

## Limits of rational functions at $x \rightarrow \pm\infty$

Example:  $f(x) = \frac{1}{x}$ .

$\lim_{x \rightarrow \infty} \frac{1}{x} = 0$  When  $x$  gets large (positively),  $1/x$  gets close to 0.

$\lim_{x \rightarrow -\infty} \frac{1}{x} = 0$  When  $x$  gets large (negatively),  $1/x$  gets close to 0.

What about?

$$\lim_{x \rightarrow \infty} \frac{2}{x}$$

$$\lim_{x \rightarrow \infty} \frac{-6}{x}$$

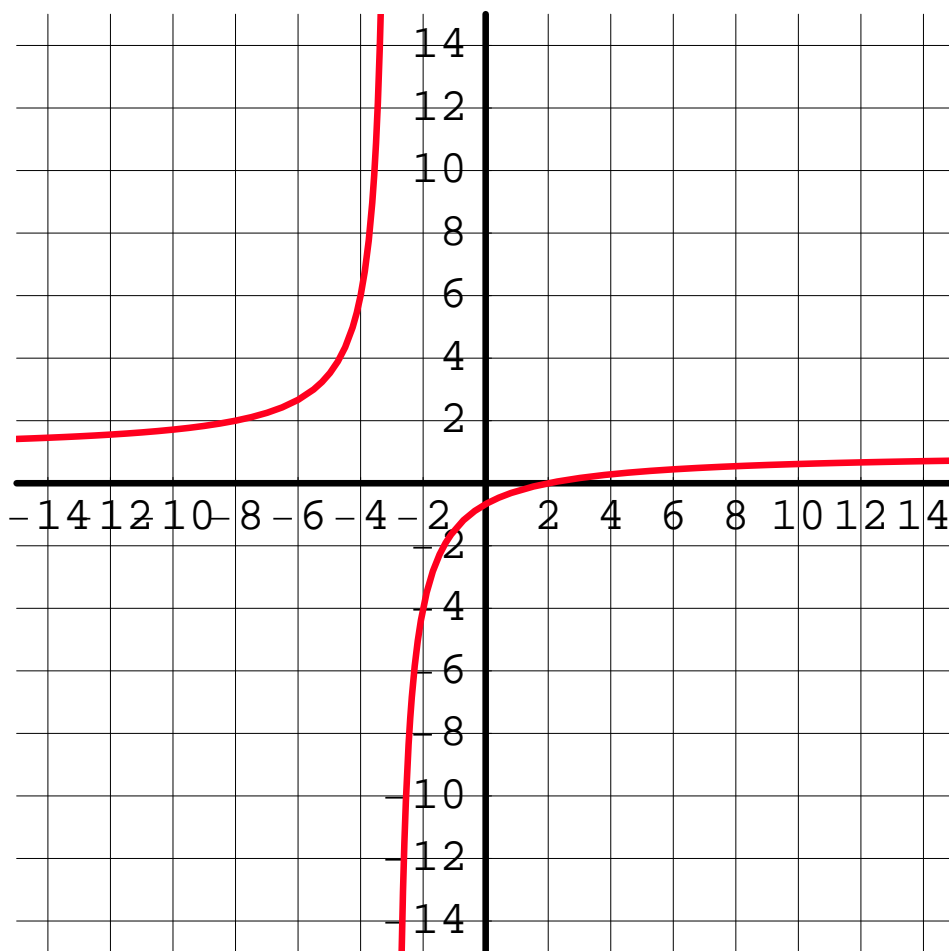
$$\lim_{x \rightarrow -\infty} \frac{5}{x}$$

$$\lim_{x \rightarrow -\infty} \frac{8}{x}$$

## Limits of rational functions at $x \rightarrow \pm\infty$

Example:  $f(x) = \frac{x - 2}{x + 3}$ .

