

# ***Technical and Practical Considerations in applying Value Added Models to estimate teacher effects***

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# Purpose VAM

- What motivates the use of Value Added Models?
  - Improve student performance
  - Teacher pay/licensure decisions
- School Accountability
  - Free rider problem
- Teacher Evaluation
  - Elements of teacher evaluation
    - Inputs as indicator
    - Observations as indicators

# VAM Introduction

- Vocabulary
- What are VAMs trying to measure?
  - Teacher contribution to student learning and attempting to make this a causal estimate
- Is there a plausible alternative? Unlike experimental setting where plausible alternative is specified, use average as basis for comparison.
  - Average of what?
  - School
  - District
  - State

# Technical considerations

- Bias
  - Student sorting
- Precision
- Reliability
- Stability

# Practical considerations

- Assessments and scale
- Available data and linkages
- Tested and non-tested subjects
- Components of evaluation system

# Models

- Models vary in complexity and assumptions
- Models vary in application
  - Education basis
    - generally random effects models
  - Economist basis
    - Generally fixed effects models
  - State accountability based models

# A First Approximation

- $A_{ti} = XB + S\Gamma + T\Phi + e_{ti}$
- $X$  = vector of student and family inputs
- $S$  = vector of schooling inputs
- $T$  = vector of teacher inputs
- Assumes  $e_{tj}$  is orthogonal to covariates, which is highly unlikely

# Rationale Underlying Value Added Models

- The underlying assumption for value added models is:

$$A_{it} = f(B_{it}, P_{it}, S_{it}, I_{it}, E_{it}), \quad (1)$$

- where for student  $i$  at time  $t$  Achievement  $A$ , is some function of:
  - *Student background (B)*
  - *Peer and other influences (P)*
  - *School/teacher inputs (T/S)*
  - *Innate/general ability (I)*
  - *And luck (E).*
- Model is cumulative and past inputs may affect current Achievement.
- Also would need independent measure of innate ability, gathered before any S has occurred.



# Specification

- If we assume that (1) holds for any time  $t$ , then we can consider change in achievement from  $t$  to  $t'$ .

$$A_{it'} - A_{it} = f(.)$$

# VAM Specified

- Simplified specification

$$A_{it} = \delta A_{it-1} + T_{tj} + \alpha_i + e_{it}^*$$

where  $e_{it}^* = e_{it} - e_{it-1}$

$T_j$  is teacher  $j$ 's effect and

$\alpha_i$  is an individual student time-invariant effect.

If assume  $\delta = 1$  then use common “gain” model

$$A_{it} - A_{it-1} = T_{tj} + \alpha_i + e_{it}^*$$

# Assumes

- age independence
- additive separability
- fixed family inputs
- geometric decay in previous inputs
- homogenous teacher effectiveness
- sorting based on fixed student covariates
- OLS produces biased estimates because  $A_{t-1}$  and that part of  $e^*$  related to  $e_{t-1}$  - not orthogonal.

# Student Effects

- Student fixed effect  $\alpha_i$  can be modeled by:
  - Dummy variables
    - Empirical evidence suggests that this biases teacher effects downwards.
  - Using student time invariant covariates
  - Assuming adequately captured by  $A_{t-1}$
  - Use instrumental variables or additional test scores for  $A_{t-1}$

# Teacher effects

- Can estimate  $T_j$  by
  - Using dummy variables for each teacher
  - Demeaning by teacher means
    - Generating within unit estimates

# Fixed effects

- Avoids bias (if sorting based on static factors)
  - Some evidence that sorting is based on dynamic factors.
- Limitations
  - Can't estimate time invariant effects
  - ignores between teacher variability
    - Teacher effects will be less precise than when using random effects models

# Random Effects Models

- Common approach used by educational researchers/statisticians
- Can be residualized gain, or growth specification.

# ANCOVA Specification

- $A_{tij} = \delta A_{t-1ij} + \phi T_j + e_{tij}$
- Where  $\phi$  is a random teacher effect where:  
 $\phi = \gamma + U_j$   
Hence  $A_{tij} = \delta A_{t-1ij} + \phi(\gamma + U_j) + e_{tij}$   
and  $U_j \sim N(0, \tau)$  and assumed orthogonal to student covariates that may be in the model  
Generally use EB estimates which “shrink” estimates towards mean (effect depends on reliability of teacher effect estimate).



