10. a) Let $x \in \text{int} A$. By def $\exists \delta(x) > 0 : (x-\delta, x+\delta) \subset A$. $\therefore$ int A is open by the def of an open set.

b) Let $x \in B$ where $B \subset A$ and $B$ is open. Then, by the def of an open set $\exists \delta(x) > 0$ st. $(x-\delta, x+\delta) \subset B \subset A$. $\therefore$ $x \in \text{int} A$ by the def of int A.

c) Let $x \in \text{int} \,(A \cap B)$. Then by def $\exists \delta(x) > 0 : (x-\delta, x+\delta) \subset A \cap B$.

$\Rightarrow (x-\delta, x+\delta) \subset A$ and $(x-\delta, x+\delta) \subset B$ 

$\Rightarrow x \in \text{int} A$ and $x \in \text{int} B$ by def 

$\Rightarrow x \in \text{int} \,(A \cap B)$ 

$: \therefore \text{int} \,(A \cap B) \subset \text{int} A \cap \text{int} B$.

Conversely, let $x \in \text{int} A \cap \text{int} B$.

$\Rightarrow \begin{cases} x \in \text{int} A \Rightarrow \exists \delta_1 > 0 : (x-\delta_1, x+\delta_1) \subset A \\ \text{and} \\ x \in \text{int} B \Rightarrow \exists \delta_2 > 0 : (x-\delta_2, x+\delta_2) \subset B \end{cases}$

Let $\delta = \min(\delta_1, \delta_2)$. Then $(x-\delta, x+\delta) \subset A$ and $(x-\delta, x+\delta) \subset B$.

$\Rightarrow (x-\delta, x+\delta) \subset A \cap B$ 

$\Rightarrow x \in \text{int} \,(A \cap B)$ by def 

$: \therefore \text{int} A \cap \text{int} B \subset \text{int} \,(A \cap B)$.

11. b) $A = (0,1]$ 

i) neither b/c $A$ does not satisfy the def of open or closed. 

ii) connected by thm 3-14. 

iii) not compact since it is not closed. 

i) Im pts of $A = [0,1]$ using: every open neighborhood of $x$ contains $\infty$ many pts of $A$. 

ii) $\text{int} A = (0,1)$ by def of interior. 

iii) $\bar{A} = [0,1]$ by the def of closure. 

iv) $\partial A = \{0,1\}$ by the def of a boundary pt.
11.c) \( A = \left\{ \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots \right\} \)

not closed since \( 0 \notin A \) \((0 \text{ is a lim pt})\), and obviously not open.

i) not connected, e.g., use \( U = (-\infty, \frac{\sqrt{2}}{2}) \), \( V = (\frac{\sqrt{2}}{2}, \infty) \).

ii) not compact since not closed.

i) \( \lim pt = \{0, 3\} \) using: every neighborhood of \( x \) contains \( \infty \)-many pts of \( A \).

ii) \( \text{int } A = \emptyset \), since \( \forall x \in A, \exists \epsilon > 0 : (x-\delta, x+\delta) \notin A \), e.g., \( x = \frac{1}{2} \) and \( \delta = \frac{1}{8} \)

\( \Rightarrow (\frac{1}{2} - \frac{1}{8}, \frac{1}{2} + \frac{1}{8}) = (\frac{3}{8}, \frac{5}{8}) \) \( \notin A \).

iii) \( \overline{A} = \{0, 3\} \) using def of closure.

iv) \( \partial A = \overline{A} \), using def of a \( \partial \) pt.

d) \( A = \{0, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots \} \)

i) closed since it contains all its \( \lim pt \).

ii) not connected.

iii) compact since closed and bounded.

i) \( \lim pt = \{0\} \)

ii) \( \text{int } A = \emptyset \)

iii) \( \overline{A} = A \)

iv) \( \partial A = A \)
e) \( A = \mathbb{R} \)

i) closed since \( A^c \) is open.

ii) not connected, e.g., use \( U = (-\infty, \pi) \); \( V = (\pi, \infty) \).

iii) not compact since not bounded.

i) \( \lim pt = \emptyset \) since no element of \( A \) satisfies def.

ii) \( \text{int } A = \emptyset \) since no element of \( A \) satisfies def.

iii) \( \overline{A} = A \) since the set of \( \lim pt \) of \( A = \emptyset \)

iv) \( \partial A = A \) since every pt is a boundary pt by def of a \( \partial \) pt.
f) \( A = (0, 1) \cup (1, 2) \)

i) not closed since \( 0 \notin A \) (\( 0 \) not open since \( 2 \notin A \)).

ii) not connected since not an interval.

iii) not compact since not closed.

i) \( \lim pt = \{0, 1, 2\} \)

ii) \( \text{int } A = (0, 1) \cup (1, 2) \)

iii) \( \overline{A} = [0, 2] \)

iv) \( \partial A = \{0, 1, 2\} \).