Evaluate  $\lim_{x \rightarrow \infty} \frac{x - 2}{x + 3}$



$$\frac{x - 2}{x + 3} = \frac{1 - 2/x}{1 + 3/x}$$

Examples:

Evaluate:

Horizontal asymptotes

$$\lim_{x \rightarrow \infty} \frac{2x - 7}{3x + 1}$$

$$\lim_{x \rightarrow \infty} \frac{-x + 4}{2x + 3}$$

$$\lim_{x \rightarrow -\infty} \frac{3x - 10}{-2x + 1}$$

$$\lim_{x \rightarrow -\infty} \frac{x - 1}{-5x + 11}$$

# Math 103 Section 10.2: Continuity

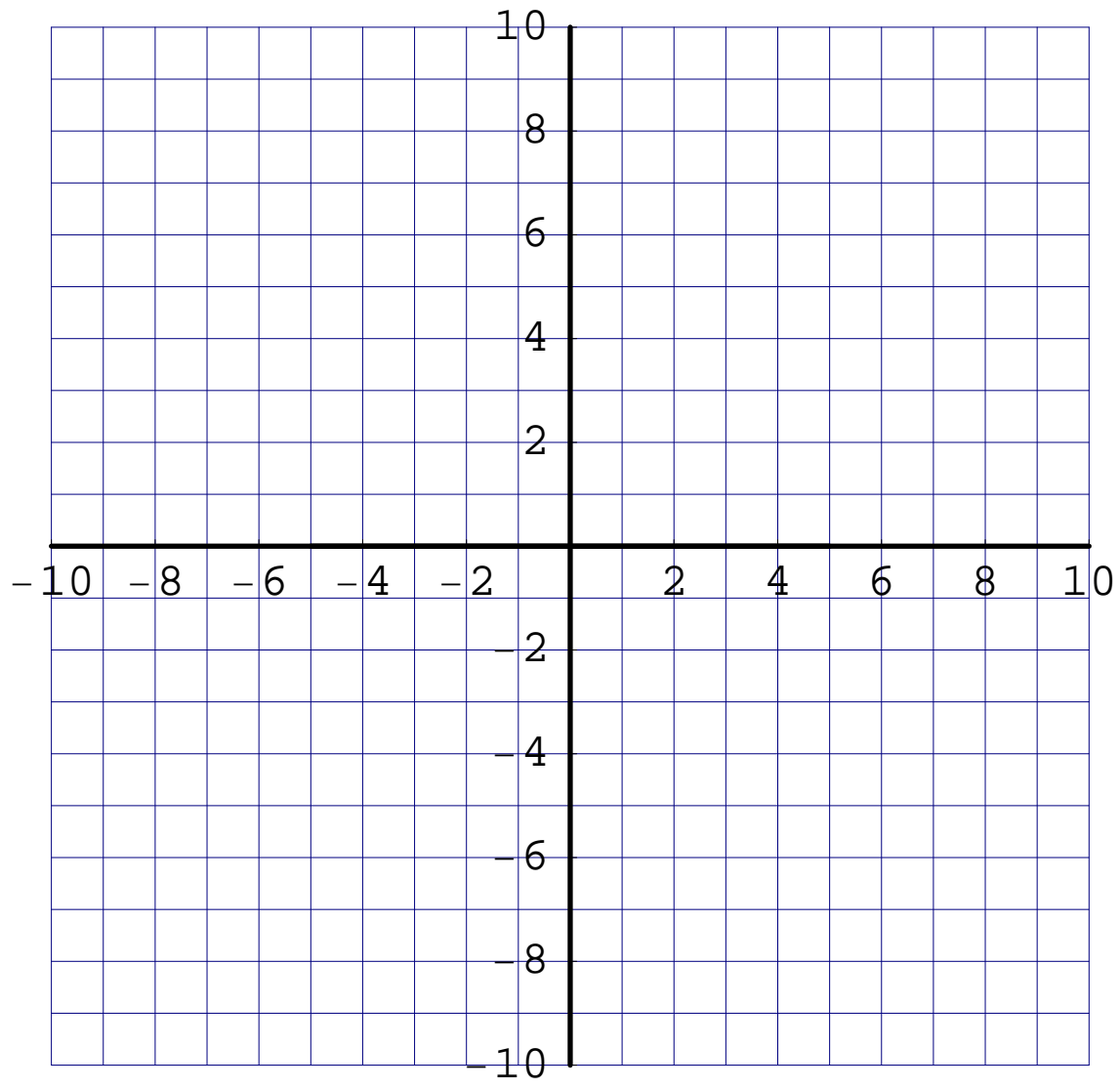
- definition
- examples

## Definition of continuity

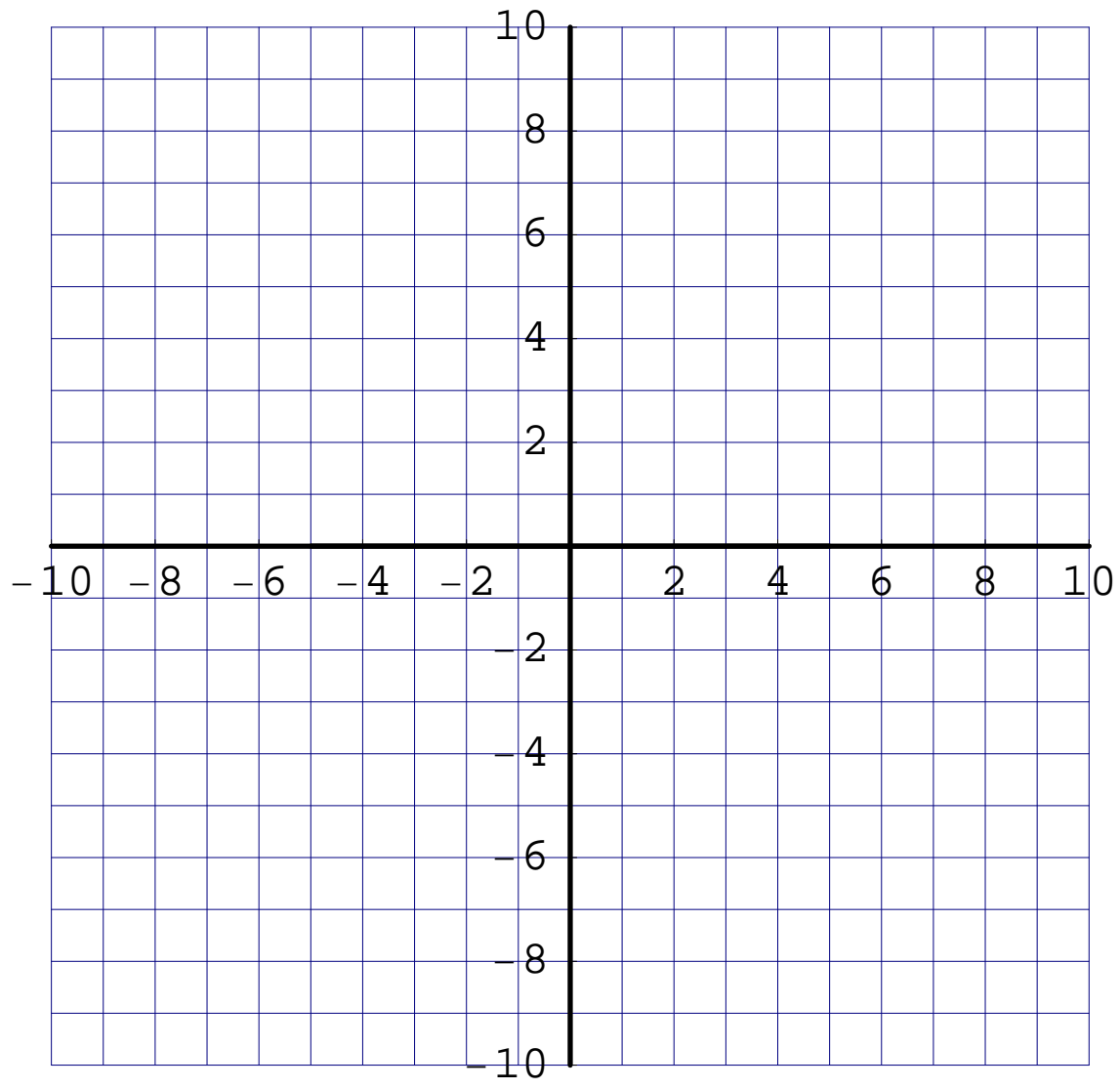
Let  $f(x)$  be a function and  $c$  a number. The function  $f(x)$  is *continuous* at the point  $x = c$  if

1.  $\lim_{x \rightarrow c} f(x)$  exists,
2.  $f(c)$  exists,
3.  $\lim_{x \rightarrow c} f(x) = f(c)$ .