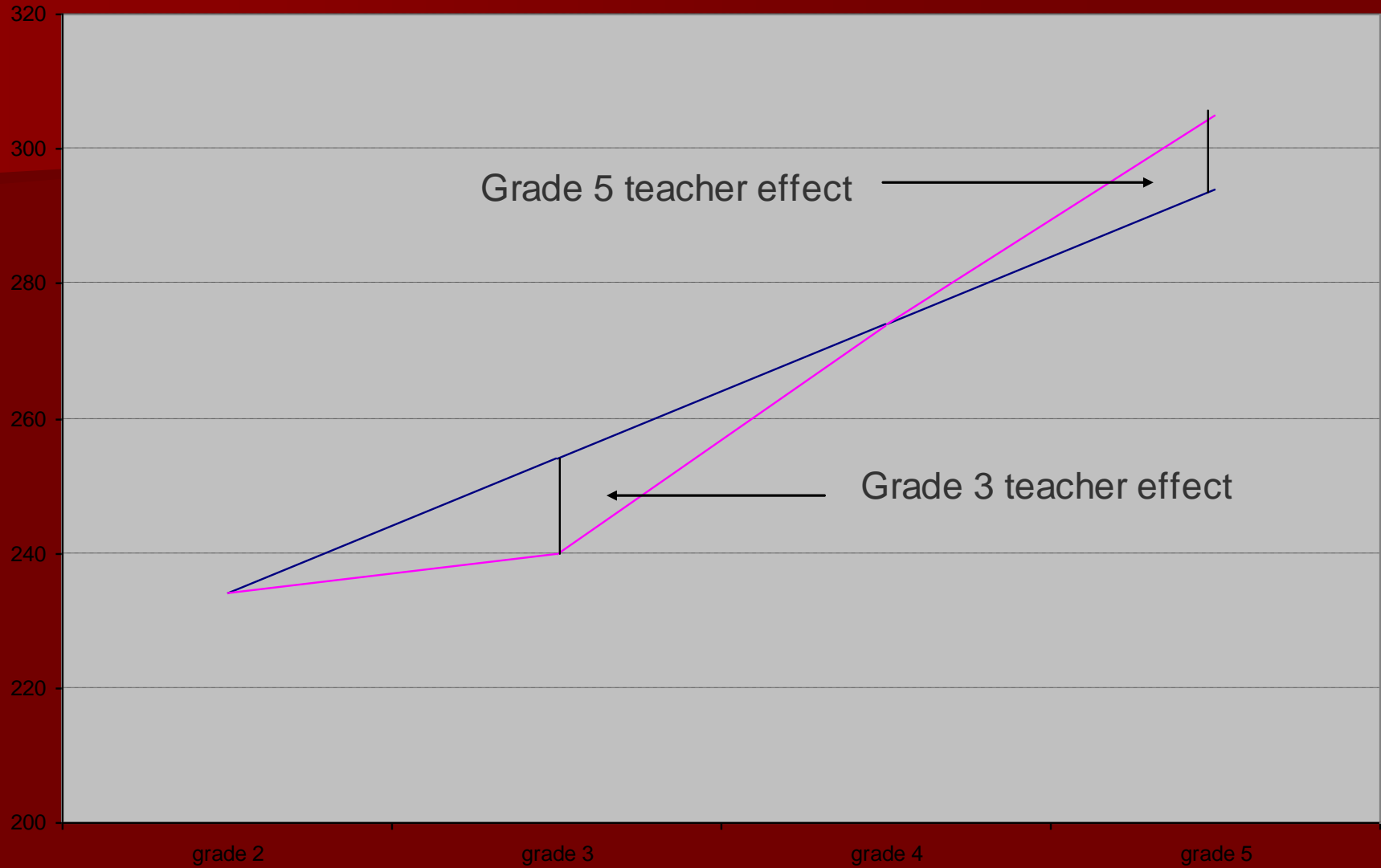
# Random Effects tradeoff

- Random effect teacher estimates are more precise
- Random effects may be biased if more restrictive (compared to fixed effects models) assumptions are not met.
- Random effect models can include time invariant covariates.
- Models can be “centered” around group means to recreate fixed effects estimates.

