Instructions: (a) Round your answers to four decimal places; (b) Answers to all questions should be written in complete sentences; (c) For partial credit show all your work for every problem.

Problem 1. (15 points) A class survey in a large class for first-year college students asked: “About how many minutes do you study on a typical weeknight?” The mean response of the 269 students was $\bar{x} = 137$ minutes. Suppose that we know that the study time follows a Normal distribution with standard deviation $\sigma = 65$ minutes in the population of all first-year students at this university. Use the survey result to give a 99% confidence interval for the mean study time of all first-year students.

Problem 2. (15 points) The goal of an experiment is to estimate the mean healing rate of skin wounds in newts within a margin of error of ±1 micrometer per hour with 98% confidence. From historical data it is known that the standard deviation of healing rates for newts is $\sigma = 8$ micrometers per hour. How large a sample is needed?

Problem 3. (20 points) An environmentalist group collects a liter of water from each of 45 random locations along a stream and measures the amount of dissolved oxygen in each specimen. The mean is 4.62 milligrams. Is this strong evidence that the stream has a mean oxygen content of less than 5 milligrams per liter? Suppose we know that dissolved oxygen varies among locations according to a Normal distribution with $\sigma = 0.92$ milligrams. State the hypotheses and calculate an appropriate p-value to answer the question. (Use $\alpha = 5\%$.)

Problem 4. (25 points) A patient is classified as having gestational diabetes (high blood glucose levels during pregnancy) if her blood glucose level is above 140 mg/dl one hour after a sugary drink is ingested. Based on historical data blood glucose level varies according to a Normal distribution with $\mu = 125$ mg/dl and $\sigma = 10$ mg/dl.

a. What is the probability that a patient is diagnosed as having gestational diabetes based on a single blood glucose measurement?

b. What is the probability that a patient is diagnosed as having gestational diabetes based on the average of 4 blood glucose measurements (made on 4 separate days)?

Problem 5. (25 points) A sample of 31 7th-grade girls in a Midwest school district had a mean IQ score of $\bar{x} = 105.8387$. Test the 31 girls as an SRS of all 7th-grade girls in the school district. Suppose that the standard deviation of IQ scores in this population is known to be $\sigma = 15$.

a. Give a 95% confidence interval for the mean IQ score $\mu$ in this population.

b. Is there significant evidence that the mean IQ score in the population differs from 100? State the hypotheses and calculate an appropriate p-value to answer the question. (Use $\alpha = 5\%$.)