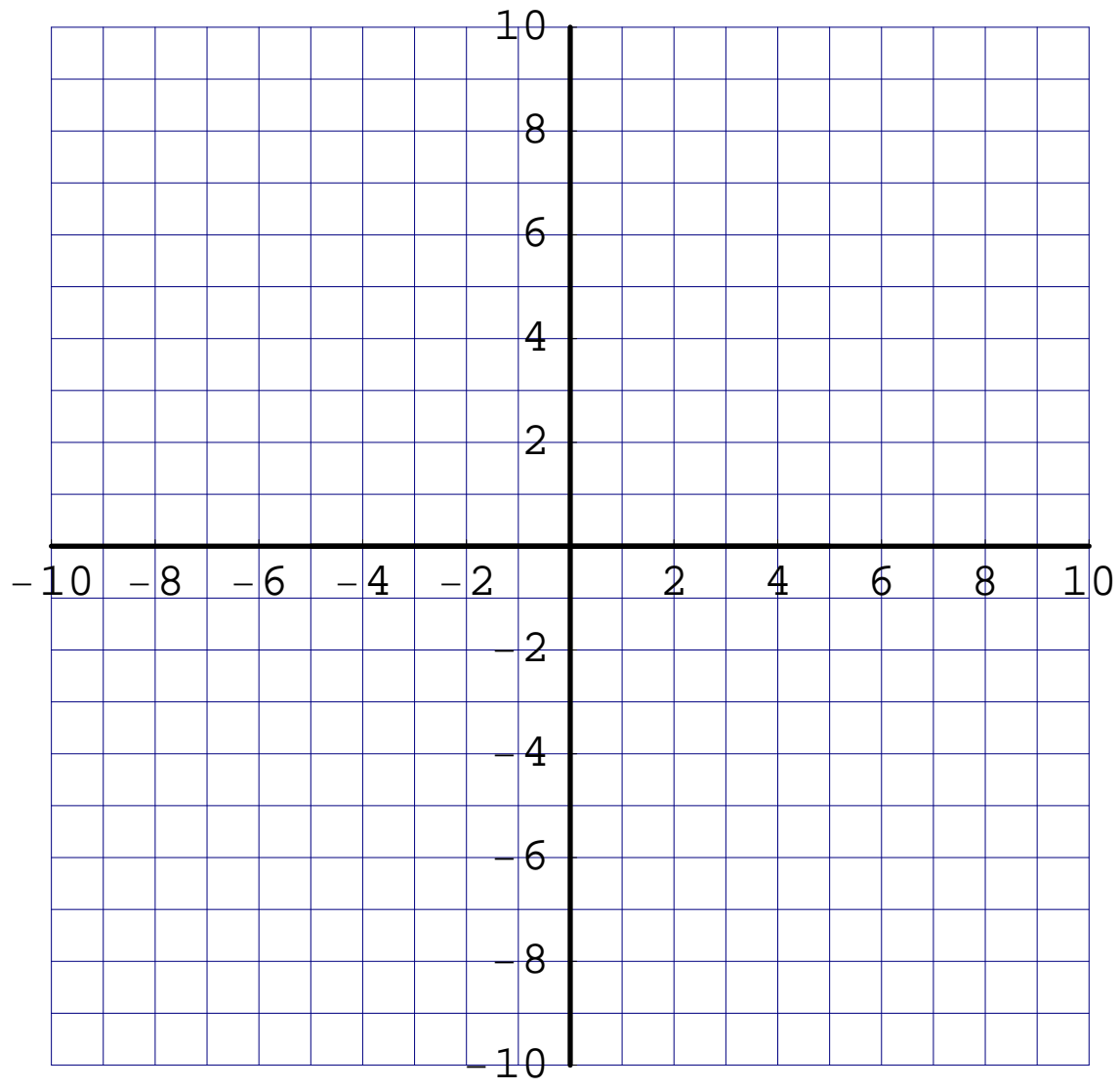
Example:



