

Math 103 Section 2.2: Elementary Functions and Transformations

1. A beginning library of elementary functions
2. Graphs of elementary functions
3. Shifts and stretches
4. Piecewise -defined functions

Beginning Library

- identity function $f(x) = x$
- absolute value function $f(x) = |x|$
- square function $f(x) = x^2$
- square-root function $f(x) = \sqrt{x}$
- piecewise defined functions

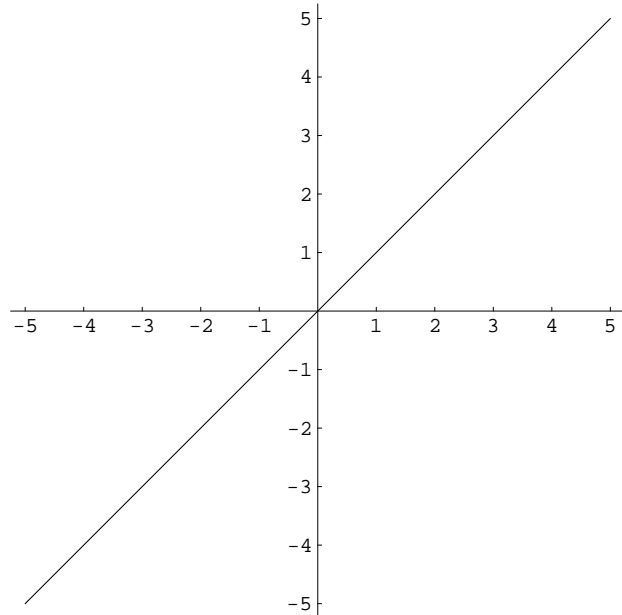
Identity and Absolute value functions

Identity function

Expression $f(x) = x$

Domain: all numbers $(-\infty, \infty)$

Range: all numbers $(-\infty, \infty)$

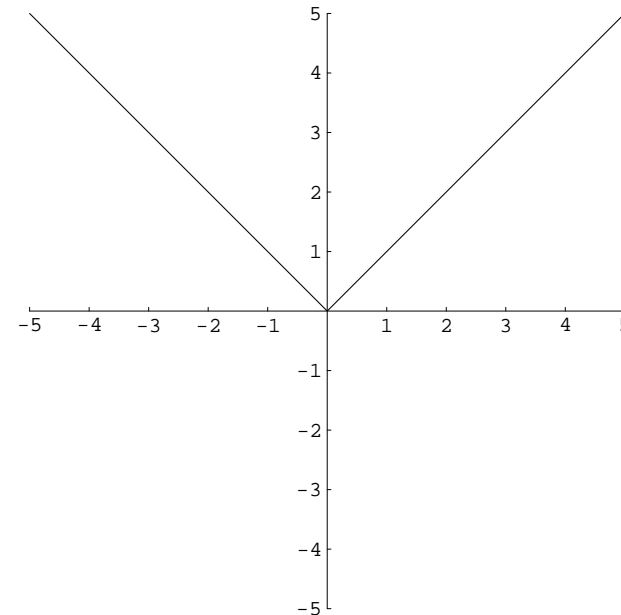


Absolute-value function

