Information & Entropy

Comp 595 DM
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**Information & Entropy**

- Information Equation

\[ I(p) = -\log_b(p) \]

- \( p \) = probability of the event happening
- \( b \) = base

(base 2 is mostly used in information theory)

*unit of information is determined by base

- base 2 = bits
- base 3 = trits
- base 10 = Hartleys
- base e = nats
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• Example of Calculating Information

Coin Toss

There are two probabilities in fair coin, which are head(.5) and tail(.5).

So if you get either head or tail you will get 1 bit of information through following formula.

\[ I(\text{head}) = - \log (.5) = 1 \text{ bit} \]
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• Another Example

Balls in the bin

The information you will get by choosing a ball from the bin are calculated as following.

I(red ball) = - log(4/9) = 1.1699 bits
I(yellow ball) = - log(2/9) = 2.1699 bits
I(green ball) = - log(3/9) = 1.58496 bits
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• Then, what is Entropy?
  - Entropy is simply the average(expected) amount of the information from the event.

• Entropy Equation

\[
\text{Entropy} = - \sum_{i=1}^{n} p_i \log_b(p_i)
\]

\(n = \text{number of different outcomes}\)
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• How was the entropy equation is derived?

\[ I = \sum_{i=1}^{n} (N \times p_i) \times \log_b(p_i) \]

I = total information from N occurrences
N = number of occurrences
(N*Pi) = Approximated number that the certain result will come out in N occurrence

\[ \text{Entropy} = - \sum_{i=1}^{n} p_i \log_b(p_i) \]

So when you look at the difference between the total Information from N occurrences and the Entropy equation, only thing that changed in the place of N. The N is moved to the right, which means that I/N is Entropy. Therefore, Entropy is the average(expected) amount of information in a certain event.
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• Let’s look at this example again...
Calculating the entropy....
In this example there are three outcomes possible when you choose the ball, it can be either red, yellow, or green. (n = 3)
So the equation will be following.

\[
\text{Entropy} = - \sum_{i=1}^{3} p_i \log_b(p_i)
\]

\[
\text{Entropy} = -(4/9) \log(4/9) + (2/9) \log(2/9) + (3/9) \log(3/9)
\]

= 1.5304755

Therefore, you are expected to get 1.5304755 information each time you choose a ball from the bin
Clear things up.

• Does Entropy have range from 0 to 1?
  – No. However, the range is set based on the number of outcomes.
  – Equation for calculating the range of Entropy:
    \[ 0 \leq \text{Entropy} \leq \log(n), \text{ where } n \text{ is number of outcomes} \]
  – Entropy 0 (minimum entropy) occurs when one of the probabilities is 1 and rest are 0’s
  – Entropy \( \log(n) \) (maximum entropy) occurs when all the probabilities have equal values of 1/n.
If you want more information...

  - Look at pages from 15 to 34. This is what I read and prepared all the information that are on the current powerpoint slides. Very simple and easy for students to understand.

- [http://ee.stanford.edu/~gray/it.pdf](http://ee.stanford.edu/~gray/it.pdf)
  - Look at chapter two of this pdf file, it has very good detailed explanation of Entropy and Information theory.