

**Math 103 Section 3.1, 3.2:**

**Math of Finance:  
solving for time**

## Three ways to compute future value

Simple interest  $A = P(1 + rt)$

Compound interest  $A = P(1 + i)^n$

Continuous compounded interest  $A = Pe^{rt}$

These formulas can also be used to compute the present value required to attain a given future value.

**Example:** What present value  $P$  is required for a future value  $F$  of \$4,000? Interest is compounded semiannually for 5 years at a rate of 8%.

Solve the equation for  $P$ :

$$\begin{aligned}4000 &= P(1 + .08/2)^{10} \\ &= P(1.48024) \\ P &= 4000/1.48024 \\ &= 2702.26\end{aligned}$$

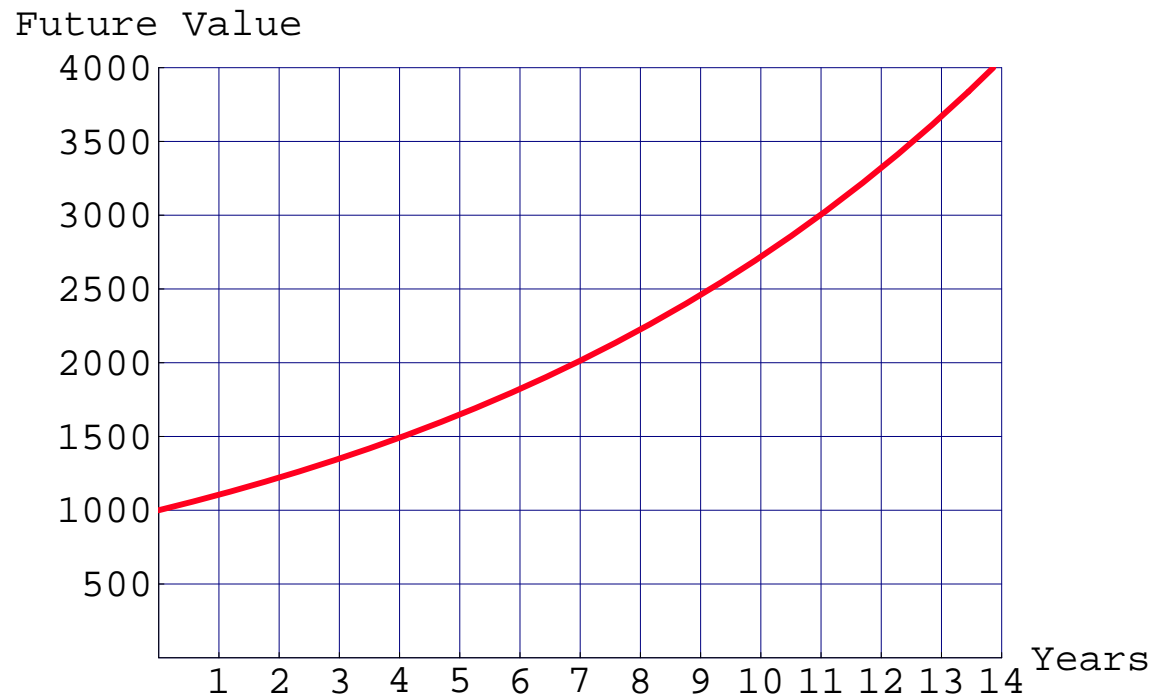
**Summary:** A present value of \$2702.26 is required for a future value of \$4000 if interest is compounded semiannually for 5 years at a rate of 8%.

## Solving the future/present value formula for time $t$

**Example:** A present value of \$1000 is invested at 10% compounded continuously. How many years are required for a future value of \$3000?

Use the graph to solve the equation for the number of years  $t$ :

$$3000 = 1000e^{(.10)t}$$



## Solving the future/present value formula for time $t$

Use logarithms graph to solve the equation for the number of years  $t$ :