Expression $f(x) = |x|$ or $\text{abs}(x)$

Domain: all numbers $(-\infty, \infty)$

Range: $x \geq 0, [0, \infty)$



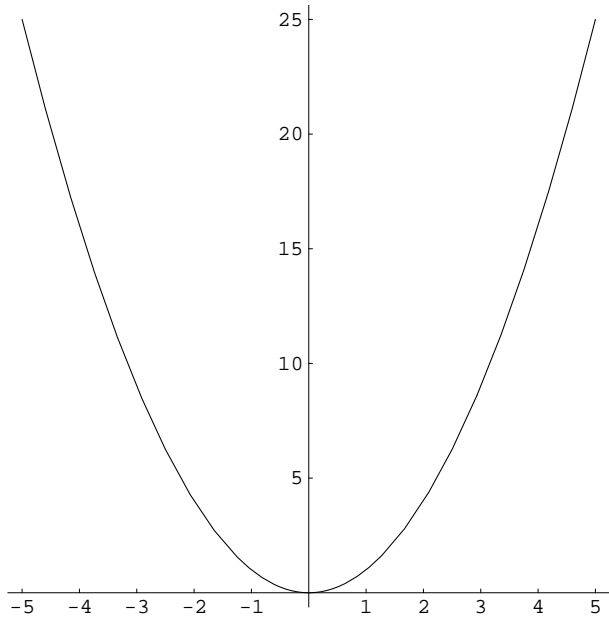
Square and Square-root functions

Square function

Expression: $f(x) = x^2$

Domain: all numbers $(-\infty, \infty)$

Range: $x \geq 0, [0, \infty)$

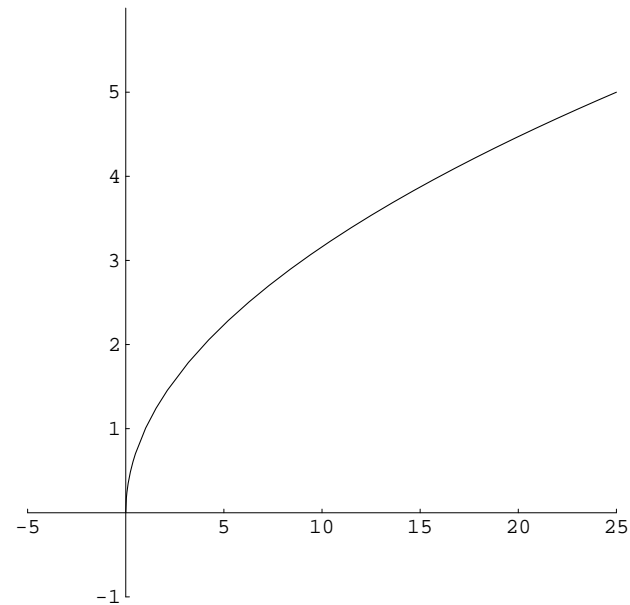


Square-root function

Expression: $f(x) = \sqrt{x}$

Domain: $x \geq 0, [0, \infty)$

Range: $x \geq 0, [0, \infty)$

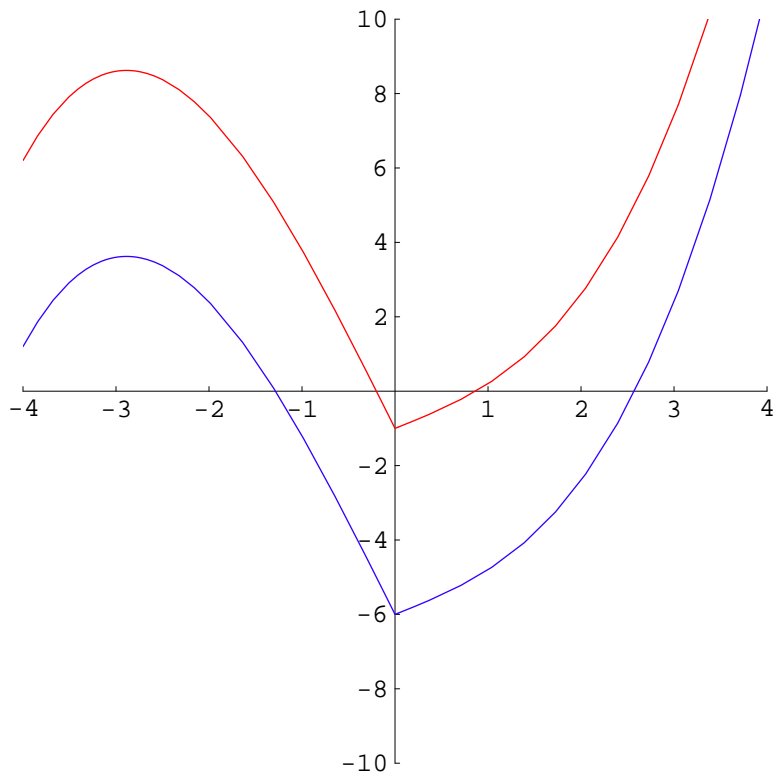


Transformations

- vertical translations (shift)
- vertical stretch
- horizontal translation (shift)
- reflections (lab sessions)

