ADJUSTMENT OF MODIFIED INTERNAL RATE OF RETURN FOR SCALE AND TIME SPAN DIFFERENCES

David Cary, California State University, Northridge
david.cary@csun.edu
Michael Dunn, California State University, Northridge
mdunn@pacbell.net

ABSTRACT

The use of the Internal Rate of Return (IRR) method of capital budgeting is popular as many managers prefer a rate of return method as a decision-making criterion for capital budgeting. However, the Net Present Value (NPV) method is preferred by academics since the rankings of mutually exclusive projects by IRR may not always select the project which will maximize the value of the firm, due to an implied reinvestment rate assumption by IRR. In response to this weakness, the Modified Internal Rate of Return (MIRR) was developed. However, MIRR may also lead to erroneous rankings when projects require different initial outflows to start the project, the scale problem, or the projects have different lives, the time span problem. This paper demonstrates how MIRR can be adjusted to give rankings that are consistent with NPV for projects in the same risk class, even with scale differences and for some types of time span differences. A secondary contribution is a simplified method of computing MIRR with examples to show the consistency with the NPV and with the goal of maximizing the value of the firm to its shareholders.

INTRODUCTION

While rates of return methods, in general, and the Internal Rate of Return (IRR) method in particular, have been found to be favored by a majority of companies (Gitman & Forrester, 1997), it has also been shown that the IRR method can lead to erroneous rankings of mutually exclusive projects when compared to the Net Present Value (NPV) method of capital budgeting (Fisher, 1930). The differences in rankings may be caused by the implied reinvestment rate assumption of the IRR method (Fisher, 1930), or by differences in the size of the projects, the *scale problem*, or in the life of the projects, the *time span problem*. Differences in the risk classes of the projects and capital rationing can also cause ranking differences. This paper will assume that all projects are in the same risk class as the firm and that capital rationing does not exist. The Modified Internal Rate of Return (MIRR) method of capital budgeting, or similarly the Financial Management Rate of Return method (Findlay & Messner, 1973), was developed to overcome the problem of the implied reinvestment rate assumption (Bierman & Smidt, 1984, Hirshleifer, 1970, Solomon, 1956). However, when scale or time span differences exist, the MIRR method may still give rankings of mutually exclusive projects that are different than NPV (Brigham & Gapenski, 1988). This paper presents an adjustment to the MIRR method that will give rankings that are consistent with NPV for scale differences and for non-repeatable projects, for time span differences. In addition, a simplified method of calculating MIRR is developed.

MODIFIED INTERNAL RATE OF RETURN

This paper assumes that an initial outflow is followed by a series of inflows. The method of calculating MIRR presented here can be used when there are future outflows by either discounting the outflows back to the present, at the cost of capital, and including them as part of the initial outflow or by letting future inflows offset the future outflows. As long as the largest initial outflow is used in the calculation of the MIRR's for all projects, the ranking between MIRR and NPV will remain consistent.

The MIRR is computed in two steps by first compounding the future cash flows of a project to the end of the project at an explicitly assumed reinvestment rate "k" to get a Terminal Value of the Future Cash Flows (TVFCF).

For firms which are not subject to capital rationing the reinvestment rate should be the cost of capital which represents the rate of return generally available for projects of equivalent risk (Dudley, 1972, Meyer, 1979, Nicol, 1981). The essence of the project is then represented in the Initial Outflow (IO) and the Terminal Value. Then the MIRR, the implied rate of return which equates these two values over time, is calculated. For "N" periods:

$$TVFCF = \sum_{t=1}^{N} CF_t \, (1 + k)^{N - t}$$

$$TVFCF = IO \, (1 + MIRR)^N \quad thus$$

$$MIRR = \left(\frac{TVCFF}{IO} \right)^{1/N} - 1 \qquad (Equation \ \#1)$$

As presently used, MIRR, like other rates of return, is subject to problems. Rate of return is per dollar invested and per year. A small project with a high rate of return may contribute less to the wealth of the firm than a large project with a lower rate of return. A project which will last many years may be superior to a shorter duration project with a higher rate of return. As will be discussed below, a critical factor in the analysis of projects with different lives will be whether a project can be repeated in the future and if so, on what terms. Additional problems exist if the projects are in different risk classes and therefore have different costs of capital or if the discount rate is not constant over time.

