AN INTEGER PROGRAMMING MODEL FOR SETTING SERVICE OPERATING SCHEDULES

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ABSTRACT

Setting the hours during which a service operation is available to its customers is a basic business decision common to all services. Surprisingly, the way services select their hours of operation has received hardly any attention in the service research literature. In this paper we propose to view the selection of operating hours as part of an overall employee-scheduling problem. We demonstrate this idea by modifying a standard work-shift scheduling model to create a more general service-scheduling model where operating hours and employee scheduling are considered simultaneously.

I. INTRODUCTION

Setting the days and hours during which a service is available to its customers is a basic business decision common to all services. The evidence that this decision is not trivial is all around us as there seem to be an endless variety of operating schedules ranging from the all-encompassing 24/7 availability to services that are available for only a few hours during a given day, a few days during a given week, or even a few weeks during a given year. The operating schedule decision is a significant demand and capacity management decision [Klassen and Rohleder, 2002], yet it received little attention in the literature. Further discussion on demand and capacity management in services may be found, for example, in [Heskett, Sasser and Hart, 1990, Crandel and Markland, 1996].

The three cost categories that are affected by the operating schedule are: Facility fixed costs (e.g., electricity, security, supervision), which obviously go down with the number of operating hours; the costs of the demand lost as a result of limiting the number of operating hours (e.g., lost profit, ill-will cost), which go up when the number of operating hours decreases; and variable labor costs, which are affected by ability of the service to schedule employees efficiently, which, in turn, depends on the effect of the operating schedule on the demand.

In this paper, we use the relationship between the operating schedule and employee scheduling by proposing a generalized service-scheduling model where operating hours and employee scheduling are determined simultaneously. The model is developed through modifying a standard work-shift scheduling model in several ways. First, the decision of whether each hour during the planning horizon should, or should not, be included in operating schedule is added. Second, the facility fixed costs and the lost demand costs, which are not part of standard work-shift scheduling models, are included. Finally, the model includes a representation of the effect of selecting operating hours on the demand.

The remainder of the paper is organized as follows. In the next section we discuss a model where work-shift scheduling and operating hours are determined simultaneously. The modeling of demand shifting is discussed next, followed by a service-scheduling model under the assumption that no demand is lost. We conclude with a summary and some suggestions for further research.

II. A BASIC OPERATING SCHEDULE MODEL

Employee work-shift scheduling has been one of the most common tools for capacity management in service operations. With this method, employees are scheduled to work separate shifts, which may vary from one employee to another in their starting and ending times (or even the timing of relief breaks), to create a minimum cost aggregate capacity needed to meet a given pattern of demand. Numerous models and solution techniques have been proposed for obtaining optimal, or near-optimal, work-shift schedules [e.g., Bechtold and Jacobs, 1990, Aykin, 2000]. The first integer programming formulation of the work-shift scheduling problem was suggested by [Dantzig, 1954] who suggested that the problem could be modeled as a set-covering problem. A variation of this model is given below:
Minimize \[ \sum_{i \in T} c_i \sum_{j \in J} a_{ij} x_j \]
Subject to
\[ b \sum_{j \in J} a_{ij} x_j \geq d_i, \quad t \in T \]
\[ x_j \geq 0 \text{ and integer}, \quad j \in J, \]

where \( J \) represents the set of permissible shifts, \( T \) is the set of planning periods in the planning horizon, \( d_i \) is a measure of the demand (e.g., number customers) during period \( t \), \( b \) is the maximum number of customers that can be handled by a single employee during each period, \( a_{ij} \) is equal 1 if period \( t \) is a work period for shift \( j \) and 0 otherwise, \( c_i \) is the cost of scheduling an employee to work during period \( t \), and \( x_j \) is an integer variable indicating the number of employees assigned to shift \( j \).

In order to add the selection of hours of operation to the work-shift scheduling model, it has to be augmented in several ways. First, we add a set of binary variables, \( y_t, t \in T \), such that \( y_t = 1 \) if the service facility is open during planning period \( t \), and 0 otherwise. Second, the cost function should include not just the variable cost incurred when employees are assigned to work, but also any fixed costs required just to keep the facility open and the cost of not meeting the demand during periods when the facility is closed. Finally, constraints must be added to ensure that employees are not assigned to work when the facility is closed. The modified model is:

Minimize \[ \sum_{i \in T} c_i \sum_{j \in J} a_{ij} x_j + \sum_{i \in T} f_i y_i + \sum_{i \in T} r_i d_i (1 - y_i) \]
Subject to
\[ b \sum_{j \in J} a_{ij} x_j \geq d_i, \quad t \in T \]
\[ \sum_{j \in J} a_{ij} x_j \leq U y_i, \quad t \in T \]
\[ x_j \geq 0 \text{ and integer}, \quad j \in J \]
\[ y_i \in \{0,1\}, \quad t \in T, \]

where, in addition to the notation defined above, \( f_i \) is the fixed cost of keeping the service facility open during period \( t \), \( r_i \) is the cost of losing one unit of demand during period \( t \), and \( U \) is an upper bound on the number of employees that could be assigned to work during any period. The purpose of the second set of constraints is to guarantee that no employee is assigned to work during periods when the service facility is closed.