Vertical shift

Vertical shift 5 units up



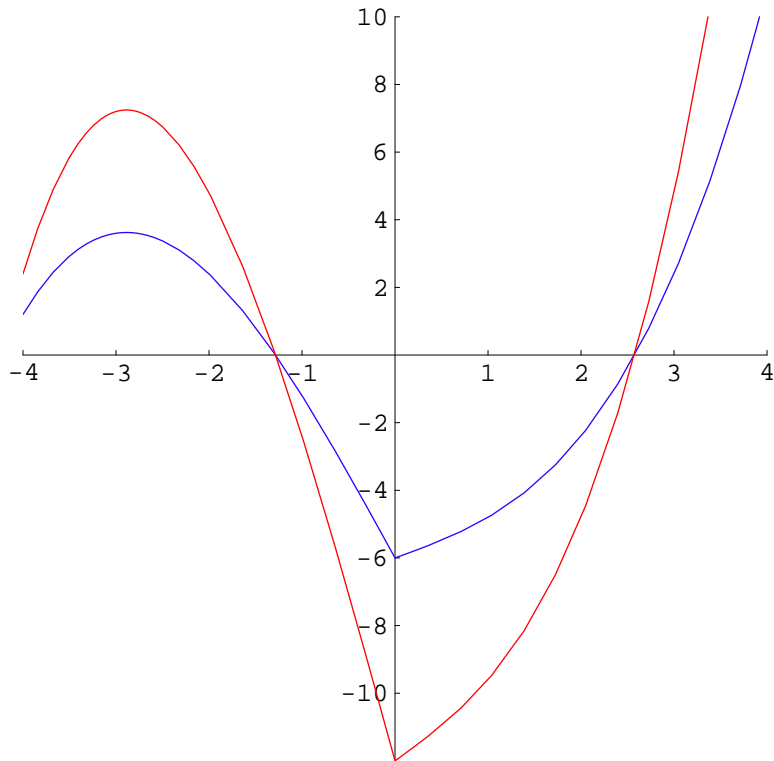
The graph of $f(x)$ is blue (dark line).

The graph of $f(x) + 5$ is red (light line).

The vertical distance between the curves is 5.

Vertical stretch

Vertical stretch by a factor of 2:



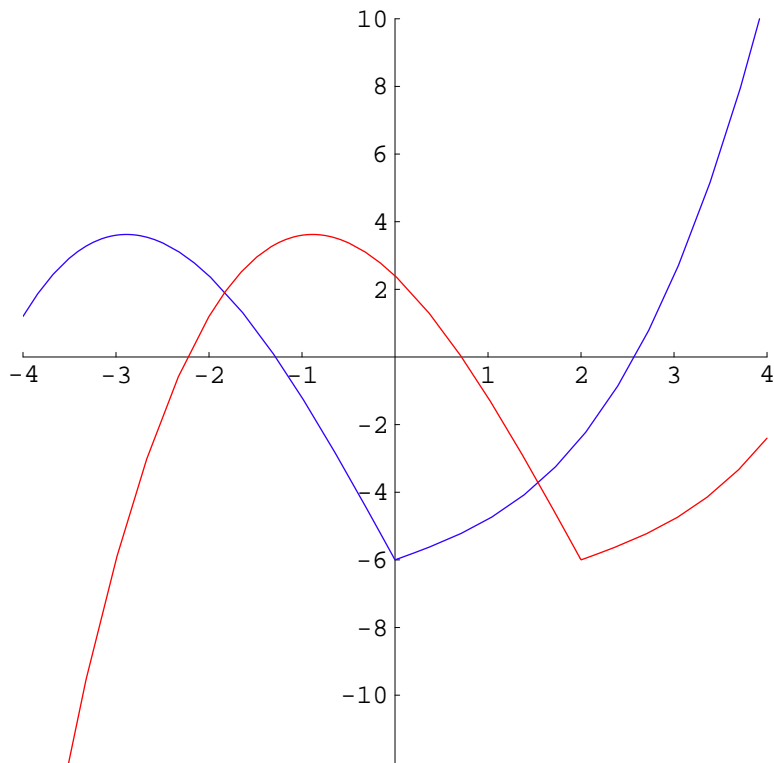
The graph of $f(x)$ is blue (dark line).