SCALE DIFFERENCES

Exhibit 1 presents an example of the ranking conflict between NPV and IRR for two mutually exclusive projects because of scale differences. Ranking is important for mutually exclusive projects to determine which otherwise acceptable project should be chosen. Ranking may also be important if there is capital rationing. In all other cases, all acceptable projects should be selected. Although Project B is vastly superior to project L in terms of wealth maximization, it has a lower IRR.

Exhibit 1
Comparison of Projects with Different Initial Outflows
(Scale Differences)

	Project L	Project B
Initial Outflow	($100.00)	($1,000.00)
Period 1	40.00	350.00
Period 2	50.00	450.00
Period 3	60.00	550.00
Period 4	70.00	650.00
Cost of Capital	10.00%	10.00%
Net Present Value	70.58	547.26
(Ranking)	(2)	(1)
Internal Rate of Return	36.44%	30.72%
(Ranking)	(1)	(2)
(Rankings are reversed)		

MIRR, as conventionally computed, has the same problem. Exhibit 2 illustrates the conventional calculation of Terminal Value (TVFCF) for project L.

Exhibit 2
Calculation of Terminal Value for Project L

Initial Outflow	($100.00)				
Period 1	40.00	x	1.3310	=	53.24
Period 2	50.00	x	1.2100	=	60.50
Period 3	60.00	x	1.1000	=	66.00
Period 4	70.00	x	1.0000	=	70.00
Terminal Value					$249.74

$$MIRR_L = \left(\frac{249.74}{100.00}\right)^{1/4} - 1 = 0.2571$$

The Terminal Value may be calculated in a way which is often more convenient. This simplified computational formula will be shown to also facilitate adjustments for differences in the size or temporal span of projects. The mathematics of time value of money imply that the Terminal Value is equal to the present value of the future cash flows compounded, at the cost of capital, to the terminal period. Reinvestment rates other than the cost of capital can be used to calculate the terminal value. However, if funds cannot be reinvested at the cost of capital, then the use of NPV may not be appropriate and the ranking between MIRR and NPV may not be consistent. Letting k be the reinvestment rate:

$$TVFCF = PVFCF\ (1+k)^N, \quad and$$

$$NPV = PVFCF - IO, \quad thus$$

$$PVFCF = (IO + NPV), \quad thus$$

$$TVFCF = (IO+NPV)\ (1+k)^N \qquad (equation\ \#2)$$

$$Substituting\ \#2\ into\ \#1$$

$$MIRR = \left(\frac{(IO+NPV)\ (1+k)^N}{IO}\right)^{1/N} - 1 \qquad (equation\ \#3)$$

$$MIRR_L = \left(\frac{(100+70.58)\ (1+0.1)^4}{100}\right)^{1/4} - 1 = 0.2571$$

A similar calculation for Project B results in a $MIRR_B$ of 0.2268 which is less than the $MIRR_L$ (= 0.2571). Note that this conventional calculation of MIRR, unadjusted for size differences, gives a ranking that is inconsistent with NPV.

The solution to the ranking problem lies in the insight that the acceptance of the smaller project L also implies the acceptance of a shadow investment, equal to the difference in size between the smaller and larger projects, which earns the cost of capital. In this case, to accept project L (an investment of $100) is to reject B (an investment of $1,000). Having assumed a non-capital rationing situation, the firm should be accepting all non-mutually exclusive investments with a positive NPV. Thus, marginal project would earn the cost of capital. Since rates of return are per dollar and per year, they are only comparable for projects of the same size and the same time span. The MIRRs for L and B may be made comparable by assuming that taking the smaller project (L) also implies accepting a shadow investment which earns the cost of capital (has a zero NPV) with an initial outflow equal to the difference between the two projects.

Exhibit #3
Inclusion of a Shadow Investment
to Adjust for Scale Differences

	Project L	Shadow Investment	Combined L + Shadow	Project B
Initial Outflow	$100.00	$900.00	$1,000.00	$1,000.00
Period 1	40.00	283.92	323.92	350.00
Period 2	50.00	283.92	333.92	450.00
Period 3	60.00	283.92	343.92	550.00
Period 4	70.00	283.92	353.92	650.00
Cost of Capital	10.0%	10.0%	10.0%	10.0%
NPV	70.58	0.00	70.58	547.26
(Ranking)			(2)	(1)
IRR	36.44%	10.00%	13.17%	30.72%
(Ranking)			(2)	(1)
MIRR	25.71%	10.00%	11.89%	22.68%
(Ranking)			(2)	(1)

