“May I please pay a higher price?”: sustaining non-simultaneous exchange through free disposal of bargaining advantage∗

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Summary. The issue of whether or not trade can be sustained when exchange is non-simultaneous is addressed. An environment in which a buyer is exposed to the possibility that a seller will not deliver an item which has been paid for is examined. Situations of this nature often arise when consumer-to-consumer trade is negotiated via the internet. Repeated interaction between a single seller and a single buyer is modelled, assuming each has incomplete information about the type of their prospective trading partner. It is possible for an agent to be better off with less relative bargaining power. Thus, if an agent can reduce his own relative bargaining power, he may choose to do so.

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1 Introduction

For most transactions trade occurs simultaneously, in that each agent parts with the item which they are giving up at the same time. For example, if an individual goes to a fast food restaurant for lunch: he decides what he wants, places an order, and then trade occurs simultaneously in that the customer pays the cashier at essentially the same time the goods are delivered. Since all aspects of the transaction occur simultaneously, the transaction cannot be “split” into separate parts (for example, the transaction cannot be divided into discrete phases of: customer delivery of payment, merchant receipt of payment, merchant delivery of goods, and customer receipt of goods). Borrowing terminology from the computer science literature, such transactions will be referred to as “atomic.” As Camp (2000) points out, such atomic transactions either fail completely or succeed completely.

Clearly not all transactions are atomic. In some instances, different phases of a transaction occur at different times. For example, in many cases it is customary for one party to part with their item first. In such “non-simultaneous” exchange, the buyer is often required to pay for an item before delivery. Such transactions will be referred to as “non-atomic.” While non-atomic transactions could either fail completely or succeed completely, there is also the possibility of partial success or failure (in which one party satisfies the terms of the agreement, while the other does not).

When a consumer purchases an item by mail order, over the telephone, or over the internet, the transaction is almost always non-atomic. However, not all non-atomic transactions differ fundamentally from atomic transactions, since in many instances neither party is exposed to the possibility that the other party might not “hold up his end of the bargain.” This is due to the fact that if one party does not satisfy the terms of the agreement, the other party is often able to take legal action in order to have the transaction either enforced or voided in its entirety.

With the rapid growth of the internet, such non-atomic trade is becoming more common. Further, in many situations the transactions are not only non-simultaneous, but also the buyer is exposed to the possibility that the seller will not deliver an item which has been paid for. This could be the case for at least two different reasons. First, it may be that the transaction which is agreed upon is not clearly a legally binding contract. In such instances, if

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1See Tygar (1996) for a discussion of atomicity of data processing in regards to electronic commerce.

2It may be that the terms of the agreement are illegal. For example, in many jurisdictions, it is illegal to sell tickets to concerts or sporting events at prices considerably above “face value.” However, such agreements are often reached over the internet. Further, even
the buyer must make a payment before the item is delivered, it is not clear that the buyer has any legal recourse if the seller does not deliver the item. Second, this could effectively be the case even if there is a legally binding contract between the buyer and seller. If the costs of enforcing the contract are extremely high relative to the value of the item being traded, it will not be worthwhile for the buyer to attempt to have the contract enforced if the seller chooses not to honor the agreement.

By its very nature, the internet facilitates transactions between parties that are geographically separated by great distances. These are often mutually beneficial transactions that would not otherwise occur. As such, economists typically view this matching of trading partners favorably. However, it is precisely this feature of internet transactions (matching parties that are geographically separated from each other) that often makes the enforcement of a bargain between the two parties more difficult. It is much easier for parties to commit fraud in such an environment.

Of the 16,775 complaints of fraud registered with the Internet Fraud Complaint Center (IFCC) during 2001, 20.3% were specifically categorized as instances of “non-delivery of merchandise and payment.” Only one other category, “auction fraud,” received more complaints (42.8% of all complaints). It should be noted that “auction fraud” consists of either “non-delivery of merchandise” or “misrepresentation” (which occurs when the seller intentionally misleads the bidder about the value of the item being sold). Further, since only one in ten incidents of fraud are ever brought to the attention of regulatory or enforcement agencies,4 such instances of failure are more common than these figures would initially suggest.