To illustrate the application of this model, consider the example, where the expected number of calls to a call center during each hour in day (24 hours) is given in figure 1(a). A single employee can handle up to 8 calls per hour and may be scheduled to any 8-hour long shift that starts and ends within the same day. Each hour an employee is scheduled to work costs $20. There is also a fixed cost of $100 for every hour that the call center is open. Finally, the cost of losing one unit of demand (one call) in any period is $10. An optimal solution is shown in figure 1(b).
FIGURE 1 -- AN HOURLY DEMAND PATTERN AND AN OPTIMAL SCHEDULE

In this example, it is optimal to operate the service for only 17 hours. However, this is true for the given demand pattern and cost structure. Figure 2 shows the optimal number of hours in the operating schedule for different combinations of fixed costs and lost sales costs.

FIGURE 2 -- OPERATING HOURS VS. FIXED COSTS AND LOST DEMAND COSTS

III. MODELING DEMAND REDISTRIBUTION

As pointed out above, in many cases it would not be correct to assume that all the demand in the periods selected for closure is lost. Rather, this demand may be shifted to other periods. In such cases, the basic model of the last section would not be correct and it must be modified to reflect the expected redistribution of the demand. In this section we show how demand redistribution may be represented by linear constraints, which are in turn incorporated in the integer LP model of the last section.
We assume that the entire demand of the closed periods shifts to other periods, i.e., none of it is lost. The way the demand of a given period is distributed to other periods is assumed to be dependent on two factors: **direction** and **spread**. Direction simply refers to whether the demand moves forward or backward. In some cases shifting is possible in only one direction. Spread is the degree to which demand of a closed period is distributed equally among all open periods. In order to represent these factors in the model, we adopt a network flow approach where, the demand of closed periods is redistributed among the open periods by way of flowing between neighboring periods. These flows are shown in the diagram of Figure 3. In this diagram, $u_t$ and $v_t$ represent the forward flow and the backward flow of demand from period $t$, respectively.

![FIGURE 3 – A NETWORK FLOW DIAGRAM OF DEMAND DISTRIBUTION](image)

The demand flows are controlled by two parameters: a "direction coefficient" $\alpha$, and "spread coefficient" $\beta$. The coefficient $\alpha$ is the fraction of the demand of a closed period, which moves forward. The remaining fraction $(1-\alpha)$ of the demand moves backward. The coefficient $\beta$ is the fraction of the demand shifted into an open period, which remains in this period (i.e., is added to the demand of that period). In addition to the flow variables defined above, let $d'_t \geq 0$ be a variable representing the modified demand in period $t$. Finally, let $U$ be a sufficiently large upper bound.

We now discuss the constraints that will guarantee that the original demand $d_t$, $t \in T$ is redistributed according to the rules described above. First, the following constraints calculate the modified demands and make sure that the modified demand for all "closed" periods is equal to zero:

$$
\begin{align*}
    d'_t &= d_t + u_{t-1} + v_{t+1} - u_t - v_t, & t \in T \\
    d'_t &\leq U \gamma_t, & t \in T
\end{align*}
$$

The next two sets of constraints handle the forward flow out of a given period $t$:

$$
\begin{align*}
    \alpha d_t + u_{t-1} - u_t &\geq 0, & t \in T \\
    \alpha d_t + u_{t-1} - u_t &\leq U \gamma_t, & t \in T
\end{align*}
$$
If period \( t \) is "open" (i.e., \( y_t = 1 \)), then its outgoing forward flow is equal to a fraction \( 1-\beta \) of its incoming forward flow:

\[
\begin{align*}
    u_t - (1 - \beta)u_{t-1} &\geq 0, \quad t \in T \\
    u_t - (1 - \beta)u_{t-1} &\leq U(1 - y_t), \quad t \in T
\end{align*}
\]

The backward flows require separate, but similar constraints, which are given below:

\[
\begin{align*}
    (1 - \alpha)d_t + v_{t+1} - v_t &\geq 0, \quad t \in T \\
    (1 - \alpha)d_t + v_{t+1} - v_t &\leq Uy_t, \quad t \in T \\
    v_t - (1 - \beta)v_{t+1} &\geq 0, \quad t \in T \\
    v_t - (1 - \beta)v_{t+1} &\leq U(1 - y_t), \quad t \in T
\end{align*}
\]

Finally, two constraints are needed to "connect" the flows of the first and last periods of the planning horizon (\( P \) in the number of periods in the planning horizon):

\[
\begin{align*}
    u_0 &= u_P \\
    v_1 &= v_{P+1}
\end{align*}
\]

For any specific assignment of values to the binary "open/close" variables \( y_t, t \in T \), simultaneously satisfying the constraints listed above results in a unique assignment of values to the flow variables \( u_t \) and \( v_t \), and, therefore in a unique assignment of values to the modified demand variables \( d_t, t \in T \). The resulting modified demand depends on the values of the coefficients \( \alpha \) and \( \beta \).