The appropriate comparison is between the MIRR of B and the MIRR of L plus the implied shadow investment. Other authors (Sweeney & Mantripragada, 1987) have suggested paired comparisons analogous to Fisher's Defender/Challenger approach to reconciling IRR with NPV. This is clearly a more complex process, particularly if numerous alternatives are under consideration. Exhibit 3 shows that in this example, the adjusted MIRR ranks consistently with NPV. (Appendix 1 has a general proof for the consistency) In practice, it is not necessary to estimate and include the shadow as in Exhibit 3. The shadow will always have a zero NPV. Using Equation #3, the calculation of MIRR for the smaller project would simply require the replacement of the initial outflow (IO) of the smaller project with the IO of the larger project in both the numerator and denominator. In fact, any number of mutually exclusive alternatives may be compared by simply assigning to each the same initial outflow as the largest of them.

$$MIRR_L = \left(\frac{(NPV_L + IO_B)(1 + k)^N}{IO_B} \right)^{1/N} - 1$$

$$MIRR_L = \left(\frac{(70.58 + 1{,}000.00)(1.10)^4}{1{,}000} \right)^{1/4} - 1$$

$$MIRR_L = 11.89\%$$

DIFFERENCES IN TIME SPANS

The adjustment for time span differences depends on the repeatability of the projects. For projects that can be repeated in the future, either a replacement chain to a common ending point or truncation of the longer project is necessary for a proper calculation of both NPV and MIRR.

However, for non-repeatable projects, Equation #3 may be used by using the life of the longest project for N in the calculation of MIRR for shorter-lived rival projects. The proof of this assertion is directly analogous to that present in Appendix 1 for scale differences.

Exhibit 4
Adjustment of MIRR for Time Span Differences

	Project P	Project Q
Initial Outflow	($1,000.00)	($1,000.00)
Period 1	300.00	500.00
Period 2	350.00	600.00
Period 3	400.00	700.00
Period 4	450.00	-
Period 5	500.00	-
Period 6	550.00	-
Cost of Capital	10.00%	10.00%
NPV	790.79	476.33
(Ranking)	(1)	(2)
IRR	31.09%	33.87%
(Ranking)	(2)	(1)
MIRR's		
With Actual Life	21.22%	25.25%
(Ranking)	(2)	(1)
With Adjusted Life	21.22%	17.38%
(Ranking)	(1)	(2)
(Agrees with NPV)		

Exhibit 4 compares two non-repeatable, mutually exclusive projects, P and Q. It shows that the shorter project Q has a lower NPV but both a higher IRR and conventional MIRR than the longer project P. The adjusted life MIRR for project Q is calculated using Equation #3, with NPV equal to $476.33, IO equal to $1,000, and N equal to six, and gives a ranking that is consistent with that of NPV. In effect, using a six year life for project Q assumes that the terminal value at the end of period three is compounded at the reinvestment rate (k) to the end of period six.

BOTH SCALE AND TIME SPAN DIFFERENCES

The analysis of two or more mutually exclusive, non-repeatable projects with both scale and time span differences may be easily accomplished by using the largest initial outflow and the largest number of periods in Equation #3 for the computation of MIRR for each project. This means that IO, k and N will be the same for all alternative projects. The NPV in Equation #3 will be the actual NPV for each project.

Exhibit 5 illustrates three mutually exclusive, non-repeatable projects with different initial outflows and time spans. Note that based on NPV, the rankings are Z, Y, and then X. However, based on IRR and the unadjusted MIRR, the rankings are Y, X, and then Z. Using Equation #3 and adjusting for scale and time span differences, the adjusted MIRR is consistent with NPV.

Exhibit 5
Adjustment of MIRR for Scale and Time Span Differences

	Project X	Project Y	Project Z
Initial Outflow	($500.00)	($1,000.00)	($2,000.00)
Period 1	150.00	500.00	750.00
Period 2	150.00	500.00	750.00
Period 3	150.00	500.00	750.00
Period 4	150.00	-	750.00
Period 5	150.00	-	-
Period 6	150.00	-	-
Cost of Capital	10.00%	10.00%	10.00%
NPV	$153.29	$243.43	$377.40
(Ranking)	(3)	(2)	(1)
IRR	19.91%	23.38%	18.45%
(Ranking)	(2)	(1)	(3)
Unadjusted MIRR:			
Initial Outflow	$500.00	$1,000.00	$2,000.00
Number of Periods	6	3	4
Unadjusted MIRR	15.01%	18.29%	14.86%
(Ranking)	(2)	(1)	(3)
MIRR adjusted for scale and time differences:			
Initial Outflow	$2,000.00	$2,000.00	$2,000.00
Number of Periods	6	6	6
Adjusted MIRR	11.36%	12.13%	13.22%
(Ranking)	(3)	(2)	(1)
(Agrees with NPV Rankings)			