Consider the transactions negotiated through the internet site WebTix (www.tixs.com). This site lists classified advertisements for tickets to concerts, theatre, and sporting events. WebTix does not sell the items listed on their site. They merely act as an intermediary, matching sellers and buyers. WebTix cannot be held responsible if a party to the transaction does not hold up his end of the bargain. Further, many of the transactions agreed upon through WebTix are likely “consumer-to-consumer” transactions because

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3“One of the components of fraud committed via the Internet that makes investigation and prosecution difficult is that the offender and victim may be located thousands of miles apart...a unique characteristic not found with many other types of ‘traditional’ crime.” (IFCC 2001 Internet Fraud Report, page 15)

4This figure is from the National Public Survey on White Collar Crime, as reported in the IFCC 2001 Internet Fraud Report.
tween buyers and sellers with no previous interaction with each other. According to the IFCC these are precisely the types of internet transactions that are most likely to result in unsuccessful outcomes.\(^5\)

The issue of whether or not trade can be sustained in such an environment is addressed. Specifically, non-simultaneous transactions between a single buyer and a single seller are considered. It is assumed that the buyer must send his payment for the item before the seller chooses whether to deliver the item. When no binding contracts can be written, the buyer is exposed to the possibility that the seller will not deliver the item.

The general framework to be analyzed is presented in Section 2. The primary focus is a situation in which there is incomplete information on each side of the transaction. As a result, neither party knows precisely when future exchange is no longer beneficial. Characterizing this interaction as a dynamic game of incomplete information, a perfect Bayesian equilibrium is identified in Section 3. A detailed examination of the conditions under which exchange is sustainable, as well as the equilibrium net gains from such interaction, is provided in Section 4. In Section 5, the possibility of allowing agents to bargain at less than their full ability is considered. If an agent is able to act in this manner, he may wish to do so. In such instances one of the following apparently counterintuitive outcomes occurs: either the buyer chooses to pay a higher per unit price than he could bargain for or the seller chooses to accept a lower per unit price than he could bargain for. In either case, exchange will be sustained when it otherwise would not have been.

### 2 Basic Framework

Consider a situation in which gains from trade between two agents potentially exist. Let the two agents be denoted by \(B\) (the buyer) and \(S\) (the seller).

\(S\) has a marginal valuation of \(V_{s,n}\) for the \(n^{th}\) unit sold. Suppose \(V_{s,n}\) is equal to either \(v_s\) or \(C\). A seller of type \(n_s \geq 1\) has: 
\[
V_{s,n} = v_s \quad \text{for each} \quad n = 1, \ldots, n_s \quad \text{and} \quad V_{s,n} = C \quad \text{for each} \quad n > n_s.
\]
A seller of type 0 has \(V_{s,n} = C\) for every unit.

\(B\) has a marginal valuation of \(V_{b,n}\) for the \(n^{th}\) unit purchased. Suppose \(V_{b,n}\) is equal to either \(v_b\) or 0. A buyer of type \(n_b \geq 1\) has: 
\[
V_{b,n} = v_b \quad \text{for each} \quad n = 1, \ldots, n_b \quad \text{and} \quad V_{b,n} = 0 \quad \text{for each} \quad n > n_b.
\]
A buyer of type 0 has \(V_{b,n} = 0\)

\(^5\)“Nearly 76% of alleged fraud perpetrators tend to be individuals (as opposed to businesses)...” (IFCC 2001 Internet Fraud Report, page 3); “...most complaints probably involve complainants and perpetrators that did not have a relationship prior to the incident.” (IFCC 2001 Internet Fraud Report, page 15)
for every unit.

Assume $0 < v_s < v_b < C$. Under this assumption, gains from trade exist on each unit for which $V_{s,n} = v_s$ and $V_{b,n} = v_b$.

The buyer and seller attempt to exchange one unit at a time, sequentially over a number of discrete periods. Each period $n$ consists of three stages of interaction as follows:

**Stage 0 (in period $n$):** The seller and buyer bargain in order to reach agreement on the terms of trade. Bargaining leads to an agreed upon price 

$$p = \alpha v_b + (1 - \alpha) v_s,$$

with $0 < \alpha < 1$. $\alpha$ should be thought of as measuring the “inherent bargaining skill” of the seller relative to the buyer.