The graph of $2f(x)$ is red (light line).

The vertical distance from the x -axis of the graph of $2f(x)$ is twice that of $f(x)$.

Horizontal shift

Horizontal shift two units to the right



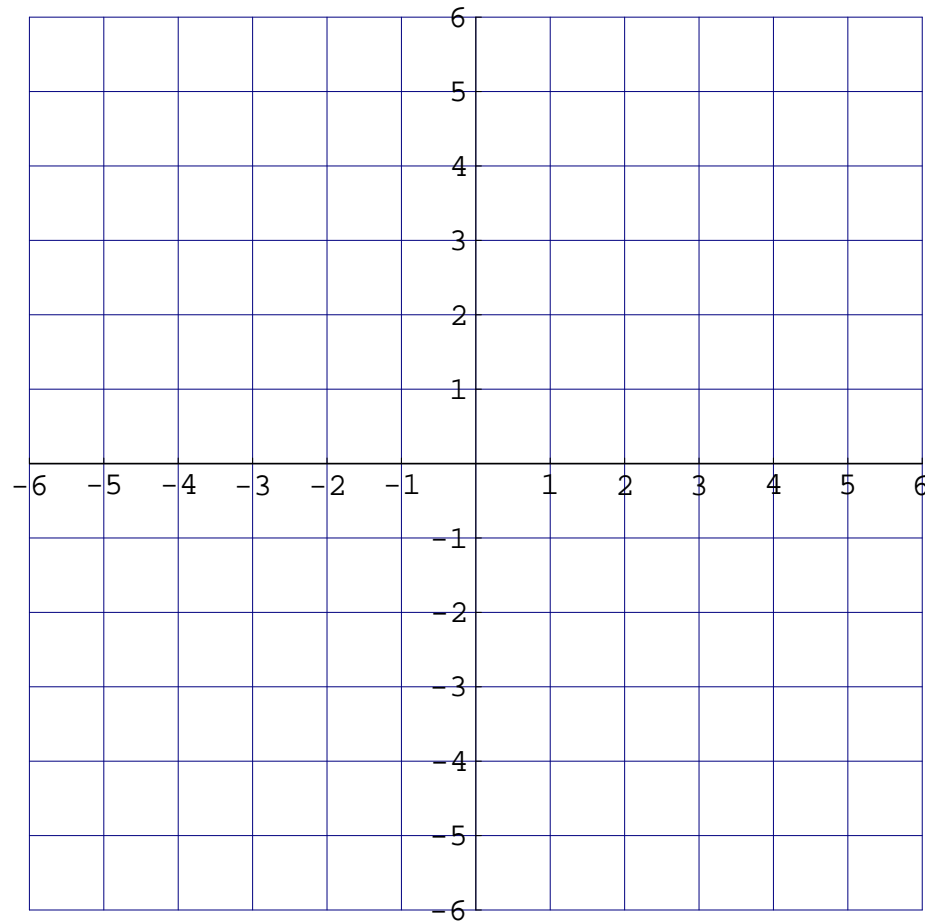
The graph of $f(x)$ is blue (dark).

The graph of $f(x - 2)$ is red (light).

