Information & Entropy

Comp 595 DM

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Information & Entropy

- Information Equation

$$I(p) = -\log_b(p)$$

p = probability of the event happening

b = base

(base 2 is mostly used in information theory)

*unit of information is determined by base

base 2 = bits base 3 = trits

base 10 = Hartleys base e = nats

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- Example of Calculating Information

  Coin Toss

  There are two probabilities in fair coin, which are head(.5) and tail(.5).

  So if you get either head or tail you will get 1 bit of information through following formula.

  I(head) = - log (.5) = 1 bit

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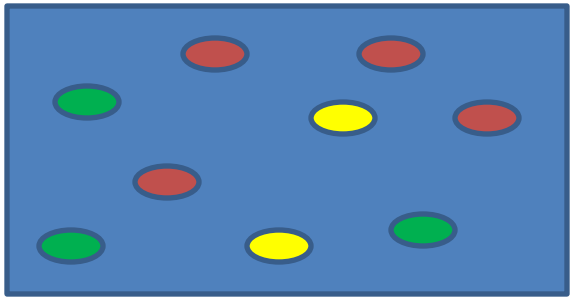
- Another Example

  Balls in the bin



  The information you will get by choosing a ball from the bin are calculated as following.

  I(red ball) = - log(4/9) = 1.1699 bits

  I(yellow ball) = - log(2/9) = 2.1699 bits

  I(green ball) = - log(3/9) = 1.58496 bits

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- Then, what is Entropy?

  - Entropy is simply the average(expected) amount of the information from the event.

- Entropy Equation

$$\text{Entropy} = -\sum_{i=1}^{n} p_i \log_b(p_i)$$

n = number of different outcomes

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- How was the entropy equation is derived?

$$I = -\sum_{i=1}^{n} (N * p_i) * \log_b(p_i)$$

I = total information from N occurrences

N = number of occurrences

(N*Pi) = Approximated number that the certain result will come out in N occurrence

$$\text{Entropy} = -\sum_{i=1}^{n} p_i \log_b(p_i)$$

So when you look at the difference between the total Information from N occurrences and the Entropy equation, only thing that changed in the place of N. The N is moved to the right, which means that I/N is Entropy. Therefore, Entropy is the average(expected) amount of information in a certain event.

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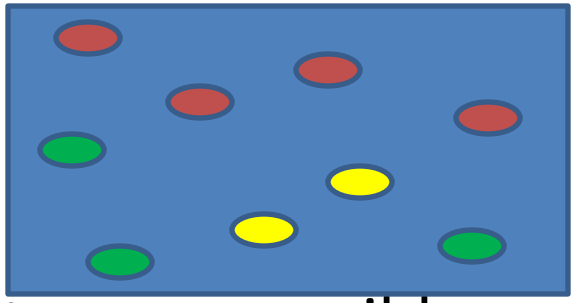

- Let's look at this example again…

  Calculating the entropy….

  In this example there are three outcomes possible when you choose the ball, it can be either red, yellow, or green. (n = 3)

  So the equation will be following.

$$\text{Entropy} = -\sum_{i=1}^{3} p_i \log_b(p_i)$$

Entropy = - (4/9) log(4/9) + -(2/9) log(2/9)
+ - (3/9) log(3/9)

= 1.5304755

  Therefore, you are expected to get 1.5304755 information each time you choose a ball from the bin

Clear things up.

- Does Entropy have range from 0 to 1?
  - No. However, the range is set based on the number of outcomes.
  - Equation for calculating the range of Entropy:

    $0 \leq \text{Entropy} \leq \log(n)$, where n is number of outcomes
  - Entropy 0(minimum entropy) occurs when one of the probabilities is 1 and rest are 0's
  - Entropy log(n)(maximum entropy) occurs when all the probabilities have equal values of 1/n.

If you want more information…

- http://csustan.csustan.edu/~tom/sfi-csss/info-theory/info-lec.pdf
  - Look at pages from 15 to 34. This is what I read and prepared all the information that are on the current powerpoint slides. Very simple and easy for students to understand.

- http://ee.stanford.edu/~gray/it.pdf
  - Look at chapter two of this pdf file, it has very good detailed explanation of Entropy and Information theory.