3.1 DIMENSIONAL ANALYSIS

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INTRODUCTION (FOR THE TEACHER)

Many teachers tell their students to solve "word problems" by "thinking logically" and checking their answers to see if they "look reasonable". Although such advice sounds good, it doesn't translate into good problem solving. What does it mean to "think logically?" How many people ever get an intuitive feel for a coluomb , joule, volt or ampere? How can any student confidently solve problems and evaluate solutions intuitively when the problems involve abstract concepts such as moles, calories, or newtons?

To become effective problem solvers, students need to develop sound problem solving techniques and strategies. The most useful problem solving technique in science is **dimensional analysis**, also known as **factor-label method** and **unit analysis**. Dimensional analysis is a general problem solving method that uses the dimensions (units) associated with numbers as a guide in setting up and checking calculations. Dimensional analysis is a consistent and predictable technique, yet many teachers are not experienced enough with the technique to teach others. If you are not comfortable with your ability to use or teach dimensional analysis, work through the exercises in this chapter, paying particular attention to section 3.1.7 in which there are numerous sample problems. Students learn well by seeing patterns in examples, so you will be wise to photocopy the sample problems and give them to your students as a reference.

If teachers are not consistent in their use of units, students will not be either. Many teachers forget to use units when solving problems for their students, and this gives students license to do the same. To avoid this problem, consider giving students extra credit for catching each time the teacher omits units in a measurement or calculation.

Dimensional analysis is predicated upon the fact that all measurable quantities have dimensions (units). To specify that the speed of light is 3.0×10^8 is meaningless. Is it 3.0×10^8 miles/hour, 3.0×10^8 kilometers/second, or 3.0×10^8 fathoms/fortnight?! The number 3.0×10^8 has no inherent meaning. However, when we specify that the speed of light is 3.0×10^8 meters/second, we provide all the information needed to compare it with the speed of other things, and to solve equations that involve the speed of light. Before solving a problem, make certain that all numbers have units, with the exception of the following : trigonometric functions (sine, cosine, tangent, etc.) logarithms, and certain special numbers such as e and π.

Just like a golfer searches for the flag of the next hole before teeing off, so the good problem solver must determine the dimensions of the final answer (<u>the unknown</u>) before starting to solve the problem. If you do not know the units of your answer before you begin, you will be like a golfer who tees off without any knowledge of direction of the hole. The likelihood that either you or the golfer will achieve your goals will be minimal! If, however, you specify the desired units of the answer before proceeding, you will know how to set up your problem to achieve the correct answer.

Once you have identified the units of the answer you seek (the unknown), carefully list the values and units of all things that you know (<u>the knowns</u>). If appropriate, draw a diagram of the problem and label the parts with the appropriate units.

Examine the units of the values that you have and the ones you desire in the answer, and specify all relevant <u>conversion factors and formulas</u>. For example, if your answer has units that include hours, and your known values are only expressed in minutes, you will need the conversion factor of 60 minutes/hour. Without such conversion factors you will be unable to get the answer expressed in the units you desire. Conversion factors can be expressed as ratios. For example , since sixty minutes is the same as one hour, you can express this equivalency as 60 minutes/hour or 1 hour/60 minutes.

Dimensional analysis uses what is known to determine what is unknown. It is necessary to establish an <u>equation</u> that shows how the unknown is related to the knowns. Customarily, one places the units of the unknown on the right side of the equation, and manipulates the knowns on the left side to determine the value of the unknown.

Before a <u>calculation</u> of the unknown is made, it is essential to make sure that the units on the left side of the equation are equivalent to those on the right. If the units on the left are not equivalent to those on the right, then the equation is set up incorrectly and you must re-examine the known side of the equation and manipulate the terms as necessary to get the desired units. When working to balance the units, make certain that you keep all conversions factors and formulas intact, even if it is necessary to invert them. For example, if your equation involves the use of the 60min/hour conversion factor, you can multiply by it (60min/hr) or divide by it (hr/60 min), but you can' t move the units independently (1/60 min·hr). Once the units on the left side of the equation equal those on the right, you may proceed with calculations. The following example illustrates the process of dimensional analysis.

<u>Example 1 (physiology)</u>

If the average heart rate is 72 beats per minute, how many times will a heart beat in a year? (figure xx)

- <u>Unknown</u> An analysis of the problem shows that the unknown (the number of times a heart will beat in a year) must have units of beats/year.
- <u>Knowns</u>: We know that the average heart rate is 72 beats per minute
- <u>Conversion factors and formulas</u>: To convert minutes to years, we will need the following conversion factors: one hour is equal to 60 minutes; one day is equal to

24 hours; and one year is equal to 365 days. Since conversion units are always equal to one, you can multiply by them, or divide by them (invert and multiply) without changing their meaning.

- Equation: The units of the unknown (beats/min) become the target units and are set up on the right side of the equation. The left side of the equation is assembled so that units will cancel and leave only the target units.
- Calculation: After the units are canceled, the equation yields the answer in beats/year.

Activity 3.1.1 (Using appropriate unit measures) illustrates the importance of using units. Students will experience difficulty completing part B because of the absence of units. Show that this difficulty is reduced when units are consistently reported, and illustrate how the problem solver can make reasonable inferences even when the data is incomplete.

Activity 3.1.2 (Understanding Fundamental Quantities) can be performed as a pre-test and post-test in a unit on problem solving. Most students have never heard of some of the units in this activity, but can still make logical inferences. For example, since "candela" sounds like "candle", it may be inferred to be a measure of luminous (light) intensity. Similarly, "sidereal day" (the time it takes for the Earth to make one complete revolution in relation to a given star; equal to 23 hours, 56 minutes, 4.1 seconds), may be inferred as a measure of time because it contains the word "day."

Your students may have difficulty realizing that there are more than one unit for the same quantity. For example, the quantity "length" can be measured in meters, feet, cubits, hands, etc (table xx). To help them with this concept, it may be useful to introduce the concept of an alias. You may have noticed that some authors, politicians, and celebrities write or work under assumed names known as aliases, pen names or pseudonyms, creating similar confusion to the one students experience when they see pressure measured in torr, mm Hg, newtons per square meter and PSI. For example, American author Samuel Clemens wrote "Tom Sawyer" and the "Adventures of Huckleberry Finn" under the pen name, Mark Twain, but most people would assume that Samuel Clemens and Mark Twain are two different individuals unless someone explained this relationship. To liven things up, play the "alias game" by displaying the list of aliases on the right on an overhead or data projector, and then reading a few of the given names to see if your students can guess who's who. Although aliases are not the same thing as multiple units (each unit has a different value), this shows students that things are less confusing when you realize that there may be multiple names related to the same or similar basic concept..

Given name	*Assumed name (Alias)*
William H. Bonney	Billy the Kid, notorious gunfighter
Kunk Tze	Confucius, ancient Chinese educator
Theodore Geisel	Dr. Seuss, author of children's books
Eric Arthur Blair	George Orwell, English novelist
Leslie Lynch King, Jr.	Gerald R. Ford, 38th US president
Ehrich Weiss	Harry Houdini, escape artist
Nguyen Tha Thanh	Ho Chi Minh, Vietnamese revolutionary

John Griffith Chaney	Jack London, American author
Karol Wojtyla	John Paul II, first pope from a Slavic country
Samuel Langhorne Clemens	Mark Twain, American author
Agnes Gonxha Bojaxhiu	Mother Teresa, champion of the poor
Tatanka Iotake	Sitting Bull, Sioux Indian chief
Eldrick Woods	Tiger Woods, one of the world's greatest golfers
William Claude Dukinfield	W.C. Fields, Famous comedian

Table xx Different units for the same quantity

Quantity	*SI base units*	*other units*
distance	meters	centimeters, nanometers, miles, inches, feet, fathoms, Ångstroms, microns, kilometers, yards, light-years, femtometers, mils, astronomical units
mass	kilograms	grams, centigrams, kilograms , milligrams, micrograms, atomic mass units, carats, ounces, slugs, tons, metric tons
time	seconds	hours, days, minutes, centuries, decades, millennia, nanoseconds, milliseconds
temperature	kelvin	degrees centigrade, degrees Celsius, degrees Fahrenheit, degrees Rankine
volume	cubic meters	milliliters , cubic centimeters, liters, bushels, gallons, cups, pints, quarts, pecks, tablespoons, teaspoons, cubic yards, barrels, board feet
density	kilograms per cubic meter	grams per milliliter, grams per cubic centimeter, grams per liter, pounds per cubic foot, ounces per gallon
pressure	newtons per square meter	pascals, kilopascals , bars, millibars , dynes/cm^2 , bayres, torrs, millimeters Hg, centimeters H_20, atmospheres (atm), pounds per square inch (PSI)
energy	joules	joules, kilojoules, ergs, dynes, Calories, kilocalories, kilowatt-hours, British thermal units, therms, electron volts

Activity 3.1.3 (Understanding Unit Definitions) helps students see the logic behind the measurement conventions that have been adopted. The astute student may look at the answers to this activity and notice that the original SI definition of mass required a measurement of length (*a kilogram is the mass of one cubic decimeter of water*), and the original SI definition of length required a measurement of time (*a meter is the length of a pendulum having a period of one-half second).* The original SI unit of time, however, was not as intuitive, being defined as 1/*86 400 of the mean solar day*. To integrate history and science, you may wish to show that the customary (English) system's measurements of length were rooted in natural dimensions of the adult human body. Although such

measurements were approximate, they were easy to conceptualize. For example, the *inch* represented the width of a thumb, the *foot* was the length of a human foot, the *yard* was the distance from the tip of the nose to the end of the middle finger, the *fathom* was the total arm span, and the *mile* as the length of 1000 paces (where a pace is defined as two steps). If students are not familiar with a unit name, they may rush to a dictionary to identify it. We suggest that you discourage this, and encourage them to talk in groups to see if they can figure out the answers based upon their collective experience. English language learners may to do better if they realize that some of the terms have cognates (words with same etymological roots and similar sounds) in their native languages. For example, *kilometer* in English is *chilometro* in Italian, *kilómetro* in Spanish, *kilomètre* in French and *Kilometer* in German.