# Growth Model Approach

- Can model both  $A_t$  and  $A_{t-1}$  on the LHS of the equation avoiding correlations of error and  $A_{t-1}$  or other covariates.
- Can model multiple assessment occasions over time as a function of time.

# Conceptualization of teacher effect as deviation from average trajectory



# Estimating True Status and Gain

(an example of an optional approach, based on\*)

- $A_{tij} = a_{1tij}\pi_{1ij} + a_{2tij}\pi_{2ij} + e_{tij}$
- Here, for student  $i$ , with teacher  $j$ , at time  $t$ , the assessment scale score is denoted as  $A_{tij}$ . Time in this instance refers to the  $A_{t-1}$ ,  $t = 0$ , and the  $A_t$   $t = 1$ .
- This estimates two parameters, student's initial status for the  $A_{t-1}$  ( $\pi_{1ij}$ ) and gain on the  $A_t$ , ( $\pi_{2ij}$ )

\*Bryk, A., Thum, Y. M., Easton, J., & Luppescu, S. (1998). Assessing school academic productivity: The case of Chicago school reform. *Social Psychology of Education*, 2, 103-142

The error,  $et_{ij}$ , is assumed to be  
 $N \sim (0, \sigma^2)$

Can conceive as the student growth part of the model as a measurement model and incorporate precision (SEM) to identify the model.

- $A_{tij}^* = a_{1\, tij}^* \pi_{1ij} + a_{2\, tij}^* \pi_{2ij} + e_{tij}^*$
- In this way  $e_{ijt}^*$  is distributed  $N \sim (0,1)$ , and  $\pi_{1ij}$  and  $\pi_{2ij}$  now estimate a student's true initial status and true gain, respectively. Can estimate teacher effect using estimate of true gain.
- \* indicates parameter is weighted by precision ( $1/SEM_{ij}$ ).

- In order to replicate fixed effects estimates group mean center
- $A_{tij}^* = a1_{tij}^* \pi_{1ij} + (a_{2_{tij}}^* - a_{2_{.ij}}^*) \pi_{2ij} + e_{tij}^*$
- where  $e_{tij} \sim N(0,1)$ , and  $a_{2_{.ij}}^* = S_{t=1,2} a_{2_{tij}}^* / 2$  (2 in this case because we have a  $A_t$  and  $A_{t-1}$ ).

- *Hence, combined model is:*
- $A_{tij}^* = \gamma_{1tij}\pi_{1ij} + \gamma\pi_{2ij} + r_{ij} + u_j$
- where  $r_{ij} = r_1a_1^* + r_2(a_{2tij}^* - a_{2.ij}^*)$ ,  
and  $u_j = u_1a_1^* + u_2(a_{2tij}^* - a_{2.ij}^*)$ .
- $U_j$  is estimated teacher effect