$$3000 = 1000e^{(.10)t}$$

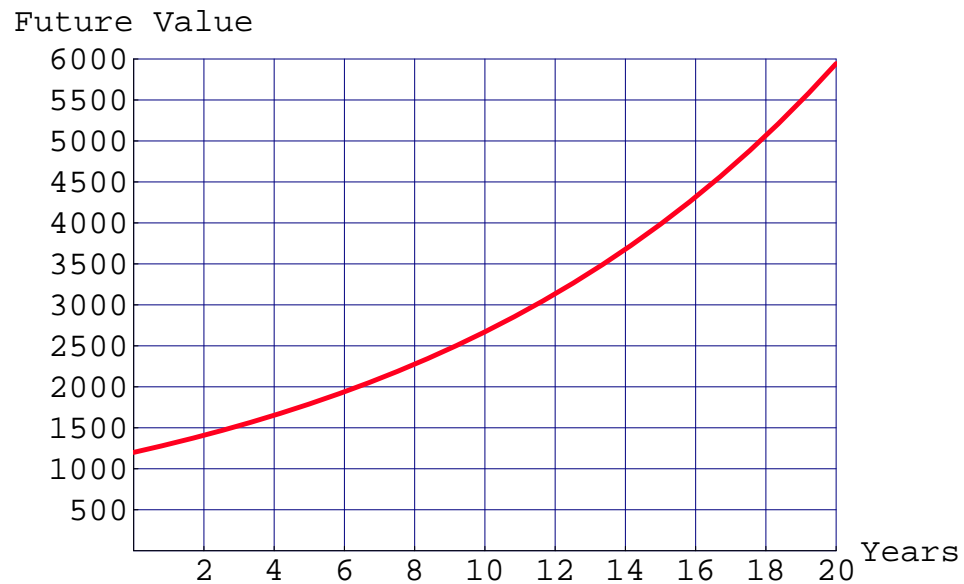
$$\begin{aligned}e^{(.10)t} &= 3 \\ \ln(e^{(.10)t}) &= \ln(3) \\ (.10)t &= \ln(3) \\ t &= \ln(3)/.10 \\ &= 10.98612\end{aligned}$$

**Summary:** It takes 10.98612 years for a present value of \$1000 to grow to a future value of \$3000 at a rate of 10% compounded continuously

# Solving the future/present value formula for time $t$

Use logarithms graph to solve the equation for the number of years  $t$ :

$$5000 = 1200e^{(.08)t}$$



## Solving the future/present value formula for time $t$

Use logarithms graph to solve the equation for the number of years  $t$ :

$$2500 = 1000e^{(.09)t}$$

$$3600 = 1000e^{(.05)t}$$

## Two facts about the natural logarithm, ln:

$$\ln(e^x) = x \quad (1)$$

$$\ln(a^x) = x \ln(a) \quad (2)$$

Fact (1):	Fact (2):
$\ln(e^{(.03)t}) = (.03)t$	$\ln((1.02)^n) = n \ln(1.02)$ $= (0.0198026)n$
$\ln(e^{(.09)t}) = (.09)t$	$\ln((1.10)^n) = n \ln(1.10)$ $= (0.0953102)n$
$\ln(e^{(.06)t}) =$	$\ln((1.045)^n) = n \ln(1.045)$ $= (0.0440169)n$
$\ln(e^{(.10)t}) = (.10)t$	$\ln((1.01)^n) = n \ln(1.01)$ $=$

## How to use Fact (2):

Compound Interest Formula:  $A = P(1 + i)^n$

**Problem:** Deposit \$100 into an account earning 4.5% interest compounded annually. How many years will it take to have a future value of \$200? Solve for  $n$ :

$$200 = 100(1 + .045)^n.$$

$$(1.045)^n = 2$$

$$\ln((1.045)^n) = \ln(2)$$

$$n \ln(1.045) = \ln(2)$$

$$n = \ln(2) / \ln(1.045)$$

$$n = = 15.747302$$

**Summary:** It takes 15.75 years to have a future value of \$200 if a present value of \$100 earns 4.5% interest compounded annually.

## How to use Fact (2):

Compound Interest Formula:  $A = P(1 + i)^n$

**Problem:** Deposit \$100 into an account earning 9% interest compounded semiannually. How many years will it take to have a future value of \$200? Solve for  $n$ :

$$200 = 100(1 + .045)^n.$$

This is the same equation as in the previous slide. The answer is still  $n = 15.747302$ , but it must be interpreted differently.  $n$  is the number of compounding periods.  $n$  isn't always the number of years. In this example, interest is compounded semiannually. So 15.747302 periods is  $15.747302/2 = 7.873651$  years.

**Summary:** It takes 7.87 years to have a future value of \$200 if a present value of \$100 earns 9% interest compounded semiannually.

**Example:** The present value is \$200. Interest is compounded quarterly at a rate of 10%. How many years does it take for a future value of \$500?

Compound interest formula:  $A = P(1 + i)^n$

$i = 0.10/4 = 0.025$  Solve for  $n$ :

$$200(1.025)^n = 500$$

$$(1.025)^n = 2.5$$

$$\ln((1.025)^n) = \ln(2.5)$$

$$n \ln(1.025) = \ln(2.5)$$

$$n = \ln(2.5) / \ln(1.025)$$

$$n = 37.107890$$

There are 37.108 periods. Each period is a quarter (of a year). So that's  $37.108/4 = 9.277$  years.

**Summary:** It takes 9.277 years to have a future value of \$500 if a present value of \$200 earns 10% compounded quarterly.

**Problem:** The present value is \$1200. Interest is compounded monthly at a rate of 8%. How many years does it take for a future value of \$2000?

**Problem:** The present value is \$1800. Interest is compounded quarterly at a rate of 12%. How many years does it take for a future value of \$3200?