CONCLUSION

This paper has shown how to calculate a rate of return measure that will give rankings that are consistent with NPV for mutually exclusive projects, even if the projects are of different sizes and in some cases, different lives. Since managers tend to rely on rate of return calculations in capital budgeting, the consistency with NPV is an important contribution. An additional contribution is the simplified calculation formula for MIRR as presented in Equation #3.

APPENDIX 1

For the proof that the shadow investment will give consistent rankings between NPV and MIRR, assume that $NPV_A > NPV_B$ and $IO_A > IO_B$. Note that if $IO_B > IO_A$ then use IO_B in place of IO_A in the proof below.

$$NPV_A > NPV_B$$
$$NPV_A + IO_A > NPV_B + IO_A = (NPV_B + IO_B) + (IO_A - IO_B)$$
$$(NPV_A + IO_A)(1 + K)^N > ((NPV_B + IO_B) + (IO_A - IO_B))(1 + K)^N$$
$$TVFCF_A > TVFCF_B + (IO_A - IO_B)(1 + K)^N$$

The left hand side is equal to the Terminal Value of the Future Flows ($TVFCF_A$) for project A. The right hand side (RHS) is equal to the Terminal Value of the Future Cash Flows for project B plus the Terminal Value of the shadow investment that has an initial outflow equal to difference between the initial outflows of projects A and B. Call the RHS the Modified Terminal Value of project B (MTV_B). Using the terminal values and equation #1:

$$MIRR_A = \left(\frac{TVFCF_A}{IO_A}\right)^{1/N} - 1 > \left(\frac{MTV_B}{IO_A}\right)^{1/N} - 1 = *MIRR_B$$

where $*MIRR_B$ is the Modified Internal Rate of Return for project B, adjusted for the difference in the size of the initial outflows of the two projects. Note that the inequality is preserved and thus the MIRR methodology, when adjusted for differences in the initial outflow, will always agree with the NPV method.

BIBLIOGRAPHY

Bierman, Harold and Seymour Smidt, (1984) "The Capital Budgeting Decision, Sixth Edition," New York, Macmillen Publishing Company, 64-65.

Brigham, Eugene F. and Louis C. Gapenski, (1988) "Financial Management, Theory and Practice, fifth edition," Chicago, Illinois: The Dryden Press, 272-273.

Dudley, C.L., Jr. (1972) "A Note on Reinvestment Assumption in choosing between Net Present Value and Internal Rate of Return," *The Journal of Finance,* September 1972, 907-915.

Findlay, M. Chapman and Stephen D. Messner, (1973) "Determination and Usage of FM Rate of Return." Realtron Corporation, Detroit.

Fisher, Irving, (1930) "The Theory of Interest" (The Macmillan Company; reprinted (Kelly) 1955).

Gitman, Lawrence J. and John F. Forrester, Jr., (1977) "A Survey of Capital Budgeting Techniques Used by Major U.S. Firms" *Financial Management*, Fall 1977, 66-71.

Hirshleifer, J., (1970) "Investment, Interest, and Capital," Englewood Cliffs, New Jersey, Prentice-Hall, Inc.

Meyer, R.L., (1979) "A Note on Capital Budgeting Techniques and the Reinvestment Rate," *Journal of Finance*, December 1979, 1251-1254.

Nicol, D.J., (1981) "A Note on Capital Budgeting Techniques and the Reinvestment Rate: Comment," *The Journal of Finance*, March 1981, pp. 193-195.

Solomon, Ezra, (1956)"The Arithmetic of Capital-Budgeting Decisions," *Journal of Finance*, April, 1956, 124-129.

Sweeney, L.E. & Mantripragada, K.G., (1987) "Ranking Mutually Exclusive Investments: A Modified Internal Rae of Return Approach," *Akron Business and Economic Review*, Spring 1987, 19-25.