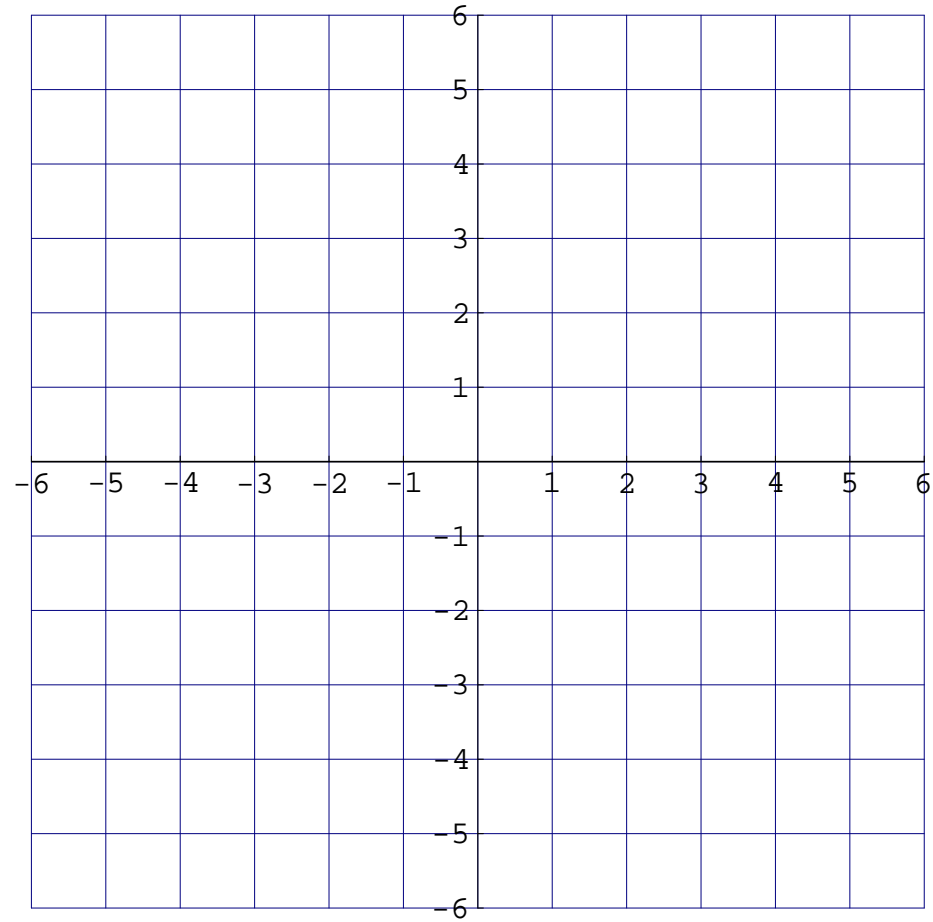
The horizontal distance between the curves is 2.

Practice: Graph these functions

$$y = 2|x|$$



$$y = |x + 3|$$



Practice: Each function corresponds to geometric description

$f(x - 5)$	horizontal shift 5 units to the right
$f(x) + 7$	
$3f(x)$	
$f(x - 3) - 1$	
	vertical shift 2 units up
	vertical shrink by a factor of $1/2$
	horizontal shift 4 units to left

Piecewise defined functions, an example

A car rental agency charges \$30 per day (or partial day) or \$150 per week, whichever is least. What is the rental cost $C(x)$ for x days?

Fill in the charges for the values of x :

x	1.0	2.0	2.6	3.0	3.1	4.0	4.2	5.0	6.0	7.0	7.1
$C(x)$											

Example from business continued:

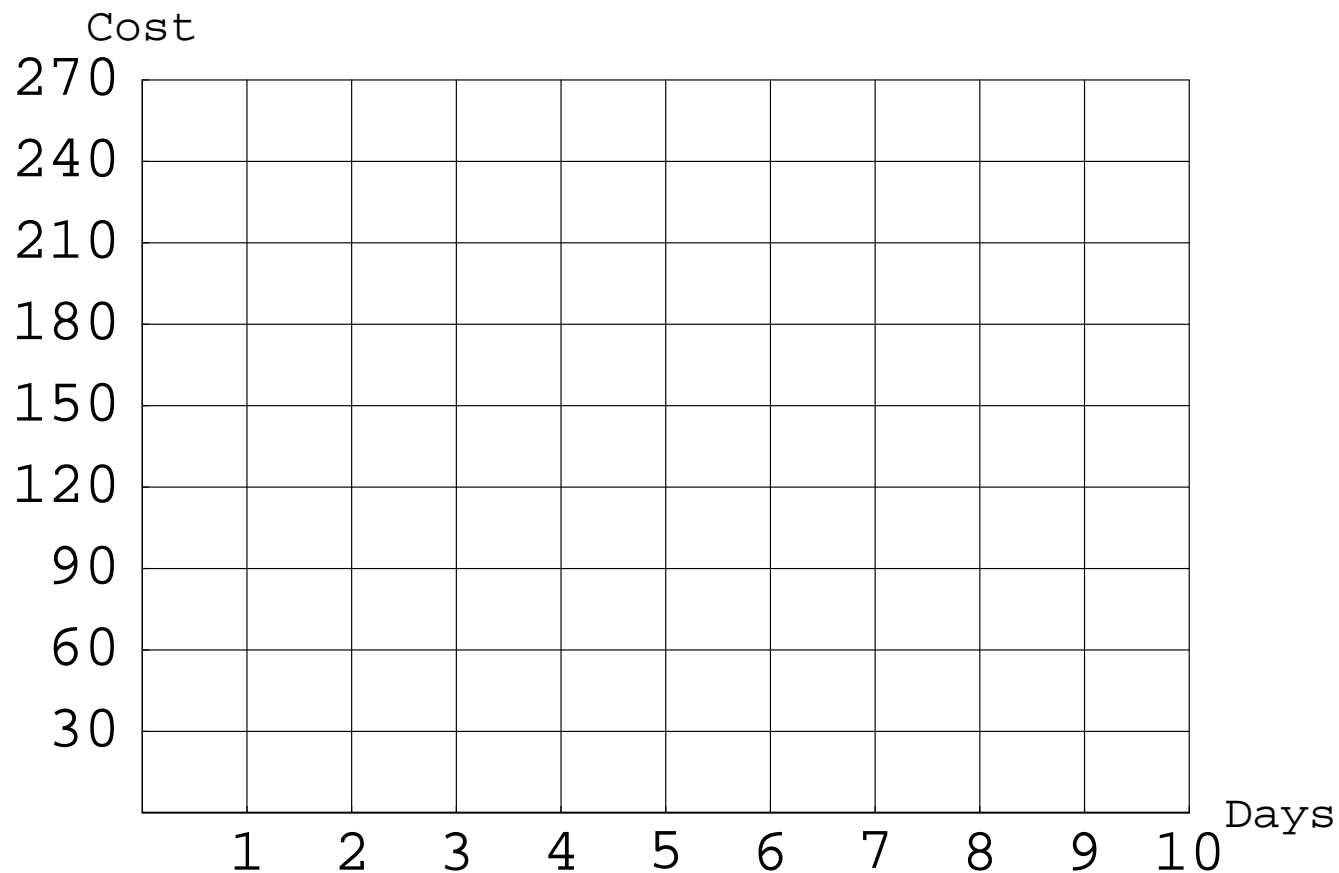
A car rental agency charges \$30 per day (or partial day) or \$150 per week, whichever is least. What is the rental cost $C(x)$ for x days?

Fill in the charges for the values of x :

x	1.0	2.0	2.6	3.0	3.1	4.0	4.2	5.0	6.0	7.0	7.1
$C(x)$	30	60	90	90	120	120	150	150	150	150	180

Example from business

A car rental agency charges \$30 per day (or partial day) or \$150 per week, whichever is least. Graph the cost function $C(x)$.



Example from business $T(x)$ is the tax on taxable income of x .