Example:



Example:

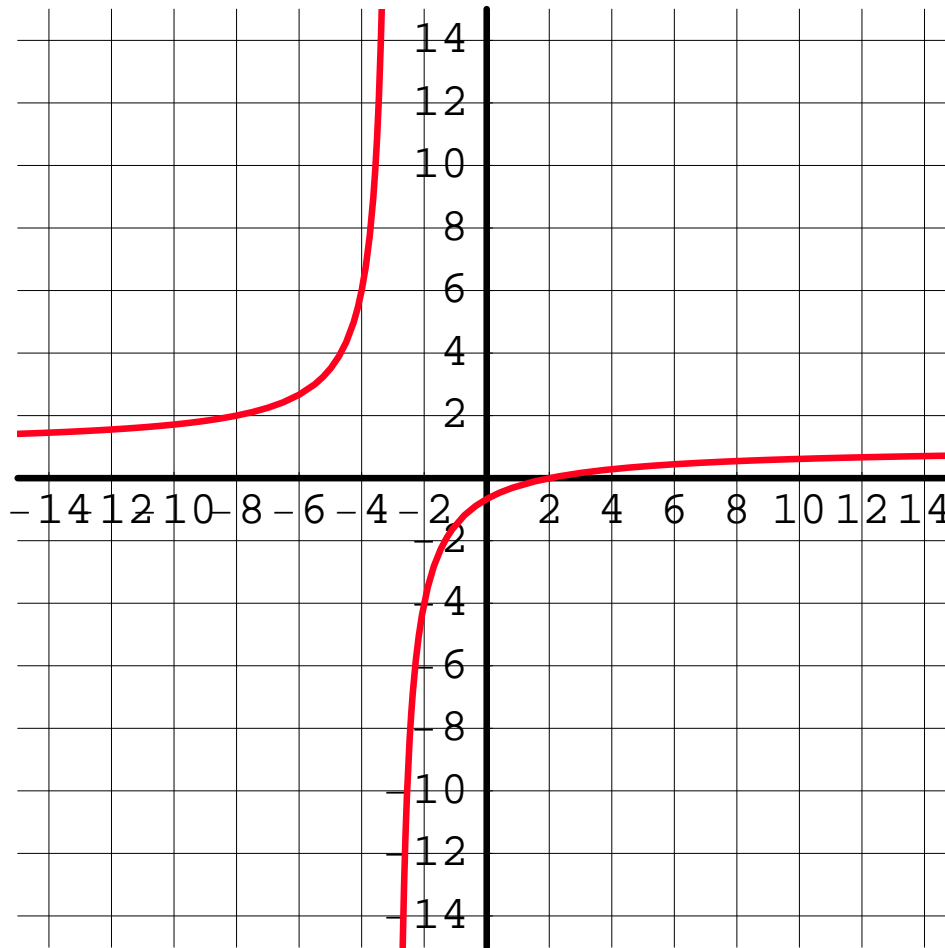


Example: Is the function  $f(x) = \frac{x - 2}{x + 3}$  continuous at

$c = 0$ ?

$c = -3$ ?

$c = 6$ ?



## Example from business

A car rental agency charges \$30 per day (or partial day) or \$150 per week, whichever is least. What is the rental cost  $C(x)$  for  $x$  days?

Fill in the charges for the values of  $x$ :

$x$	1.0	2.0	2.6	3.0	3.1	4.0	4.2	5.0	6.0	7.0	7.1
$C(x)$											

Example from business continued:

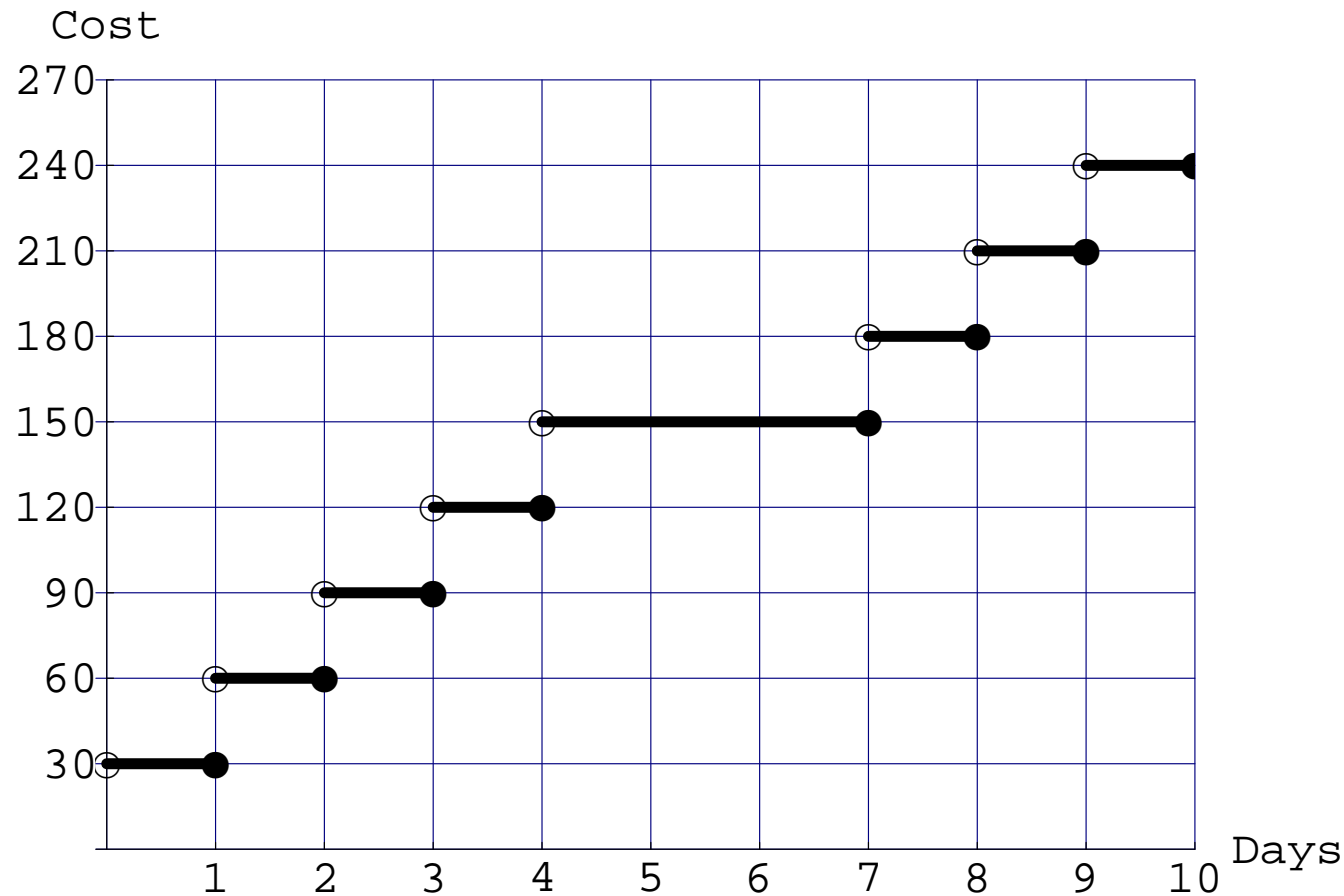
A car rental agency charges \$30 per day (or partial day) or \$150 per week, whichever is least. What is the rental cost  $C(x)$  for  $x$  days?

Fill in the charges for the values of  $x$ :

$x$	1.0	2.0	2.6	3.0	3.1	4.0	4.2	5.0	6.0	7.0	7.1
$C(x)$	30	60	90	90	120	120	150	150	150	150	180

## Example from business

A car rental agency charges \$30 per day (or partial day) or \$150 per week, whichever is least. Graph the cost function  $C(x)$  and discuss continuity.



Example from business  $T(x)$  is the tax on taxable income of  $x$ .

