

Math 510B**Exercises: Due Wednesday, April 30, 2008**

1. Use the companion matrix of $q(x)$ to compute the resultant of the polynomials $p(x), q(x)$ below:

$$\begin{aligned}p(x) &= 2x^3 - 3x + 5 \\q(x) &= x^2 - x + 2.\end{aligned}$$

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2. Let

$$p(x, y) = 2xy^3 + 5y^3 - x^2y^2 + y^2 + 3xy + x + 2.$$

- a. Write $p(x, y)$ as a cubic polynomial $a(y) = a_3y^3 + a_2y^2 + a_1y + a_0$, where the coefficients, q_i , are polynomials in x .
- b. Write the companion matrix for the polynomial $a(x)/a_3$.

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3. Use two methods to eliminate y from this system of two polynomials equations:

$$\begin{aligned}y^2 - x &= 0 \\y^2x + 2y + 1 &= 0.\end{aligned}$$

- a. From the first equation we have $y = \pm\sqrt{x}$. Replace y with $\pm\sqrt{x}$ in the second equation and use algebra to find a *polynomial* in x that the x -coordinate of a solution must satisfy.
- b. Write the companion matrix for $y^2 - x$ as a polynomial in y and use it to eliminate y .

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4. Find all solutions to the system of equations:

$$\begin{aligned}yx^2 + x^2 - y^2x + 4yx - 4x + y^2 - 4y + 4 &= 0 \\yx^2 - 2x^2 + 2y^2x + 2yx + 6x + y - 2 &= 0.\end{aligned}$$

5. The graph of $x^2 + y^2 - r = 0$ is a circle centered at the origin with radius \sqrt{r} . For which value(s) of r is this circle tangent to the circle

$$(x - 3)^2 + (y + 2)^2 - 13?$$

Try to solve this problem using algebraic techniques.