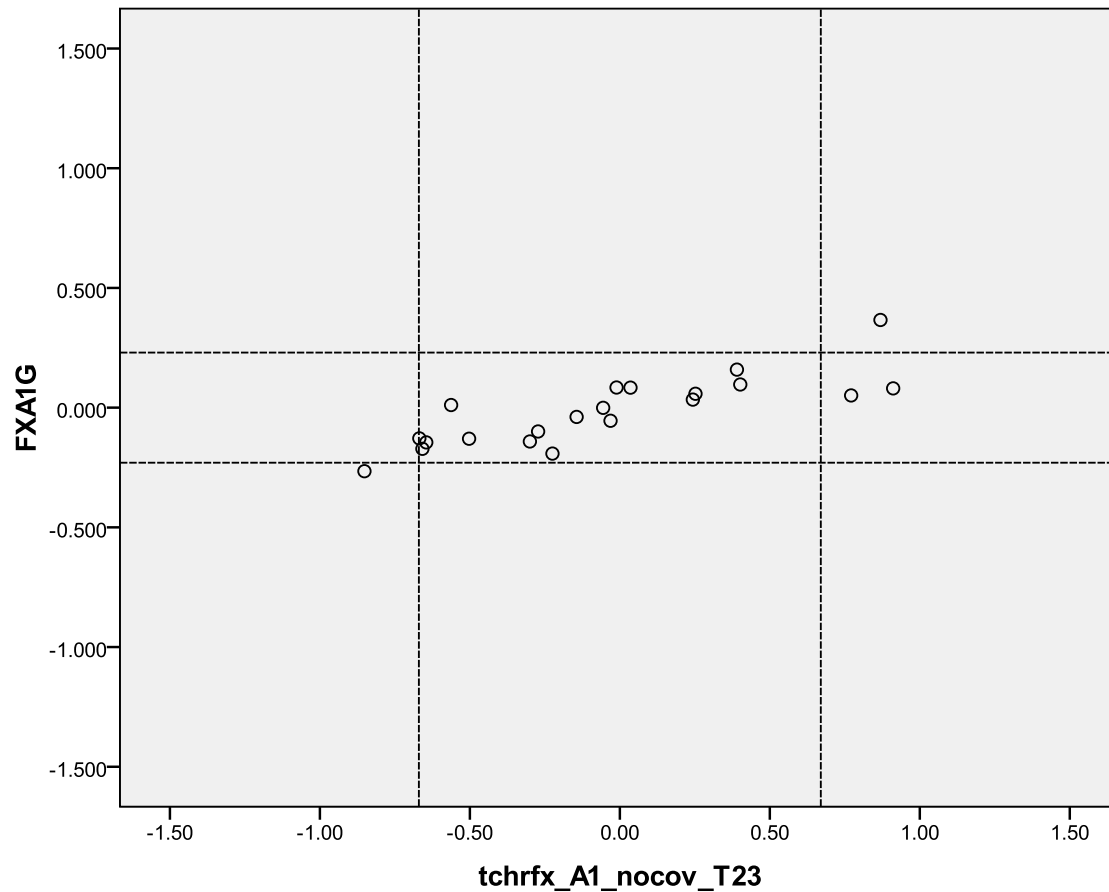


- Advantage of including student covariates:
  - Can estimate initial achievement gaps
  - Can estimate time to close gap
  - These estimates likely too imprecise at the teacher level, but are useful at the school level