The federal income tax rate is

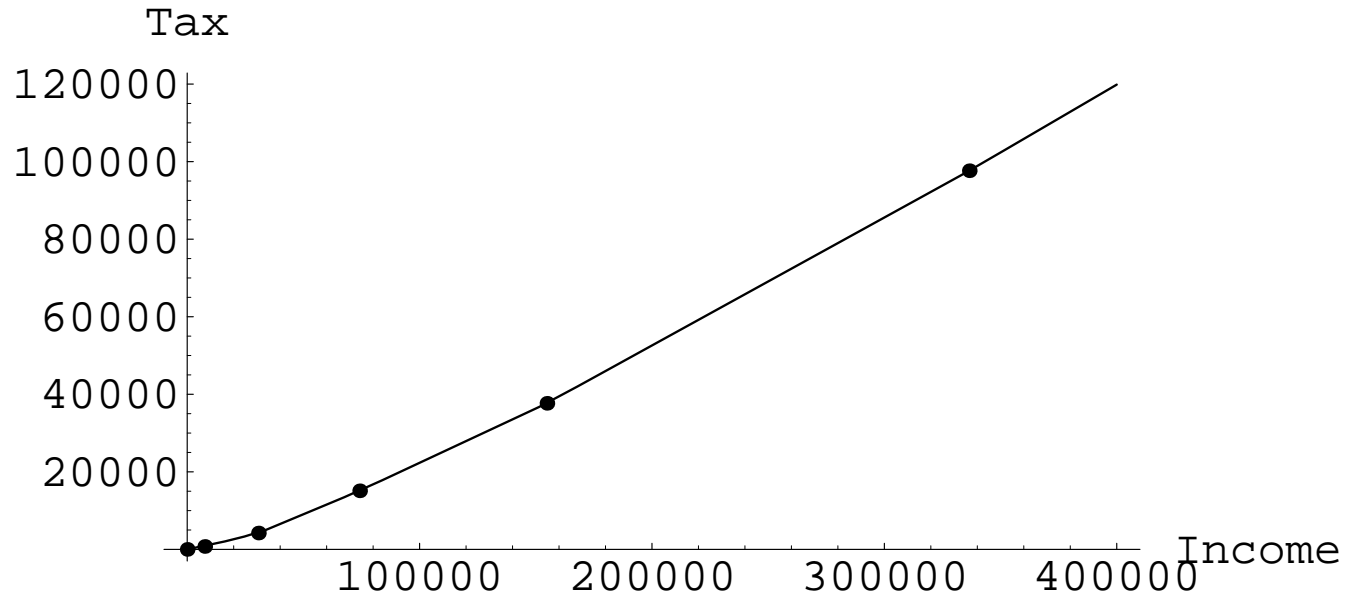
Between	But Not Over	Base Tax	Rate	Of the Amount Over
\$0	\$7,550	0	10%	\$0.00
\$7,550	\$30,650	\$755.00	15%	\$7,550
\$30,650	\$74,200	\$4,220.00	25%	\$30,650
\$74,200	\$154,800	\$15,107.50	28%	\$74,200
\$154,800	\$336,550	\$37,675.50	33%	\$154,800
\$336,550		\$97,653.00	35%	\$336,550

If you have a taxable income of  $x = \$110,000$ , your tax is

$$\begin{aligned} T(110,000) &= \text{Base Tax} + (\text{Rate} \times \text{Amount Over}) \\ &= 15,107.50 + [.28 \times (110,000 - 74,200)] \\ &= 15,107.50 + [.28 \times 35,800] \\ &= 15,107.50 + 10,024.00 \\ &= 25,131.50 \end{aligned}$$

Is  $T(x)$  continuous? What does that mean?

The graph of  $T(x)$ :



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The equations for  $T(x)$ :

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\$336,550		\$97,653.00	35%	\$336,550

For income between \$74,200 and \$154,800:

Line 4 in the table.

$74200 \leq x \leq 154800$ :

The equations for  $T(x)$ :

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\$336,550		\$97,653.00	35%	\$336,550

For income between \$30,650 and \$74,200:

Line 3 in the table.

$30650 \leq x \leq 74200$ :