Figure 4 demonstrates the effect of the spread coefficient \( \beta \). In both cases we use \( \alpha = 0.5 \) but in Figure 4(a) \( \beta = 0.8 \) (small spread) while in Figure 4(b) \( \beta = 0.2 \) (large spread).

**FIGURE 4 – THE EFFECT OF THE SPREAD COEFFICIENT \( \beta \)**
IV. SELECTING OPERATING HOURS – THE DEMAND REDISTRIBUTION CASE

A model for the optimal selection of operating hours under the assumption that the demand of closed periods is shifted to open periods is obtained by augmenting the basic model of Section II with the additional variables and constraints discussed above. An optimal schedule for the case where $\alpha = 0.5$ and $\beta = 1$ is shown in figure 5. As can be seen in the graph, it is optimal in this case to open the service for only 16 hours starting at period 7. The total number of employees is again 14. The total cost of this schedule is $3,840.

\[\text{Minimize } \sum_{i \in T} c_i \sum_{j \in J} a_{ij} x_j + \sum_{i \in T} f_i y_i\]

\[\text{Subject to}\]

\[b \sum_{j \in J} a_{ij} x_j \geq d_i, \quad t \in T\]

\[\sum_{j \in J} a_{ij} x_j \leq U y_i, \quad t \in T\]

\[d_i' = d_i + u_{i-1} + v_{i+1} - u_i - v_i, \quad t \in T\]

\[d_i' \leq U y_i, \quad t \in T\]

\[ad_i + u_{i-1} - u_i \geq 0, \quad t \in T\]

\[ad_i + u_{i-1} - u_i \leq U y_i, \quad t \in T\]

\[u_i - (1 - \beta) u_{i-1} \geq 0, \quad t \in T\]

\[u_i - (1 - \beta) u_{i-1} \leq U (1 - y_i), \quad t \in T\]

\[(1 - \alpha) d_i' + v_{i+1} - v_i \geq 0, \quad t \in T\]

\[(1 - \alpha) d_i' + v_{i+1} - v_i \leq U y_i, \quad t \in T\]

\[v_i - (1 - \beta) v_{i+1} \geq 0, \quad t \in T\]

\[v_i - (1 - \beta) v_{i+1} \leq U (1 - y_i), \quad t \in T\]

\[u_0 = u_p\]

\[v_1 = v_{p+1}\]

\[x_j \geq 0 \text{ and integer, } \quad j \in J\]

\[y_i, \in \{0,1\}, \quad t \in T\]

\[d_i' \geq 0, u_i \geq 0, v_i \geq 0, \quad t \in T\].
The effect of $\beta$ on the optimal schedule is shown in figure 6. In this case, when the value of $\beta$ is lowered, it becomes optimal to shorten the workday, and the total cost decreases. This apparently is because a smaller $\beta$ causes the demand shifted from closed periods to be distributed more evenly, resulting in a smoother demand pattern.

![Table showing the effect of $\beta$ on the optimal operating schedule](image)

**FIGURE 6 – THE EFFECT OF $\beta$ ON THE OPTIMAL OPERATING SCHEDULE**

**V. CONCLUSION AND FURTHER RESEARCH**

Using a simplified work-shift scheduling model as a basis, we develop an integer LP model for selecting operating hours under given demand pattern, scheduling constraints, and cost structure. A small, synthetic, example is used to demonstrate that the solutions obtained by the models confirm the following insights:

1. Higher facility fixed costs lead to shorter operating hours
2. Higher lost demand costs lead to longer operating hours
3. Greater willingness of customers to shift their demand from non-operating hours to operation hours leads to shorter operating hours
4. More even re-distribution of demand from non-operating hours to operation hours leads to shorter operating hours

The use of the model with actual data, as part of future research, is needed to provide further support to the findings of this study.

The optimal selection of hours of operation clearly depends on the pattern of the daily demand. Most service operations experience seasonal, including weekly, variations in demand, which means that the optimal set of operating hours could change over time (for example for different days of the week). Most services would prefer to keep fixed operating hours (except perhaps for different hours for weekdays and the weekend) to avoid confusing their customers and to make it easier for customers to remember the operating schedule. The proposed model can easily be used to find fixed operating hours by considering demand data that include an entire week or even a longer period. Such a model can also be used to explore the question of days of operation in addition to hours of operation.
VI. REFERENCES


