1$

   b. $y = 4x^2 + 12x + 5$

   c. $y = 9x^2 - 12x + 4$

23.

a. If an object is thrown vertically with an initial velocity of v_o from an initial height of h_o feet, then neglecting air friction its height h(t) in feet above the ground t seconds after the ball was thrown is given by the formula

$$h(t) = -16t^2 + v_o t + h_o$$

If a ball is thrown upward from the top of a 144 foot tower at 96 feet per second, how long will it take for the ball to reach the ground if there is no air friction and the path of the ball is unimpeded?

b. The boiling point of water depends on air pressure and air pressure decreases with altitude. Suppose that the height H above the ground in meters can be deduced from the temperature T at which water boils in degress Celsius by the following formula:

$$H = 1000 (100 - T) + 580 (100 - T)^2$$

1. If water on the top of a mountain boils at 99.5 degrees Celsius, how high is the mountain?

2. What is the approximate boiling point of water at sea-level (H=0 meters) according to this equation? Round your answers to the nearest 10 degrees.

24 a. Verify to your own satisfaction, by direct calculation, the correctness of the following equations (do not submit your calculations on this exam):

$$3 = \frac{3}{2} (3^1 - 1)$$

$$3 + 3^2 = \frac{3}{2} (3^2 - 1)$$

$$3 + 3^2 + 3^3 = \frac{3}{2} (3^3 - 1)$$

$$3 + 3^2 + 3^3 + 3^4 = \frac{3}{2} (3^4 - 1)$$

1. Using inductive reasoning, propose a formula that gives the sum for $3 + 3^2 + 3^3 + ... + 3^n$ for any counting number n.

2. Does the sequence of formulas above prove that your answer to part 1 is correct? Explain your answer.

24

[CONTINUED]

b. Consider the following mathematical statement:

If y is a positive integer, then $1 + 1141y^2$ is not a perfect square.

1. Write the hypothesis of this statement.

2. Write the conclusion of this statement.

3. Use whole number arithmetic to prove that the conclusion is correct when y = 1.

4. It has been shown by mathematicians that the conclusion is correct for each positive integer y up to and including 30,693,385,322,765,657,197,397,207. However, if this number is increased by 1 so that

   y = 30,693,385,322,765,657,197,397,208

   then the positive square root of $1 + 1141y^2$ is

   1,036,782,394,157,223,963,237,125,215

   Is the statement, "If y is a positive integer, then $1 + 1141y^2$ is not a perfect square" correct? Explain your answer.

25

[CONTINUED]

a. Prove, using basic properties of algebra, or disprove by finding a counterexample, each of the following statements:

1. The set of even numbers is closed under addition.

2. The sum of any two odd numbers is even.

3. For any positive real number x, $\sqrt{x} \leq x$

b. Find all possible pairs of numbers a and b which satisfy the equation $a^2 + b^2 = (a + b)^2$. Explain your reasoning.

c. Identify the step below in which a fallacy occurs:

Step 1: Let $a = b = 1$
Step 2: $a^2 = ab$
Step 3: $a^2 - b^2 = ab - b^2$
Step 4: $(a - b)(a + b) = b(a - b)$
Step 5: $a + b = b$
Step 6: $2 = 1$

Answer: Step _____
Explain why the step you have chosen as the fallacy is incorrect.

25

[CONTINUED]

d. Is the following equation true for some values of x, no values of x or all values of x?

$$16 \left(x - \frac{1}{4} \right)^2 - 1 = 16x^2 - 8x$$