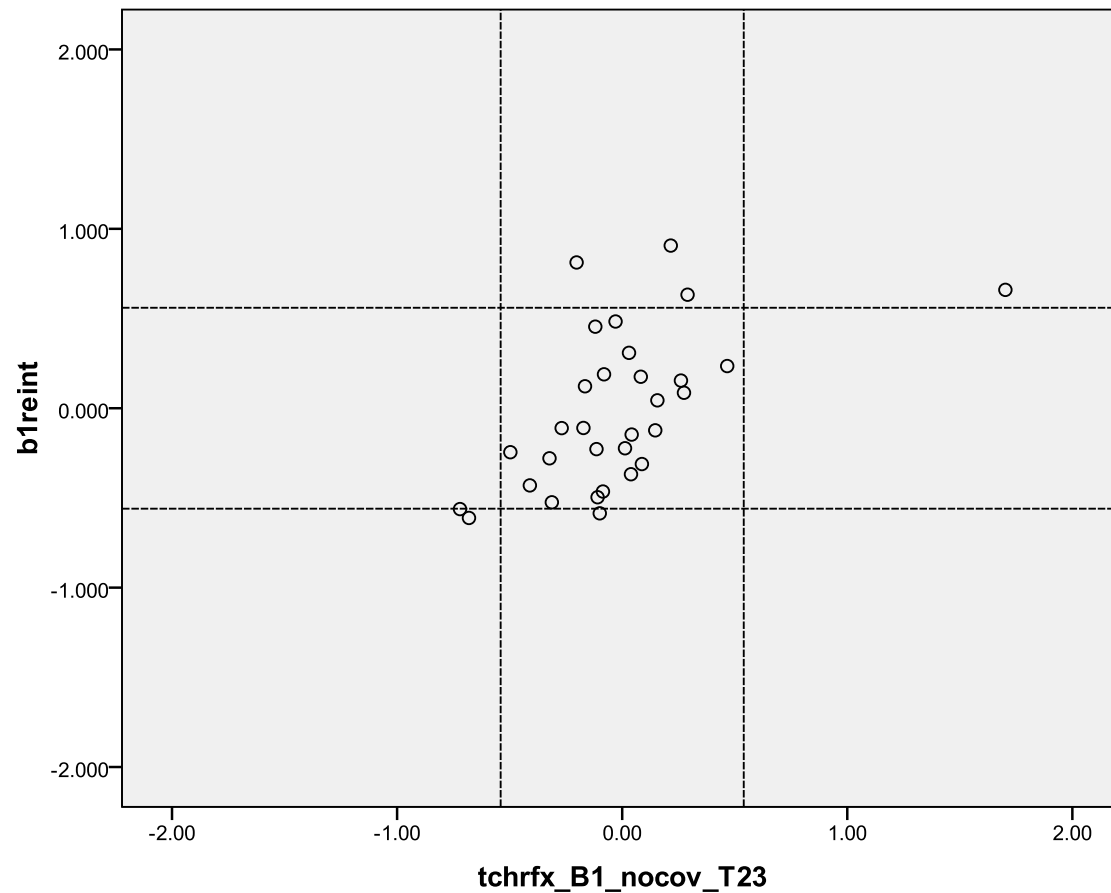
# Technical considerations

- Bias
  - Sorting
  - Measurement error
  - Equating error
- Precision
  - Classification categories– e.g. highly effective
  - Better precision than existing teacher evaluation measures?  
multiple assessments
- Reliability
  - Ability to detect true between-teacher difference
- Stability
  - Classifications stable over time
  - Multiple year (congruent with policy?)

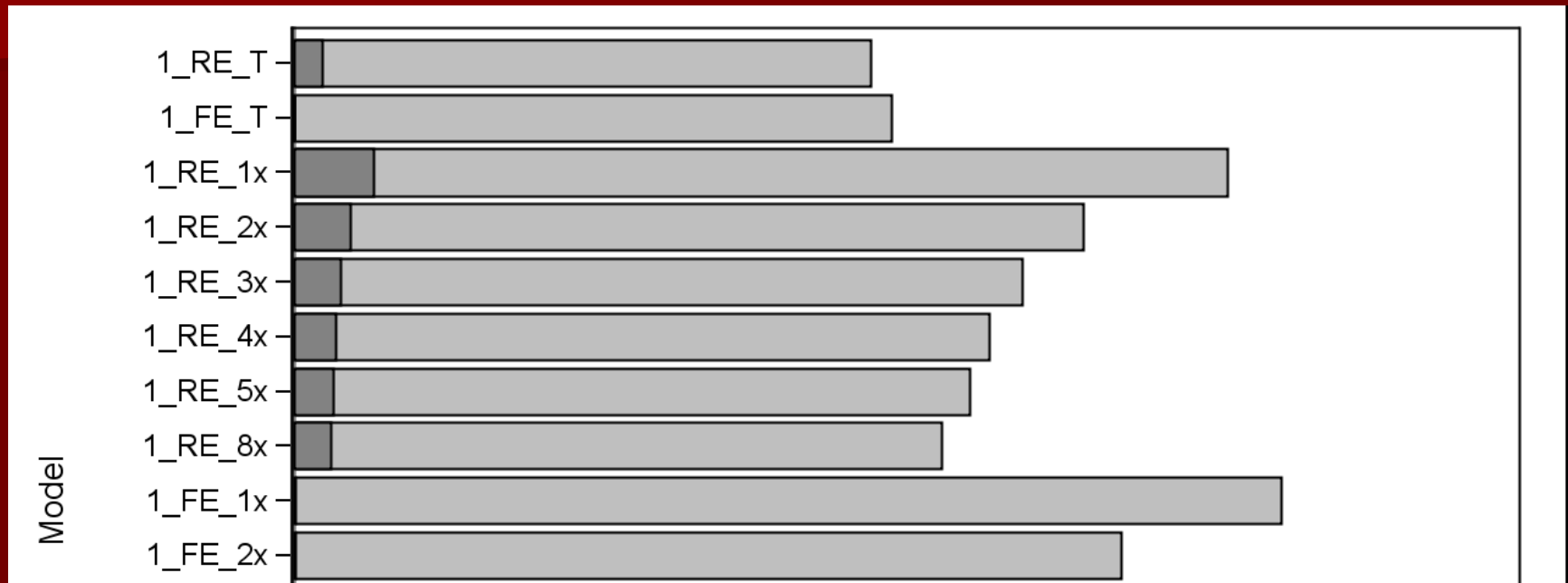
# Effect of Model and Precision



# Effect of Model and Precision



# Effect of measurement error



From Wright, P, 2008

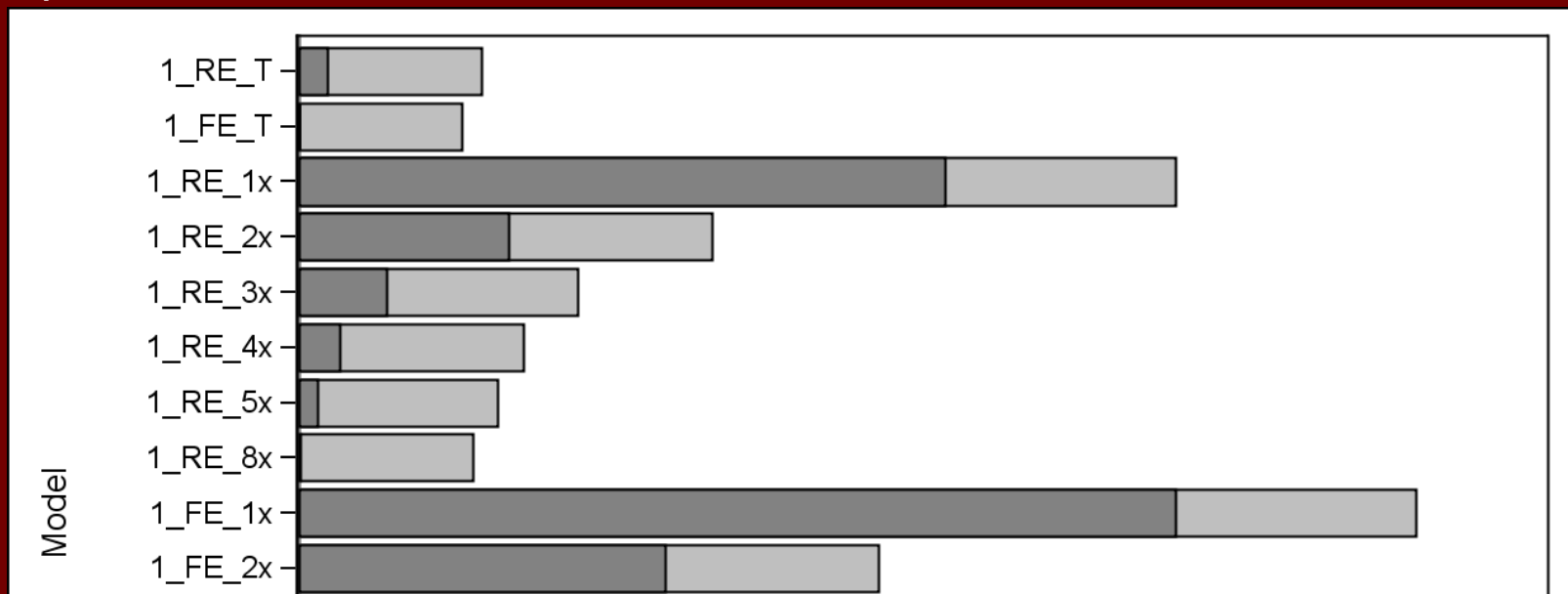
Results with no measurement error in assessment

Light gray = se

Dark gray = bias

# Measurement Error and Sorting bias

- Overall, half of the students were free/reduced-price lunch eligible.
- For individual students,  $\text{Prob}(P_{ij}=1)$  decreased with increasing true pre-test score ( $\xi_j$ ).
- Lower performing students (lower  $\xi_j$ ) were more likely to be assigned to a poorer teacher.



Results with measurement error in assessment

From Wright, P, 2008

Light gray = se

Dark gray = bias

# Practical considerations

- Assessments and scale
- Available data and linkages
  - Spill over
  - Persistence
- Tested and non-tested subjects
- Components of evaluation system

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