5. I will denote the thrust by $\hat{T}$ and the radius of the propeller by r. Then, the dimensions are as follows.

$$\hat{T} \to \frac{M\cdot L}{T^2} \quad ; \quad V \to \frac{L}{T} \quad ; \quad \upsilon \to \frac{L^2}{T}$$

$$r \to L \quad ; \quad g \to \frac{L}{T^2} \tag{1}$$

$$n \to \frac{1}{T} \quad ; \quad \rho \to \frac{M}{L^3}$$

a) Here we use the procedure outlined in the text. Hence, we should consider $\hat{T}/\rho r^2 V^2$ as a function of model variables and parameters (note that we don't have the DE for this prob.). Thus, based on the given information:

$$\frac{\hat{T}}{\rho r^2 V^2} = \Phi(r, n, V, g, \rho, \upsilon) \tag{2}$$

b) Dimensional Analysis: Assume Φ has the general form,

$$\Phi = \pi_1^{\alpha_1} \cdot \pi_2^{\alpha_2} \cdots \pi_n^{\alpha_n}$$

where each π_i is a dimensionless combination of model variables and parameters, and α_i's are arbitrary constants.

i. To get the form of each π_i (or more accurately, one possible form of each π_i since there may be more than one dimensionless combination and hence answer) we proceed as follows.

ii. Let,

$$\pi = r^{c_1} \cdot n^{c_2} \cdot V^{c_3} \cdot g^{c_4} \cdot \rho^{c_5} \cdot \upsilon^{c_6} \tag{3}$$

iii. Now, our task is to compute c_i's so that π is dimensionless. Using the dimensions identified above in (1), the dimensions in the RHS of (3) satisfy:

$$\pi = (L)^{c_1} \cdot \left(\frac{1}{T}\right)^{c_2} \cdot \left(\frac{L}{T}\right)^{c_3} \cdot \left(\frac{L}{T^2}\right)^{c_4} \cdot \left(\frac{M}{L^3}\right)^{c_5} \cdot \left(\frac{L^2}{T}\right)^{c_6}$$

$$= L^{c_1 + c_3 + c_4 - 3c_5 + 2c_6} \cdot T^{-c_2 - c_3 - 2c_4 - c_6} \cdot M^{c_5} \qquad (4)$$

iv. In order for π to be dimensionless, each of the three exponents in (4) must be equal to zero. Namely,

$$\begin{cases} \boxed{c_5 = 0} \\ c_1 + c_3 + c_4 - 3c_5 + 2c_6 = 0 \quad \Rightarrow \\ -c_2 - c_3 - 2c_4 - c_6 = 0 \quad \Rightarrow \end{cases} \begin{cases} c_1 = -c_3 - c_4 - 2c_6 \\ c_2 = -c_3 - 2c_4 - c_6 \end{cases} \qquad (5)$$

v. Plugging the results in (5) back in (3) we get the dimensionless π in it's most basic form:

$$\pi = r^{-c_3 - c_4 - 2c_6} \cdot n^{-c_3 - 2c_4 - c_6} \cdot V^{c_3} \cdot g^{c_4} \cdot \nu^{c_6}$$

$$\boxed{\pi = \left(\frac{V}{r \cdot n}\right)^{c_3} \cdot \left(\frac{g}{r \cdot n^2}\right)^{c_4} \cdot \left(\frac{\nu}{r^2 n}\right)^{c_6}} \qquad (6)$$

c) Note that each combination of model variables/parameters in (6), i.e., V/rn, g/rn^2, ν/r^2n is dimensionless. Hence, any function of these combinations will remain dimensionless. For example, we can select the exponents c_3, c_4, and c_6 as we wish.

d) Our results in (6) are satisfactory. However, in order to get the requested results we further analyze the 2nd and the 3rd terms in the RHS of (6):

i. $\frac{g}{rn^2}$: $r \cdot n^2 \rightarrow L \cdot \frac{1}{T^2} = \frac{1}{L}\left(\frac{L}{T}\right)^2 \rightarrow \frac{V^2}{r}$

$\therefore$ From a dimensional pt of view: $\frac{g}{rn^2} \longleftrightarrow \frac{g}{V^2/r} = \boxed{\frac{rg}{V^2}}$ (7)

ii. $\frac{\nu}{r^2 n}$: $r^2 \cdot n \rightarrow L^2 \cdot \frac{1}{T} = L \cdot \frac{L}{T} \rightarrow rV$

$\therefore$ From a dimensional pt of view: $\frac{\nu}{r^2 n} \longleftrightarrow \boxed{\frac{\nu}{rV}}$ (8)

e) Hence, (6) is equivalent (from a dimensional pt of view) to:

$$\boxed{\Pi = \left(\frac{V}{rn}\right)^{c_1} \left(\frac{rg}{V^2}\right)^{c_4} \left(\frac{\nu}{rV}\right)^{c_6}} \quad (9)$$

i. Let,

$$\Pi_1 := \frac{rn}{V} \quad ; \quad \alpha_1 := -c_1$$

$$\Pi_2 := \frac{V^2}{rg} \quad ; \quad \alpha_2 := -c_4 \qquad (10)$$

$$\Pi_3 := \frac{rV}{\nu} \quad ; \quad \alpha_3 := -c_6$$

ii. Then,

$$\frac{\hat{T}}{\rho r^2 V^2} = \Phi(r, n, V, g, \rho, \nu)$$

$$= \phi\left(\frac{rn}{V}, \frac{rg}{V^2}, \frac{rV}{\nu}\right) \qquad (11)$$

$$= \Pi_1^{\alpha_1} \cdot \Pi_2^{\alpha_2} \cdot \Pi_3^{\alpha_3}$$

iii. e.g., wlog $\alpha_1 = \alpha_2 = \alpha_3 = 1$.