Activitiy 3.1.4 (Discovering Key Relationships Using Fundamental Units) illustrates the power of dimensional analysis in understanding important laws and principles. Dimensional analysis with fundamental units helps students see the simplicity and elegance of the physical world by showing that everything that is measurable can be expressed as a combination of seven fundamental units. A manipulation of these units reveals numerous relationships and promotes a deeper understanding of science. Students who are adept at dimensional analysis with fundamental units are able to understand and derive relationships that others blindly memorize. An analysis of the equations derived in this activity reveals important relationships in the physical world. For example, $C=Q/V$ is the definition of the capacitance of a capacitor, $V=IR$ is a statement of Ohm's Law of Resistance, $P=W/t$ is a definition of power, and $P=I^2R$ is an expression of the power consumed or expended in a circuit. After deriving these equations by dimensional analysis, students will have a better understanding of the relationship between physical quantities.

Show your students that the final four equations in this acitivity are all expressions of energy: $E=Fd$; $E=W$; $E=mv^2$; $E= mad$. The product of force and distance has units of energy, indicating that energy is the capacity to move a force through a distance, or more specifically, energy is the capacity to do work. The product of mass and speed squared has units of energy, which helps us understand why the kinetic energy of an object can be expressed as $1/2\ mv^2$ (where m is mass, and v is velocity), while the energy obtained by the conversion of mass to energy in a nuclear reaction ($E=mc^2$;where c is the speed of light) has the same units. Energy is one of the central themes of science, and dimensional analysis of fundamental units can be used to show how it appears in many equations and relationships.

Activitiy 3.1.5 (Using Dimensional Analysis for Standardizing Units) encourages science teachers and students to communicate using standard *mks-SI* units rather than *cgs-SI* or *customary* (English) units. The following table has been included to help you convert from *cgs* to *mks* units.

CGS unit	**measuring**	**SI (mks) equivalent**
barye (ba)	pressure	0.1 pascal (Pa)
biot (Bi)	electric current	10 amperes (A)
calorie (cal)	heat energy	4.1868 joule (J)

dyne (dyn)	force	10-5 newton (N)
erg	work, energy	10-7 joule (J)
franklin (Fr)	electric charge	3.3356 x 10^{-10} coulomb (C)
galileo (Gal)	acceleration	0.01 meter per second squared ($m \cdot s^{-2}$)
gauss (G)	magnetic flux density	10^{-4} tesla (T)
gilbert (Gi)	magnetomotive force	0.795 775 ampere-turns (A)
lambert (Lb)	illumination	104 lux (lx)
langley	heat transmission	41.84 kilojoules per square meter ($kJ \cdot m^{-2}$)
maxwell (Mx)	magnetic flux	10-8 weber (Wb)
oersted (Oe)	magnetic field strength	79.577 472 ampere-turns per meter ($A \cdot m^{-1}$)
poise (P)	dynamic viscosity	0.1 pascal second (Pa·s)

Activity 3.1.6 (Simplifying Calculations: The Line Method) introduces a simplifed approach for problem solving that reduces the amount of writing and the potential for error. Make certain students understand that the "Line method" is simply a formatting technique, and does not represent a difference in the mathematics.

Activity 3.1.7 (Solving Problems with Dimensional Analysis) is the culminating activity in this chapter and stresses the importance of solving problems with this technique. The problems at the beginning of each set are simpler than those at the end. We have included problems from everyday life, biology, chemistry, earth science, and physics to illustrate the importance of this problem solving technique, regardless of the field of study.

Although it may be tempting to photocopy the solutions and project them for instruction, we encourage you to solve each problem in class "from scratch." There are many potential sequences in which each may be solved, so don't be concerned if the sequences you use differ from those shown here. The important issue is that the units cancel to leave only the desired units. If you are not confident in your abilities to solve these problems in front of class, we suggest that you scan and project figures xx – xx. Show your students how units cancel to leave only the desired units.

Please note that some conversion values are precise, and therefore have an unlimited number of significant digits. For example, there are precisely one ten millimeters in a centimeter, so the 10 mm/cm conversion value does not limit the number of significant figures in any calculation. Failing to clarify this issue can cause confusion among students who are trying to express answers in the right number of significant units. The following example may help clarify this situation.

> The product of 25 cm and 2.000 cm is 50 cm^2, not 50.00 cm^2. Since there are only 2 significant digits in the first multiplier, there can be no more than 2 in the answer. By contrast, the product of 2.000 cm and 10 mm/cm is 20.00 mm. Note that the conversion factor (10 mm/cm) is a pure conversion factor with unlimited significant figures, and therefore does not reduce the number of significant figures in the answer.

Students may get lazy and try to solve problems without dimensional analysis. To impress the importance of dimensional analysis, you may give your students the following caution:

Warning! You should avoid using dimensional analysis if

- You enjoy making mistakes…
- You have a phobia of correct answers…
- You enjoy solving problems in a convoluted manner… or
- You are take pleasure in "re-inventing the wheel."

Otherwise, dimensional analysis is for you!

ANSWERS TO ACTIVITIES 3.1.1-3.1.7

3.1.1a
(1) milligrams; (2) km/hour; (3) 60 PSI; (4) IU; (5) degrees Celsius

3.1.1b
(1) °F; (2) feet, feet, miles; (3) G, CM, G; (4) km/h; (5) mm; (6) °C; (7) miles/h; (8) light years; (9) megatons; (10) meters; (11) feet, inches; (12) acres; (13) ounces; (14) meters; (15) inches.

3.1.2 Units are classified correctly as measures of one of the seven fundamental quantities.

	Mass	Length	Time	Temperature	Luminous intensity	Electric charge (current)	quantity		Mass	Length	Time	Temperature	Luminous intensity	Electric charge (current)	quantity
A (ampere)						*x*		hour			*x*				
amu	*x*							inch		*x*					
biot						*x*		Kelvin,				*x*			
Bolt (of cloth)		*x*						kilogram	*x*						
Cd (candela)					*x*			league		*x*					
cubit		*x*						meter		*x*					
day			*x*					mole							*x*
decade			*x*					neutron mass	*x*						
°C				*x*				oz (ounce)	*x*						
°F				*x*				pace		*x*					
°R (Rankine)				*x*				pica		*x*					
°Re (Reaumur)				*x*				pound ap	*x*						
dozen							*x*	pound,	*x*						
dram ap	*x*							second			*x*				
earth mass,	*x*							sidereal day			*x*				
football field		*x*						sidereal month			*x*				
fortnight			*x*					slug	*x*						
Gross							*x*	UK mile		*x*					
hand		*x*						week			*x*				
hefner candle.					*x*			yr (year)			*x*				

3.1.3 *Answer to part a: A meter is the length of a pendulum having a period of one-half second.*
Answer to part b: A kilogram is the mass of one cubic decimeter of water.

3.1.4 C=Q/V; V=IR; P=IV; $P=I^2R$; P=W/t; E=Fd; E=W; E=1/2 mv^2; E= mad.

3.1.5a (1) 0.028 kg; (2) .00015 J; (3) 0.234 Pa; (4) 0.095 N; (5) 0.0018Wb; (6) 0.0234 T; (7) 520,000 lux; (8) 7.0 C; (9) 23.4 Pa·s; (10) 7.86m

3.1.6 (Insert Figure xx)
(1) 3.3 m^2; (2) 9.47 x 10^{15} ; (3) 4.9 carats; (4) 8.8 x 10^{12} bits; (5) 1 x 10^4 N
(6) 1.01 X 10^{-47}N; (7) 9.0 c=x 10^5N.

3.1.7 The solutions to the problems in activities 1-5 are found in figures xx-xx.

3.1.1 USING APPROPRIATE UNIT MEASURES

Internet Resources

- Unit conversion calculators
- World record sites

Introduction: On September 24, 1999, the New York Times headline declared: *Mars Orbiting Craft Presumed Destroyed By Navigation Erro*r. The text of this article explained, "A $125 million robotic spacecraft, the first ever dispatched especially to investigate weather on another world, was missing on Thursday and presumed destroyed just as it was supposed to go into an orbit around Mars." After extensive investigation, it was learned that the Mars Climate Orbiter was lost, not due to system malfunction or collision with an asteroid, but rather because a few engineers were sloppy and failed to include units with their calculations. Lockheed Martin Corporation in Colorado sent daily course adjustments to the Jet Propulsion Laboratory in Pasadena with numbers that should have included English units of "pound-seconds" to describe the impulse necessary to adjust the course of the spacecraft into orbit around Mars. Unfortunately, the engineers at JPL assumed these numbers were specifying impulse in "newton-seconds". Since a newton-second is only 0.225 pound-second, the rocket motors were not given sufficient impulse to alter the orbit appropriately, and it is presumed that the craft plummeted into the Red Planet. If engineers had included or checked units, this disaster could have been avoided.

Using the correct units is important not only for scientists and engineers, but for people in all walks of life. The Federal Food and Drug Administration (FDA) estimates that medication errors cause at least one death every day and injure approximately 1.3 million people annually in the United States. The FDA cites errors in medical abbreviations or writing, including the absence of dimensions (units), as one of the chief causes for these problems. According to the National Academy of Sciences' Institute of Medicine (IoM), "drug complications" in the hospital is the leading cause of medical mistakes in

America—accounting for nearly 20 percent of all errors. The IoM's study, *To Err is Human: Building a Better Health System*, reported that 2 out of every 100 hospital admissions experienced a "preventable adverse drug event." The 1999 IoM study estimated that the cost of prescription errors is as much as $2 billion annually. Some of these costly errors could be easily avoided if physicians and pharmacists clearly indicated and checked units.

Activity A **<u>The hazards of not using units.</u>** Much confusion is caused when measurements are reported without unit, as you will see from the following problems. Examine each of the following problems, and make your best guess of the actual units.

(1) Medication: A typical adult prescription of the painkiller acetaminophen with codeine is 500 _____ ?
(a) tablets; (b) grams; (c) milligrams; (d) ounces; (f) milliliters
<u>Consequences of being incorrect</u>: Symptoms of overdose may include: cold and clammy skin, extreme sleepiness progressing to stupor or coma, general bodily discomfort, heart attack, kidney failure, liver failure, low blood pressure, muscle weakness, nausea, slow heartbeat, sweating, and vomiting.

(2) Speed Limit: The speed limit in school zones in Canada is 30 _____.
(a) miles/hour; (b) km/hour; (c) feet/second; (d) meters/second; (e) yards/second

<u>Consequences of being incorrect</u>: Speeding in a school zone may result in an accident or a significant fine.

(3) Bicycle Tire Pressure: The recommended air pressure in many mountain-bike tries is 60 _____.
(a) Pascals; (b) Kilopascals; (c) PSI; (d) Torr; (e) mm Hg

<u>Consequences of being incorrect</u>: Under-inflated tires puncture easier, wear out faster, are more difficult to pedal, and don't stop as efficiently. Over-inflated tires may blowout.

(4) Dietetics: Many physicians recommend that pregnant women should take no more than 10,000 ___ of Vitamin A per day.
(a) grams, (b) milligrams, (c) ounces, (d) IU; (5) microliters

<u>Consequences of being incorrect</u>: Excess vitamin A may give rise to birth defects, dry skin, scaly skin, headaches, fatigue, painful bones and loss of appetite.

(5) Food Storage: Food scientists recommend that produce companies store apples, cherries, apricots and most berries at 2 ____.
(a) degrees Fahrenheit, (b) Kelvin, (c) degrees Celsius (d) degree Rankine (e) degree Reaumur.

<u>Consequences of being incorrect</u>: Storing these fruits at too high of a temperature will result in ethylene production and early ripening. Storing them at too low of a temperature will damage the integrity of the fruit.

(6) **Your own example**: Develop your own hypothetical examples of problems that may arise when units are omitted in fields such as sports-casting or international trade.

Activity B: **<u>Units in everyday life:</u>** In the statements that follow you will find a variety of interesting facts, but each is missing a crucial piece of information -- the units (dimensions)! All the statements are meaningless until supplied with the appropriate units. On the basis of your experiences, try to complete the statements with the appropriate units from the list provided.

acre	kilograms	milliliters
centimeters	kilometers	millimeters
degrees Celsius	kilometers/hour	ounces
degrees Fahrenheit	kilowatt-hours	pounds
feet	light-years	square miles
G	liters	square km
grams/ml	megatons	stories
hectare	meters	tons
inches	miles	yard
kcal (Cal)	miles per hour	
Kelvin	milligrams	

(1) Coldest Place In The Known Universe
The Boomerang Nebula, a cloud of dust and gas dispensed by "white dwarf", is one of the coldest places in the known universe, with an estimated temperature of -521.6 ______.

(2) Deepest Lake
The deepest lake in the world is Lake Baikal, in Siberia, Russia. The lake has a depth of 15,371 _____, of which 3,875 ____ are below sea level. Baikal contains one-fifth of all the world's fresh surface water! The lake has a surface area of 12,200 square ______.)

(3) Highest G-Force Endured By An Animal
The click beetle (*Athous haemorrhoidalis*) moves its body in a snapping fashion to generate lift and a record-breaking acceleration of 400 ____. The beetle jumps this way to avoid predators, and can leap up to 15 _____ high. By comparison, humans can endure brief periods of up to 6 _______.

(4) Fastest Dive By A Bird
A peregrine falcon can reach a velocity of 350 ______ when diving towards its prey.

(5) Driest Place on Earth
The meteorological station in Quillagua, in the Atacama Desert, Chile records an average annual rainfall at was just 0.5 _________.

(6) Greatest Temperature Range On Earth
The greatest temperature range recorded is in Verkhoyansk, Siberia in Easter Russia. The temperature is as high as 105 _______ in the summer and as low as -68 deg C ______ to 37 ______ in the winter.

(7) Fastest Avalanche
The volcanic explosion of Washington State's Mount St Helens on May 18, 1980, triggered an avalanche with a velocity of 250 ______).

(8) Farthest Object Visible By The Unaided Eye
The remotest object visible with the unaided eye is the Great Galaxy in the constellation of Andromeda. The Great Galaxy is composed of approximately 200,000,000,000 stars and is 2,200,000 ____________ from Earth.

(9) Most Powerful Nuclear Explosion
The most powerful thermonuclear device ever tested was the Soviet's "Tsar Bomba", with an explosive force equivalent to that of 57 ________ of TNT. The shockwaves from the 1961 blast circled the earth three times!

(10) Tallest Living Tree
The tallest tree currently standing is the Mendocino Tree, a coast redwood (*Sequoia sempervirens*) at Montgomery State Reserve, in California. When measured , its height was determined to be 112.014 _______.

(11) Wood and Paper Usage
The average amount of wood and paper consumed per person each year in America is equivalent to a tree ______ tall and 16 ______ diameter.

(12) Largest Forest
The boreal forests of northern Russia, lying between Lat. 55°N and the Arctic Circle, cover a total area of area 2.7 billion _________.

(13)Longest Venomous Snake
The king cobra (*Ophiophagus hannah*) from Southeast Asia and India grows to an average length of 3.65-4.5 m (12-15 ft)! It is estimated that two fluid _________ of a king cobra's venom can kill 20 people!

(1`4) Largest Bird
The largest bird is the North African ostrich. It can grow to 2.75 ________ tall and weigh 156.5 _______ The ostrich is also the fastest bird on land, reaching speeds of 72 _______.

(15) Greatest Snowfall For A Snowstorm
In February 1959, a single snowstorm dropped 480 _______ of snow at Mount Shasta Ski Bowl, California, USA.

3.1.2 UNDERSTANDING FUNDAMENTAL QUANTITIES

Internet Resources

- Unit conversion calculators
- Databases of equations

One of the greatest inventions of all time is the *alphabet*, a set of symbols representing the basic sounds (phonemes) of spoken words. English and many other languages use the Latin alphabet, which developed from the Greek alphabet (figure XX), that in turn developed from the ancient Phoenician alphabet. A comparison of these alphabets illustrates a common heritage.

Each letter of an alphabet represents a consonant or a vowel, rather than a syllable or concept. Because alphabetic characters represent the most basic features of speech, one needs only a small number of characters to represent all of the words of the language. The English language has the greatest vocabulary of any language on earth, yet every word can be written using a set of just 26 characters. By contrast, languages that are based on larger units, such as syllables or concepts, are much more complex. Chinese, for example, employs a logographic writing system in which characters are used to represent basic words of the Chinese language. There are approximately 40,000 Chinese characters, of which nearly 2,000 must be memorized in order to be functionally literate.

The beauty and power of an alphabetic system is that it is based upon the most fundamental linguistic units. From these fundamental units, one can derive any term in the spoken language. In a similar manner, there are *fundamental units* of measurement from which all other units of measurement are derived. Just as comprehension of the Latin alphabet is necessary for reading and writing English, so comprehension of the “scientific alphabet” is essential to understanding the “language of science”.

There are 7 "letters" in the "language of measurement" from which all units of measurement are derived. These 7 "letters" are *distance , mass, time, electric charge, temperature, amount,* and *luminous intensity* (see Table xx). These are known as the *fundamental quantities* because they can not be expressed in a simpler fashion. All other measurable quantities are known as *derived quantities* because they can be derived by combining two or more fundamental quantities.

Distance is a fundamental quantity because it can be expressed in no simpler terms than distance. Volume, however, is a derived quantity because it can be expressed as the cube of distance. For example, when measuring the volume of a box you multiply its length by its width by its height. The resulting volume is expressed as a cube of distance (d^3) such as cubic feet or cubic centimeters. Density is considered a derived unit because it can be expressed in terms of fundamental quantities of mass and distance (density = mass/distance^3).

In 1960 the 11th General Conference on Weights and Measures adopted the International System of measurement (SI) and assigned base units for each physical quantity. Table XX shows some common physical quantities and their SI units. The first 7 (bold type) are the seven fundamental units while the remaining units are derived from these.

Table 1: The General Conference on Weights are Measures Definitions for the seven fundamental units of measurement		
Quantity	*SI unit*	*Definition of unit*
distance	meter (m)	"The meter is the length of the path traveled by light in vacuum during a time interval of 1/299 792 458 of a

		second."
mass	kilogram (kg)	"The kilogram is equal to the mass of the international prototype of the kilogram."
time	second (s)	"The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom."
electric current	ampere (A)	"The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2 Å~ 10-7 newton per meter of length."
temperature	kelvin (K)	"The Kelvin is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water."
amount of substance	mole (mol)	"The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12. When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles."
intensity of light	candela (cd)	"The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540 Å~ 1012 hertz and that has a radiant intensity in that direction of 1/683 watt per steradian."

Activity 1: Many ways of measuring the same thing.

Although there are only seven fundamental quantities, a variety of ways have been developed to measure these quantities, making things seem much more complex than they really are. For example, Daniel Fahrenheit the German physicist who invented the alcohol thermometer, developed a temperature scale which took as zero the temperature of an equal ice-salt mixture and selected the values of 30° and 90° for the freezing point of water and normal body temperature, respectively. (Later the temperatures of freezing water and body temperature were revised to 32° and 98.6°). In 1742 the Swedish astronomer Anders Celsius developed a competing temperature scale based on 0° for the freezing point of water and 100° for the boiling point of water. In addition, Renè Reamur, William Rankine, and William Thompson (Lord Kelvin) developed temperature scales that now bear their names. Despite the different names and ways of measuring, Fahrenheit, Celsius, Reamur, Rankine and Kelvin simply provide different ways of expressing temperature, a single fundamental quantity.

.In the same way, students of science can become confused when they assume that units with different names measure different physical quantities. The list of terms below are units for measuring the seven fundamental quantities of length, mass, time, temperature, current, luminous intensity, and quantity. Examine the list and classify each term as a measure of one of these seven quantities. If necessary, use one of the unit conversion calculators available on the website to compare values.

	Mass	Length	Time	Temperature	Luminous intensity	Electric charge (current)	quantity		Mass	Length	Time	Temperature	Luminous intensity	Electric charge (current)	quantity
A (ampere)								hour							
amu								inch							
biot								Kelvin,							
Bolt (of cloth)								kilogram							
Cd (candela)								league							
cubit								meter							
day								mole							
decade								neutron mass							
°C								oz (ounce)							
°F								pace							
°R (Rankine)								pica							
°Re (Reaumur)								pound ap							
dozen								pound,							
dram ap								second							
earth mass,								sidereal day							
football field								sidereal month							
fortnight								slug							
Gross								UK mile							
hand								week							
hefner candle.								yr (year)							

Table XX: Identify the unit as a measurement of one of the seven fundamental quantities.

3.1.3 UNDERSTANDING UNIT DEFINITIONS (SI VS NON-SI UNITS)

Internet Resources

- National Institute of Standards

Some of the most commonly measured quantities in science are distance, mass, time, temperature, volume, density, pressure, amount, concentration, energy, velocity, molarity,, and electric charge. All of these quantities can be measured in a variety of ways. For example, distance can be measured in centimeters, nanometers, miles, inches, feet, fathoms, Ångstroms, microns, kilometers, yards, light-years, femtometers and mils!

Specialized circumstances often call for specialized units. For example, interstellar distances are measured in light-years (e.g. the distance between our Sun and the next nearest star *Proxima Centurai* is 4 light-years) while intermolecular bond lengths are measured in Ångstroms (e.g. the distance between hydrogen and oxygen in water is 0.958 Å). Although light-years and Ångstroms both measures of distance, many assume they are unrelated.

In an effort to minimize confusion, the International Union of Pure and Applied Chemists (IUPAC) recommended that all quantities be measured in SI units (e.g. meters) or multiples of SI units (e.g. femtometers, nanometers, micrometers, millimeters, centimeters, kilometers). Despite growing acceptance of the SI system, many other measurement units continue to be used (e.g., miles, inches, feet, fathoms, Ångstroms, microns, yards, light-years, mils).

Activity 1: **Where did the SI base units come from?**
During the nineteenth and twentieth centuries, great advances in science and technology that lead to a variety of overlapping systems of measurements, as scientists and engineers improvised to meet the needs of their individual disciplines. To minimize the confusion of multiple systems of measurement, scientists called for a uniform international system of measurement, and over the years conferences and committees were held to establish a set of standard units, the base units of which are found in table xx. Although the definitions in table xx are precise, they are dissatisfying. Why, for example, should a meter be defined as the length of the path traveled by light in vacuum during a time interval of 1/299 792 458 of a second? It sounds like a rather random definition, and certainly is hard to conceptualize. Originally, the definitions of the basic units were easier to conceptualize, but as time progressed, the definitions became more specific so as to avoid errors or circular reasoning. None-the less, it is interesting to see what the original definitions were.

(a) Definition of length: Scientists wanted to ensure that the standard length of a meter was the same around the world, and came up with a definition based upon the motion of a pendulum. The period of a pendulum is the time it takes for a bob to complete one swing and return to its original location. The period of a pendulum is dependent only upon its length, and so, scientists argued, a length of one meter could be defined in terms of the period of a pendulum. What is that definition? Construct a one-meter long string pendulum as illustrated in figure xx, and record the time for ten complete swings. Divide the time by ten to determine the period of one pendulum swing. Repeat this process twice, take an average value, and complete the following definition.

A meter is the length of a pendulum having a period of ____ seconds.

(b) Definition of mass: Originally scientists sought to define the standard measure of mass (the kilogram) as the mass of a known volume of water. Place a one or two-liter graduated cylinder on a balance and determine its mass. Slowly add sufficient water until the mass has increased by precisely one kilogram. What volume of water has a mass of one kilogram? Remembering that one milliliter is the same thing as one cubic centimeter, and that one cubic centimeter is equal to one thousandth of a cubic decimeter or one millionth of a cubic meter, complete the following definition:

A kilogram is the mass of ______ cubic ______ of water.

Activity 2: **Different units for the measuring same quantity**. The left column in Table 3 lists some of the most commonly measured quantities and the middle column lists their SI units. Table xx provides a list of other units that are used in the measurement of one of these 8 quantities. Examine each of these terms and try to determine which quantity it measures (distance, mass, time, etc.). Place these units in the table adjacent to the quantity you believe they measure. After classifying the units, consult a dictionary, encyclopedia, chemistry text, or other resource to determine if you classification is correct.

Table xx Different units for the same quantity

Quantity	*SI base units*	*other units*
distance	meters	
mass	kilograms	
time	seconds	
temperature	kelvin	
volume	cubic meters	
density	kilograms per cubic meter	
pressure	newtons per square meter	
energy	joules	

Table xx Units of distance, mass, time, temperature, volume, density, pressure and energy

Ångstroms	dynes	mils
astronomical units	dyne per square	miles
atmospheres(atm)	electron volts	milligrams
atomic mass units	ergs	millennia
bars	fathoms	millibar
barrels	feet	milliliters
bayre	femtometers	milliseconds
board-feet	gallons	minutes
British thermal units	grams per cubic centimeter	mmHg
bushels	grams per liter	nanometers
Calories	grams per milliliter	nanoseconds
carats	grams	ounces
centigrade	hours	ounces per gallon
centigrams	inches	pascals
centimeters	joules	pecks
centuries	kilocalories	pints
cm H_20	kilograms	pounds per cubic foot
cubic centimeters	kilojoule	pounds per square inch
cubic yards	kilometers	quarts
cups	kilopascals	slugs
days	kilowatt-hours	tablespoons
decades	light-years	teaspoons
degrees Celsius	liters	therms
degrees Fahrenheit	metric tons	tons
degrees Rankine	micrograms	torrs
	microns	yards

3.1.4 DISCOVERING KEY RELATIONSHIPS USING FUNDAMENTAL UNITS (EQUATIONS)

Internet Resources

- Equation websites
- Table of fundamental and derived units

With only 26 letters, it is possible to construct all of the words in the English language. In a similar way, with only seven quantities, it is possible to express all measurable quantities in science. These seven quantities are length, mass, time, electric current, temperature, amount, and luminous intensity. These quantities are *fundamental* because they can not be expressed in a simpler fashion. Examine table xx and note that all of the derived units can be expressed in terms of the seven fundamental units.

Time is a fundamental quantity, because it can be expressed in no simpler units than those of time. Distance is a fundamental quantity because it can be expressed in no simpler terms than distance. Velocity, however, is not fundamental, because it can be expressed as a ratio of two other units, namely distance and time ($v=d/t$). Because velocity can be derived from distance and time, it is known as a derived unit.

Just as all words can be expressed as a series of letters, so all measurable quantities can be expressed in terms of their fundamental quantities. Dimensional analysis of terms expressed in fundamental units can reveal many important relationships.

For example, the farad (F), is a measure of electrical capacitance (C, the ability to store charge) and can be expressed in fundamental terms as:

$$\mathrm{F} = \frac{\mathrm{A}^2 \cdot \mathrm{s}^4}{\mathrm{kg} \cdot \mathrm{m}^2}$$

where **A** represents amps (a measure of current), **s** represents seconds (a measure of time), ***kg*** represents kilograms (a measure of mass), and **m** represents meters (a measure of length). An examination of table xx shows that the units of a farad,

$$\mathrm{F} = \frac{\mathrm{A}^2 \cdot \mathrm{s}^4}{\mathrm{kg} \cdot \mathrm{m}^2} \text{ are similar to the units of an inverse volt, } \frac{1}{V} = \frac{\mathrm{A} \cdot \mathrm{s}^3}{\mathrm{kg} \cdot \mathrm{m}^2}$$

If we multiply an inverse volt by current (amps, A) and time (seconds, s), the units are farads (F), a measure of capacitance:

$$\frac{A \cdot s^3 \cdot (A \cdot s)}{kg \cdot m^2} = \frac{A^2 s^4}{kg \cdot m^2} = F$$

In other words, a farad is equal to an amp•second divided by a volt:

$$F = \frac{A \cdot s}{V}$$

In table xx we notice that a coulomb (C, a measure of electric charge) is defined as an amp•second, and we can therefore re-write the equation to show the relation of the units:

$$F = \frac{C}{V}$$

Since a farad is a unit of capacitance (*C*), a coulomb a measure of charge (*Q*) and a volt a measure of potential difference (*V*) , the equation may be expressed in quantities as:

$$C = \frac{Q}{V}$$

Thus, by examining the fundamental units of capacitance, current, and potential difference, we have determined that capacitance is the ratio of charge to potential difference ($C=Q/V$) which is what we find in the dictionary: "capacitance is equal to the surface charge divided by the electric potential!"

Important note: It is important to distinguish quantities (e.g. capacitance, *C;* charge, *Q;* potential difference, *V*) from the SI units used to measure these quantities (e.g. farads, F; coulombs, C; and volts, V). By convention, quantities are expressed in italics, while specific units are not. If you do not realize this, much confusion can arise, particularly in a problem like this. Note, for example, that *C* (capacitance, measured in farads, F) is not the same as C (coulombs, a unit of charge, *Q*). Please see appendix xx for rules on naming quantities and units.

Activity 1: Table xx shows the most commonly used units, and illustrates how they may be expressed in fundamental terms. The seven fundamental units are shown at the top of the table in bold print. Table xx lists sets of physical quantities that are related. By examining the fundamental units of these quantities, determine their relationship and express it mathematically. It is not necessary to understand what the terms mean to discover relationships..

Table xx: Physical Quantities and Their Units
(fundamental units are in **bold** type)

	symbol	*SI measurement units*	*symbol*	*unit dimensions*
distance	***d***	**meter**	**m**	**m**
mass	***m***	**kilogram**	**kg**	**kg**
time	***t***	**second**	**s**	**s**
electric current	***I***	**ampere**	**A**	**A**
temperature	***T***	**Kelvin**	**K**	**K**
amount of substance	***n***	**mole**	**mol**	**mol**
luminous intensity	***I***	**candela**	**cd**	**cd**
acceleration	*a*	meter per second squared	m/s^2	m/s^2
area	*A*	square meter	m^2	m^2
capacitance	*C*	farad	F	$A^2 \cdot s^4/kg \cdot m^2$
concentration	*[C]*	molar	M	mol/m^3
density	*D*	kilogram per cubic meter	kg/m^3	kg/m^3
electric charge	*Q*	coulomb	C	$A \cdot s$
electric field intensity	*E*	newton per coulomb	N/C	$kg \cdot m/A \cdot s^3$
electric resistance	*R*	ohm	Ω	$kg \cdot m^2/A^2 \cdot s^3$
emf	ξ	volt	V	$kg \cdot m^2/A \cdot s^3$
energy	*E*	joule	J	$kg \cdot m^2/s^2$
force	*F*	newton	N	$kg \cdot m/s^2$
frequency	*f*	hertz	Hz	s^{-1}
heat	*Q*	joule	J	$kg \cdot m^2/s^2$
illumination	*E*	lux (lumen per square meter)	lx	cd/m^2
inductance	*L*	henry	H	$kg \cdot m^2/A^2 \cdot s^2$
magnetic flux	ϕ	weber	Wb	$kg \cdot m^2/A \cdot s^2$
potential difference	*V*	volt	V	$kg \cdot m^2/A \cdot s^3$
power	*P*	watt	W	$kg \cdot m^2/s^3$
pressure	*p*	pascal (newton per square meter)	Pa	$kg/m \cdot s^2$
velocity	*v*	meter per second	m/s	m/s
volume	*V*	cubic meter	m^3	m^3
work	*W*	joule	J	$kg \cdot m^2/s^2$

Derived Unit	**Quantity**	**expressed as fundamental units**	**complete the equation…**	**in terms of …**
volt	(potential diff., *V*)	$\frac{kg \cdot m^2}{A \cdot s^3}$	*V*=	current *(I)* resistance *(R)*
watt	(power, *P*)	$\frac{kg \cdot m^2}{s^3}$	*P*=	current *(I)* potential difference *(V)*
watt	(power, *P*)	$\frac{kg \cdot m^2}{s^3}$	*P*=	current *(I)* resistance (*R)*
watt	(power, *P*)	$\frac{kg \cdot m^2}{s^3}$	*P*=	work *(W)* time *(t)*
joule	(energy, *E*)	$\frac{kg \cdot m^2}{s^2}$	*E*=	force *(F)* distance *(d)*
joule	(energy, *E*)	$\frac{kg \cdot m^2}{s^2}$	*E*=	work *(W)*
joule	(energy, *E*)	$\frac{kg \cdot m^2}{s^2}$	*E*=	mass *(m)* velocity *(v)*
joule	(energy, *E*)	$\frac{kg \cdot m^2}{s^2}$	*E*=	mass *(m)* acceleration *(a)* distance *(d)*
N•s	(impulse)	$\frac{kg \cdot m}{s}$	*Impulse*=	force *(F)* time *(t)*
farad	(capacitance, *C*)	$\frac{A^2 \cdot s^4}{kg \cdot m^2}$	*C* =	potential difference *(V)* charge *(Q)*
Pa	(pressure, p)	$\frac{kg \cdot m}{s^2}$	*p*=	Force *(N)* Distance *(d)*

3.1.5 USING DIMENSIONAL ANALYSIS FOR STANDARDIZING UNITS

Internet Resources

- Unit conversion calculators
- World record sites

The metric system was introduced to simplify calculations and facilitate communication between scientists and engineers. Ironically, however, two separate approaches to metric measurements developed. Scientists working in laboratories preferred to work with smaller measures of mass and length than those working in the field. For example, research chemists, working with small quantities of chemicals in the laboratory, preferred to measure masses in grams, while chemical engineers analyzing production processes preferred to measure production in kilograms. As a result, two metric systems of measurement sprung up: the CGS system (based upon the **c**entimeter, **g**ram and **s**econd), and the MKS system (based upon the **m**eter, **k**ilogram and **s**econd). Table xx compares the CGS with the MKS units

In 1960, the Eleventh General Conference on Weights and Measures addressed this issue and set forth the International System of Units (SI), using the meter, kilogram, second, ampere, kelvin, and candela as basic units. Since 1960, there has been a trend among scientists and engineers to report measurements in MKS units, but many continue to report them in CGS, and some still prefer to report measurements in customary (English units). When comparing data reported in different systems, scientists generally convert all units to MKS values. To insure that this conversion process is done accurately, scientists use dimensional analysis to convert all measures to MKS.

Activity A: Converting CGS units to MKS units.

Use table xx and dimensional analysis to convert cgs units to mks units. For example, to convert 1.5 biots to amperes, simply multiply

$$1.5\text{Bi} \times \frac{10\text{amp}}{\text{Bi}} = 15\text{amps}$$

CGS unit	**measuring**	**SI equivalent**
centimeter (cm)	distance	.01 meter (m)
gram (g)	mass	.001 kilogram (kg)
second (s)	time	1 second (s)
barye (ba)	pressure	0.1 pascal (Pa)
biot (Bi)	electric current	10 amperes (A)
calorie (cal)	heat energy	4.1868 joule (J)
dyne (dyn)	force	10^{-5} newton (N)
erg (erg)	work, energy	10^{-7} joule (J)
franklin (Fr)	electric charge	3.3356 x 10^{-10} coulomb (C)
galileo (Gal)	acceleration	0.01 meter per second squared ($m \cdot s^{-2}$)
gauss (G)	magnetic flux density	10^{-4} tesla (T)
maxwell (Mx)	magnetic flux	10^{-8} weber (Wb)
phot (ph)	illumination	10^{4} lux (lx)
poise (P)	dynamic viscosity	0.1 pascal second ($Pa \cdot s$)

3.1.6 SIMPLIFYING CALCULATIONS: THE LINE METHOD

Introduction: Which of the following is the correct answer to the following problem (a) 2.10; (b) 0.43, or (c) 1.07?

[INSERT FIGURE XX]

Actually, any of them could be correct, depending upon the sequence of mathematical operations performed. If you assume that the problem is 3/2 divided by 5/7, then you could indicate this by drawing the line longer between the final operation to be performed and arrive at 2.63

[INSERT FIGURE XX]

If, however, you assume that the problem is 3/2 divided by 5 divided by 7, then you arrive at 0.43.

[INSERT FIGURE XX]

Finally, if you assume the problem is the 5/7 divided into 2 divided into 3, then arrive at 1.07

[INSERT FIGURE XX]

It is necessary to indicate the order of operation by the length of the dividing line or other convention, but unfortunately, many teachers and students are not precise in their problem setup, causing confusion and errors in calculation. For example, slight differences in line length may not be detected, resulting in a different calculation. To avoid this and other problems, it is preferable to use the "line method".

The line method requires that the problem solver place all fractions on one straight line so that there are no fractions over fractions. For example, if the problem is 3/2 divided by 5/7, it is best to write the problem as (3/2) x (7/5). Multiplication of the inverse of a fraction is the same thing as division by the fraction.

[INSERT FIGURE XX]

In the line method, each vertical line represents multiplication and the horizontal line indicates division. No fractions are placed over fractions. If a fraction exists in the numerator, its numerator is placed on top of the line, and its denominator under the line beneath it. If, however, a fraction exists in the denominator, it is inverted so that the numerator is placed under the main line and its denominator immediately above it. Compare the line method with the traditional method for the solution of the following problem:

<u>Example</u> 1: What is the approximate temperature of 3 moles of nitrogen gas contained in a 5-liter flask at 20 atmospheres (R=0.0821 L atm/mol K)? The traditional method requires that the universal gas constant, R, be placed as a fraction in the denominator:

$$PV = nRT$$

$$T = \frac{PV}{nR} = \frac{2\text{ atm}\cdot 5\text{L}}{3\text{moles}\cdot\left(\frac{0.0821\text{ L}\cdot\text{atm}}{\text{mole}\cdot\text{K}}\right)}$$

By contrast, the straight-line method requires that the universal gas constant, R, be inverted and multiplied so that there are no fractions in the numerator or denominator.

[INSERT FIGURE XX]

It is now easier to see how units cancel.

[INSERT FIGURE XX]

The line method simplifies conversion problems as can be seen in the following example.

Example 2: An ancient measurement of volume mentioned in the Bible is the ephah. Write an equation in straight-line format to express 1.00 ephah in liters using the following conversion factors:

1 ephah = 2.429 modium (a modium is an ancient Roman measure of volume)
1 modium = 0.7076 vedro (a vedro is an old Russian measure of volume)
1 vedro = 0.349bushels (a customary measure of volume)
1 bushel =2 150 cubic inches
1 cubic inch = 0.01639 liters

[INSERT FIGURE XX]

The line method generally reduces the number of pen strokes and makes the problem setup less cluttered. Students should know how to solve a problem using both the traditional and line techniques, but we recommend the straight line for most problems involving dimensional analysis.

Activity 1 Dimensional analysis with the **line method.**: Set up each of the following problems on a straight line. Note that it is not necessary to understand the context of the problems to set them up correctly. What are the final values and units in each case?

(1) A *tsubo* is a Japanese unit of area. Write an equation using the line method to express 1 tsubo in square meters given that: 1 tsubo is equal to 5124 square inches; 1 square inch is equal to 6.452 square centimeters; and 1 square centimeter is equal to 0.0001 square meters.

(2) Write an equation using the line method to express 1 light-year (the distance light travels in 365.25 days) in inches (1 inch=2.54 cm), knowing that light travels at $3.0x10^8$ m/s.

(3) Write an equation using the line method to express 1.0 gram in carats. Include all of the following conversion factors.
1 kg = 0.001 tonne; 1 g=0.001 kg; 1 tonne= 0.9842 long tons; 1 long ton= 2240 pounds; 1 pound = 16 ounces; 1 ounce= 138.3 carats

(4) A bit is a **bi**nary digi**t**, the smallest element of computer storage. It is a single binary number represented by a one or zero. The bit can be a transistor or capacitor in a memory cell, a magnetic domain on a hard disk drive, or a reflective dot on an optical disc. Write an equation in straight-line format to express 1 terabyte of memory in bits given that there are 1024 gigabytes per terabyte, 1024 megabytes per gigabyte, 1024 kilobytes per byte, 1024 bytes per kilobyte, and 8 bits per byte.

Solve for force in problems 5-7

(5) $F = \dfrac{m\Delta v}{\Delta t}$ where $m = 2\ kg;\ \Delta v = 50\dfrac{m}{s};\ \Delta t = .01 s$

(6) $F_g = Gm_1m_2/d^2$ where $G = 6.67 \times 10^{-11} N \cdot m^2/kg^2$
$m_1 = 1.67 \times 10^{-27} kg;\ m_2 = 9.11 \times 10^{-31} kg;\ d = 1.00 \times 10^{-10} m$

(7) $F_c = k\dfrac{Q_1Q_2}{d^2}$ where $k = 8.987 \times 10^9\ N \cdot m^2/C^2;\quad Q_1 = 10^{-2} C;\quad Q_2 = 10^{-2} C;\ d = 10m$

3.1.7 SOLVING PROBLEMS WITH DIMENSIONAL ANALYSIS

Internet Resources

- Spreadsheets (conversions)
- Unit calculators

For the student: Chess is one of the most popular board games in the world, and is also one of the most complex. White opens the game by selecting any of 20 possible opening moves. Black counters with 20 possible moves, bringing the number of variants to 400 at the end of two moves. The number grows rapidly as the game progresses:

in 3 first moves - 8,902
in 4 first moves - 197,281
in 5 first moves - 4,865,609
in 6 first moves - 119,060,324

Thus, at the end of just 6 moves, there are nearly 120 million possible variants of the game! Expert chess players often plan three or four moves ahead, but certainly they can not consider more than a hundred thousand options in the process! Instead of considering all scenarios, they focus on logical ones, dramatically increasing their efficiency and effectiveness. A chess player will not advance in the game until they develop good problem solving strategies that allow them to discard unlikely variants and focus only on the likely ones.

In a similar manner, scientists use problem-solving strategies that allow them to focus their attention on likely possibilities rather than all possibilities. Perhaps the most widely used problem-solving strategy among scientists is dimensional analysis (also know as unit analysis and factor-label method). Dimensional analysis allows you to set up the problem and check for logic errors before performing calculations, and allows you to determine intermediate answers in route to the solution. A student or scientist who does not use dimensional analysis is like a chess-player that has not learned key strategies of the game. He or she will spend an inordinate amount of time checking illogical possibilities, with no assurance that the steps taken are correct. Dimensional analysis involves five basic steps:

- **Unknown**: Clearly specify the units (dimensions) of the desired product (the unknown). These will become the target units for your equation.
- **Knowns**: Specify all known values with their associated units. It is often a good idea to draw a diagram of what you know about the situation, placing values with their units on the diagram.
- **Conversion factors and formulas:** Specify relevant formulas and all conversion factors (with their units).
- **Equation** : Develop an equation (using appropriate formulas and conversion factors) such that the units of the left side (the side containing the known values) are equivalent to the units of the right side (the side containing the unknown). If the units are not equal, then the problem has not been set up correctly and further changes in the setup must be made.
- **Calculation**: Perform the calculation only after you have analyzed all dimensions and are certain that both sides of your equation have equivalent units.

Example 1 (medicine) The label on the stock drug container gives the concentration of a solution as 1200mg/ mL. Determine the volume of the medication that must be given to fill a physicians order of 1600 mg. (figure xx)

- Unknown An analysis of the problem shows that the unknown (the volume of solution to be given) must have units of volume.
- Knowns: We know that the solution has a concentration of 1200 mg drug/mL medicine and that we must obtain 1600 mg of the drug. The diagram illustrates what must be done.
- Conversion factors and formulas: None necessary.
- Equation: You must divide by the concentration and multiply by the mass of the drug in order to get the desired units (mg drug).
- Calculation: After the units are canceled, the equation yields 1.333 mL of medicine.

Example 2 (physics)
On June 19, 1976, the United States successfully landed *Viking1* on the surface of the planet Mars. Twenty years later, on July 4, 1997, NASA landed another robotic probe named the *Mars Pathfinder* at a distance of 520 miles from the *Viking 1* landing site. Unlike the *Viking* mission, the Pathfinder mission included a surface rover known as *Sojourner*, a six-wheeled vehicle that was controlled by an Earth-based operator. Knowing that the distance between the landing site of the *Mars Pathfinder* and the *Viking 1* craft is 520 miles, what would be the minimum number of hours required to drive *Sojourner* to the Viking site assuming a top speed of 0.7 centimeters per second, and no obstacles. (figure xx)

- Unknown Number of hours to reach the *Viking 1* site.
- Knowns: The distance is 520 miles, and the speed is 0.7 centimeters per second.
- Conversion factors and formulas: The distance is measured in customary units (miles) while the speed of *Sojourner* is measured in metric units (centimeters/second). We will therefore need to use the following conversion units to obtain units with the same dimensions: 2.54 centimeters/inch; 12 inches/foot; 5280 feet/mile.
- Equation: The answer must have units of time. The only known factor that includes units of time is the speed of the rover (distance/time). It is therefore evident that we must divide by speed to get units of time in the numerator where they are needed . To arrive at the desired units of time, it is necessary to cancel the units of distance by multiplying by the distance that must be traveled. It is now necessary to multiply or divide by the appropriate conversion factors (you can do either since all conversion factors have a value of one, and an equation can be multiplied or divided by one without changing values) to insure that all units of distance are canceled.
- Calculation: After the units are canceled, the equation yields the answer in hours as desired. The number is changed to one significant figures since one of the

factors has only one significant figure, and you can have no greater accuracy than your least accurate factor.

Example 3 (physics)
A 2.00-L tank of helium gas contains 1.785 g at a pressure of 202 kPa. What is the temperature of the gas in the tank in Kelvin given that the molecular weight of helium is 4.002 g/mol and the universal gas constant is 8.29 x 10^3 L•Pa/mol•K? (figure xx)

- Unknown : The unknown is the temperature of the gas expressed in Kelvin
- Knowns: Volume of helium container (2.00 L), mass of helium (1.785 g), molecular weight of helium (H_2; 4.002 g/mol), pressure of helium (202 kPa); universal gas constant: 8.29 x 10^3 L•Pa/mol•K). In addition, we know the number of moles (n = 0.446 mol) of helium since $n = m/M$.
- Conversion factors and formulas: This problem requires the use of the ideal gas law equation ($PV=nRT$) which must be expressed in terms of temperature: $T=PV/nR$.
- Equation: The equation must be set up so all units cancel except the desired units, kelvin (K).
- Calculation: It is necessary to use the pressure conversion factor (1000 Pa/kPa) to insure that all units are canceled except K.

Example 4 (chemistry)
Calculate the mass of silver metal which can be deposited if a 5.12 ampere current is passed through a silver nitrate solution for 2.00 hours. Note: there are 96,500 C per mole of electrons. (figure xx)

- Unknown An analysis of the problem shows that the unknown (the mass of silver metal deposited) must have units of grams silver .
- Knowns: We know that the current is 5.12 amps for a period of 2.00 hours. We also know that 1 mole of silver is deposited per mole of electrons from the fact that silver is a plus one cation ($Ag^+ + e^- \rightarrow Ag$). From the problem description we know the experimental setup and can therefore draw a diagram. We also can acquire the gram-atomic weight of silver from the periodic table (figure xx)
- Conversion factors and formulas: This problem will require a number of conversion factors in order to get the appropriate units. One coulomb is one amp second. One mole of electrons is 96,500 coulombs. One hour is 60 minutes. One minute is 60 seconds. Because these are equalities, they can be represented as conversion factors, each of which is equal to one. For example, there are 60 seconds in a minute, so 60 s/min is equal to one. Since conversion units are always equal to one, you can multiply by them, or divide by them (invert and multiply) without changing their meaning.
- Equation: The units of the unknown become the "target units" and are set up on the right side of the equation. The left side of the equation is assembled so that units will cancel and leave only the target units.
- Calculation: Once the equation is set up so that the units cancel to leave only the target units, calculations can be performed.

Example 5 (earth science)
The island of Greenland is approximately 840,000 mi^2, 85 percent of which is covered by ice with an average thickness of 1500 meters. Estimate the mass of the ice in Greenland (assume two significant figures). The density of ice is 0.917 g/mL. (1 cm^3 = 1 mL) (figure xx)

- Unknown An analysis of the problem shows that the unknown must have units of mass. Since the specific units of mass are not specified, we will use the MKS unit of kilograms.
- Knowns: Since we know that the area of Greenland is 840,000 $miles^2$, and 85% of it is covered by ice, then 714,000 $miles^2$ must be covered by ice. We also know that the density of ice = 0.917 g/mL and the ice has an average depth of 1500 m.
- Conversion factors and formulas: Some measurements are in customary units, while others are in metric. We should always convert all units to metric unless otherwise specified. To do so, we will need to convert miles to meters using the following conversion factors: 5280 ft/mile; 0.3048 m/ft; we also need to use conversion factors to obtain consistent metric units for mass and volume. Knowing that 1 cm = 0.01 m, then 1 cm^3 = 0.000001 m^3. We also know that 1 kg= 1000 g.
- Equation: The units of the unknown become the "target units" and are set up on the right side of the equation (kg ice). The equation mass = (depth x area)/density is the basic equation, and the conversion factors are inserted to make certain all units are consistent.
- Calculation: Once the equation is set up so units cancel to leave only the target units of kilograms of ice, calculations can be performed.

Example 6 (biology)

The rate of photosynthesis is often measured in the number of micromoles of CO_2 fixed per square meter of leaf tissue, per second (μmol CO_2/m^2s). What is the rate of photosynthesis in a leaf with an area of 10 cm^2 if it assimilates 0.00005 grams of carbon dioxide each minute (MW of CO_2= 48 g/mol)? (figure xx)

- Unknown We are trying to determine the rate of photosynthesis in units of μmol CO_2/m^2s
- Knowns: rate of carbon dioxide assimilation by the leaf is 0.00005 grams of carbon dioxide each minute. We also know that the leaf area responsible for this carbon dioxide fixation is 10 cm^2.
- Conversion factors and formulas: The gram molecular weight of CO_2= 48 g/mol. This will be essential in determining the number of micromoles of carbon dioxide. We also know that there are 10^6 micromoles/mole, 100cm/m, and 60 s/min. We may multiply or divide by these unit factors because each one is an identity (equal to 1).
- Equation: Since the rate of photosynthesis is defined as the number of moles of carbon dioxide absorbed per square meter of tissue per second, the equation becomes: Rate=quantity of CO_2 per square area of tissue per second.
- Calculation: Once the equation is set up so that the units cancel to leave only the target units μmol CO_2/m^2s, then calculations can be performed.

SI AND CUSTOMARY UNITS AND CONVERSIONS

Quantity	SI Unit	Symbol	Customary Unit	Symbol	Conversion
Length	meter	m	foot	ft	1 m= 3.280 ft
Area	square meter	m^2	square foot	ft^2	1 m^2=10.76 ft^2
Volume	cubic meter	m^3	cubic foot	ft^3	1 m^3 = 35.32 ft^3
Speed	meter per second	m/s	foot per second	ft/s	1 m/s = 3.280 ft/s
Acceleration	meter per second per second	m/s^2	feet per second per second	ft/s^2	1 m/s^2 = 3.280 ft/s^2
Force	newton	N	pound	lb	1 N = 0.2248 lb
Work (energy)	joule	J	foot-pound	ft·lb	1 J = 0.7376 ft·lb
Power	watt	W	foot-pound per second	ft·lb/s	1 W = 0.7376 ft·lb/s
Pressure	pascal	Pa	pound per square inch	lb/in^2	1 Pa = 1.450 x 10^{-4} lb/in^2
Density	kilogram per cubic meter	kg/m^3	pound per cubic foot	lb/ft^3	1 kg/m^3 = 6.243 x 10^{-2} lb/ft^3

COMMON CONVERSIONS

Quantity	Customary Unit	Metric Unit	Customary/Metric	Metric/Customary
Length	inch (in)	millimeter (mm)	1 in = 25.4 mm	1 mm = 0.0394 in
	foot (ft)	meter (m)	1 ft = 0.305 m	1 m = 3.28 ft
	yard (yd)	meter (m)	1 yd = 0.914 m	1 m = 1.09 yd
	mile (mi)	kilometer (km)	1 mi = 1.61 km	1 km = 0.621 mi
Area	square inch (in^2)	square centimeter (cm^2)	1 in^2 = 6.45 cm^2	1 cm^2 = 0.155 in^2
	square foot (ft^2)	square meter (m^2)	1 ft^2 = 0.0929 m^2	1 m^2 = 10.8 ft^2
	square yard (yd^2)	square meter (m^2)	1 yd^2 = 0.836 m^2	1 m^2 = 1.20 yd^2
	acre (acre)	hectare (ha)	1 acre = 0.405 ha	1 ha = 2.47 acre
Volume	cubic inch (in^3)	cubic centimeter (cm^3)	1 in^3 = 16.39 cm^3	1 cm^3 = 0.0610 in^3
	cubic foot (ft^3)	cubic meter (m^3)	1 ft^3 = 0.0283 m^3	1 m^3 = 35.3 ft^3
	cubic yard (yd^3)	cubic meter (m^3)	1 yd^3 = 0.765 m^3	1 m^3 = 1.31 yd^3
	quart (qt)	liter (L)	1 qt = 0.946 L	1 L = 1.06 qt
Mass	ounce (oz)	gram (g)	1 oz = 28.4 g	1 g = 0.0352 oz
	pound (lb)	kilogram (kg)	1 lb = 0.454 kg	1 kg = 2.20 lb
	ton (ton)	metric ton (t)	1 ton = 0.907 t	1 t = 1.10 ton
Weight	pound (lb)	newton (N)	1 lb = 4.45 N	1 N = 0.225 lb

Activity 1: Solving problems in **everyday life** with dimensional analysis

(1) Convert the following
- a) 61.0 kilometers to miles
- b) 2.7 quarts to liters
- c) 56 grams to pounds
- d) 17 pounds to kilograms
- e) 1 million seconds to days
- f) 21 ft/minute to miles/hour
- g) 0.391 grams/ml to kg/liter
- h) 85.5 meters/day to cm/minute

(2) The food that the average American eats in one day provides 2000 Calories of energy. How many Calories per second is this?

(3) Three people estimate the height of the Washington monument in Washington DC: tourist, 555 feet; congressman, 158 yards; lobbyist, 0.173 km. Who is closest to the true height of 169.3 meters?

(4) The EPA sticker on a car states that it will obtain 30 miles per gallon on the highway. How many liters of gasoline must the driver have to insure that he/she can get home from college if there are 300 miles of highway driving between their college and their home?

(5) You want to earn $600 to buy a new bicycle. You have a job that pays $6.75/hour, but you can work only 3 hours/ day. How many days before you will have enough to buy the bike?

(6) Corn sells for $8.00/bushel. Your land yields 25 bushels/acre. How many acres should you put in corn to make $1000?

(7) A landscaper charges $7 per square meter to plant sod. How much will it cost to plant a one-acre lawn? (1 hectare = 10,000 m^2)

(8) A chemist was traveling 2850 cm/s on his way to work. Could he be cited for speeding if the speed limit was 60 miles/h ?
yes: 102.6 km/h; 63.7 miles/h

(9) Two of top fastest cars ever built were the Lingenfelter Corvette 427 biturbo (which would accelerate from 0 km/s to 100 km/s in 1.97 seconds) and the Hennessey Dodge Viper Venom 800 (0 miles/hr to 62 miles/hr in 2.40 seconds). Which car demonstrated greater acceleration?

Activity 2: Solving problems in **biology** with dimensional analysis

(1) What is the net primary productivity ($kg/m^2 \cdot y$) of a field of annual wheat crop if an average of 2500 kg is harvested each year in a plot that is 10m x 10m.

(2) Cheetahs are the fastest land mammals and are capable of sprinting at 27.8 m/s in short bursts. How long would it take a cheetah to run the length of a 100 yd football field running at this top speed (1 yd = 0.914 m).

(3) . If an artificial heart is capable of pumping at least 57,000,000 pints of blood before failure, how long will it probably last in a patient if their average heart rate is 72 beats per minutes, and average stroke volume (the amount of blood pumped with each stroke) is 70 mL?

(4) The rate of respiration in is often measured in the number of micromoles of CO_2 fixed per gram of tissue, per second (μmol CO_2/g·s). What is the rate of respiration in an organism with a mass of 1 g if it produces .002 grams of carbon dioxide each minute (MW of CO_2= 48 g/mol)?

(5) 125 pound patient is to receive a drug at a rate of 0.300 mg per 1.00 kilogram of body weight. If the drug is supplied as a solution containing 5.00 mg/mL, how many mL of drug solution should he receive?

(6) A calcium report indicated 8.00 mg/dL of calcium ions in the blood. If we assume that the patient has 6.00 quarts of blood, how many grams of calcium ions are in his/her blood? (1 dL = 0.1 L)

(7) A large dose of an antileukemia drug is to be administered to a 190. lb patient by I.V. injection. The recommended dosage is 50.0 mg per kilogram of body weight, and the drug is supplied as a solution that contains 20.0 mg per milliliter. The I.V. has a flow rate of 3.00 mL/minute. How long will it take to give the recommended dose?

Activity 3: Solving problems in **earth science** with dimensional analysis

(1) Earth has an orbital velocity of 1.0 km/s. How far will it travel in one year?

(2) When a piece of metal of mass of 4.13 g chunk of rock is dropped into a graduated cylinder containing 8.3 mL of water, the water level rises to 9.8 mL. What is the density of the metal in grams per cubic centimeter? Is this rock more likely granite (2.7 – 2.8 g/cm^3) or basalt (2.9 g/cm^3)?

(3) The Atlantic Ocean is growing wider by about 1 inch/year. There are 12 inches/ft. and 5280-ft/ mile. How long will it take for the Atlantic to grow 1 meter?

(4) The average distance between the earth and sun is approximately 93,000,000 miles. Express this distance in centimeters.

(5) A chunk of the mineral galena (lead sulfide) has a mass of 12.4 g and has a volume of 1.64 cm^3. What is its density? Will it float or sink in a pool of mercury (density =13600, kg/m^3.)

(6) A solid concrete dam measures 50 GL. How many cubic meters of concrete are in this structure?

(7) The mass of Earth is 5.97 x 10 ^{24} kilograms. What is its average density if it has a radius of 6378 km? (V_{sphere}=4/3πr^3)

Activity 4: Solving problems in **chemistry** with dimensional analysis

(1) Platinum has a density of 21.4 g/mL. What is the mass of 5.9 mL of this metal?

(2) The mass of a proton is 1.67272 x 10^{-27} kg. What is its mass in μg?

(3) What mass of silver nitrate must be used to make 2.00 cubic decimeters of a 1.00M solution?

(4) Calculate the mass of solute required to make 750 mL of a 2.50M sodium chloride solution.

(5) Calculate the molarity of a 1.50 x 10^3 cubic centimeter solution that contains 200.0 grams of $MgCl_2$

(6) What is the mass of solute in 300.0 ml of a solution if the solution is 85% water? The density of the solution is 1.60 g/cm^3.

(7) A copper refinery produces a copper ingot weighing 150 lb. If the copper is drawn into wire whose diameter is 8.25 mm. How many feet of copper can be obtained from the ingot? The density of copper is 8.94 g/cm^3.

Activity 5: Solving problems in **physics** with dimensional analysis

(1) If a beta particle travels at a speed of 112,000 miles per second, what is this value in centimeters per second? (Give three digits in your answer and also use scientific notation!) Use: 5280 ft = 1.00 mile; 12 inch = 1 ft; 2.54 cm = 1.00 inch

(2) An object is traveling at a speed of 7500 cm/s. Convert the value to kilometers per minute. (Answer: 4.5 km/min)

(3) Traffic accident investigators often discuss reaction time when trying to determine liability for an accident. If a person's reaction time is 1.5 seconds, how many meters will his or her car travel before the brakes are activated if the car is traveling at 70 miles per hour?

(4) The wavelength of visible light is 706 nm. What is its frequency in sec^{-1} (Hz) ? The speed of light, c = 3.00 x 10^8 m/sec.

(5) The acceleration due to gravity on earth is 9.8 m/s^2 while it is 3.7 m/s^2 on the surface of Mars. If you weigh 700 N on earth, how many newtons would you weigh on Mars?

(6) A light year is the distance light travels in one earth-year. Light travels at 3.0 x 10^8 m/s. How many days does it take light from our solar systems nearest star, Alpha Centauri C (4.22 light years away) to reach the earth?

(7) The escape velocity for earth is 11.2 km/s. How far will a spacecraft travel in an hour if it is traveling at 1.6 times the escape velocity?