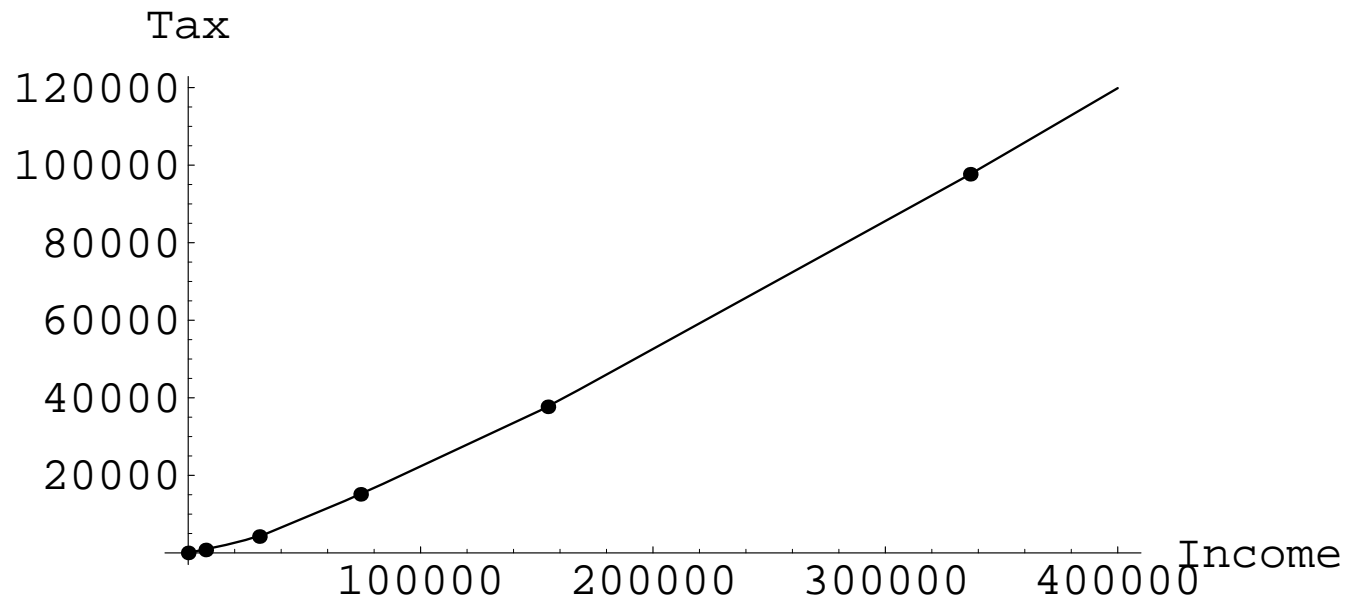
The federal income tax rate is

Between	But Not Over	Base Tax	Rate	Of the Amount Over
\$0	\$7,550	0	10%	\$0.00
\$7,550	\$30,650	\$755.00	15%	\$7,550
\$30,650	\$74,200	\$4,220.00	25%	\$30,650
\$74,200	\$154,800	\$15,107.50	28%	\$74,200
\$154,800	\$336,550	\$37,675.50	33%	\$154,800
\$336,550		\$97,653.00	35%	\$336,550

If you have a taxable income of $x = \$110,000$, your tax is

$$\begin{aligned} T(110,000) &= \text{Base Tax} + (\text{Rate} \times \text{Amount Over}) \\ &= 15,107.50 + [.28 \times (110,000 - 74,200)] \\ &= 15,107.50 + [.28 \times 35,800] \\ &= 15,107.50 + 10,024.00 \\ &= 25,131.50 \end{aligned}$$

The graph of $T(x)$:



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\$0	\$7,550	0	10%	\$0.00
\$7,550	\$30,650	\$755.00	15%	\$7,550
\$30,650	\$74,200	\$4,220.00	25%	\$30,650
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The equations for $T(x)$:

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For income between \$74,200 and \$154,800:

Line 4 in the table.

$74200 \leq x \leq 154800$:

The equations for $T(x)$:

Between	But Not Over	Base Tax	Rate	Of the Amount Over
\$0	\$7,550	0	10%	\$0.00
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\$154,800	\$336,550	\$37,675.50	33%	\$154,800
\$336,550		\$97,653.00	35%	\$336,550

For income between \$30,650 and \$74,200:

Line 3 in the table.

$30650 \leq x \leq 74200$: