5.3, #7
p.367

$$\begin{cases} X = \text{bus's } \cancel{\text{a}} \text{ arrival time} & ; \quad X = N(0, \tfrac{2}{3}) \\ Y = \text{my arrival time} & ; \quad Y = N(-1, \tfrac{1}{2}) \end{cases}$$

$8{:}10 \equiv 0$

a) 97.72 %
b) 88.49 %
c) 97.95 %

b) $P(Y < X) = ?$

$$P(Y < X) = P(Y - X < 0) = P(S < 0)$$
$$= P(Z < 6/5) = \Phi(1\tfrac{1}{5})$$
$$= \boxed{88.49\%}$$

$$S := Y - X \Rightarrow \mu_S = \mathbb{E}(Y) - \mathbb{E}(X) = -1$$
$$\sigma_S^2 = \sigma_Y^2 + (-1)^2 \sigma_X^2 = \frac{1}{4} + \frac{4}{9} = \frac{25}{36}$$

$\therefore \quad S = N(-1, 5/6)$

$$z = \frac{0 - (-1)}{5/6} = 6/5$$

Thm

$$X_i = N(\lambda_i, \sigma_i^2), \quad i = 1, \ldots, n$$
$$\Rightarrow S = \sum_{i=1}^{n} X_i =$$
$$= N\left(\sum_{i=1}^{n} \lambda_i, \sum_{i=1}^{n} \sigma_i^2\right)$$

Thm

$$X_i = N(\lambda_i, \sigma_i^2), \quad i = 1, \ldots, n$$
$$S = \sum_{i=1}^{n} \alpha_i X_i$$
$$\Rightarrow S = N\left(\sum_{i=1}^{n} \alpha_i \lambda_i, \sum \alpha_i^2 \sigma_i^2\right)$$

a) $P(Y < 0) = ?$

$$P(Y < 0) = P(Z < 2) = \Phi(2) = \boxed{97.72\%}$$
$$z = \frac{0 - (-1)}{1/2} = 2$$

c) $$P\left(X < Y \,\middle|\, Y = -1, \ X \notin (-1, 2)\right) = \frac{P(X < -1)}{P(X < -1) + P(X > 2)} =$$

$$= \frac{P(Z < -1.50)}{P(Z < -1.50) + P(Z > 3.00)}$$

$$= \frac{1 - \Phi(1.50)}{[1 - \Phi(1.50)] + [1 - \Phi(3)]}$$

$$\begin{cases} z_1 = \dfrac{-1 - 0}{2/3} = -1.50 \\ z_2 = \dfrac{2 - 0}{2/3} = 3.00 \end{cases}$$

$$= \frac{1 - .9332}{2 - .9332 - .9987} \approx \frac{.0668}{.0681} = \boxed{.9809}$$

* The answer in the back of the book is slightly off. To get the answer in the back of the book use .9986 instead of .9987, which is the z-score for 2.99 instead of 3.00.