**Stage 1 (in period $n$):** The buyer decides whether or not to give the agreed upon payment of $p$ to the seller.

**Stage 2 (in period $n$):** The seller decides whether or not to deliver the item to the buyer. If the buyer paid for the item during Stage 1, then the seller has already received a payment of $p$. The seller is able to keep this payment regardless of whether he chooses to deliver the item or not.

A transaction in period $n$ will be considered “successful” if both the seller and the buyer choose to honor the agreement that they reached during Stage 0 of period $n$. That is, the transaction is successful if the buyer sends a payment of $p$ during Stage 1 and the seller delivers the unit during Stage 2. A transaction that is not successful is “unsuccessful.” An unsuccessful transaction can be either “partially unsuccessful” (if one party honors the agreement while the other does not) or “completely unsuccessful” (if both parties do not honor the agreement).

If a single period $n$ is analyzed in isolation, it is clear that the seller has a dominant strategy of not delivering the item. As a result, the buyer would choose not to pay for the item. If each party chooses not to honor the agreement, this is the same outcome which would occur if there were no interaction between the parties during the period in question. When analyzing the entire game, the identified equilibrium strategies are Nash reversion strategies.

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This price results from the weighted Nash Bargaining Solution (Harsanyi and Selten (1972)) for the problem in which the utility of the seller is $p - v_s$, the utility of the buyer is $v_b - p$, and $S$ and $B$ have relative bargaining powers of $\alpha$ and $1 - \alpha$ respectively (with $0 < \alpha < 1$). It also results from the weighted Kalai-Smorodinsky Bargaining Solution (Thomson (1994)) for the problem in which the utility of the seller is $p - v_s$, the utility of the buyer is $v_b - p$, and $S$ and $B$ have relative bargaining powers of $\lambda = \frac{\alpha}{1 - \alpha}$ and $1$ respectively.
which call for each agent to not honor an agreement in any period for which there has been an unsuccessful transaction in some previous period. That is, in equilibrium players will use the reasonable punishment strategy of essentially never attempting to trade with someone that did not honor a previous agreement. When such strategies are used, the seller never has any incentive to deliver an item which has not been paid for. Thus, the possibility of the seller delivering an item during Stage 2 following no payment during Stage 1 need not be considered.

3 Equilibrium Behavior

If the buyer and seller each know the values of both $n_s$ and $n_b$, the situation can be modelled as a dynamic game of complete information. The resulting game is qualitatively similar to the Centipede Game introduced by Rosenthal (1981). The unique subgame perfect Nash equilibrium involves the seller choosing not to deliver the item and the buyer choosing not to pay for the item in every period $n$. The outcome is that the buyer does not deliver payment during the first period and trade is completely unsuccessful immediately.

In light of this observation, attention is focused on the case in which there is incomplete information regarding the marginal valuation of each agent. Suppose that before the first period nature determines $n_b$ and $n_s$. The buyer is of type $n_b$ with probability $\lambda^{n_b}(1 - \lambda)$, with $\lambda \in (0, 1)$. This arises when $\Pr(V_{b,1} = v_b) = \lambda$ and $\Pr(V_{b,n+1} = v_b \mid V_{b,n} = v_b) = \lambda$ for $n \geq 1$. The seller is of type $n_s$ with probability $\tau^{n_s}(1 - \tau)$, with $\tau \in (0, 1)$. Similarly, this arises when $\Pr(V_{s,1} = v_s) = \tau$ and $\Pr(V_{s,n+1} = v_s \mid V_{s,n} = v_s) = \tau$ for $n \geq 1$.

Each agent knows his own type once it is determined, but only knows the probability distribution from which the type of his rival is drawn. The resulting situation is modelled as a dynamic game of incomplete information. For this game a perfect Bayesian equilibrium is identified in which the parties do not attempt to trade during any period for which some previous exchange has been unsuccessful. Further, the identified equilibrium is such that in every period for which no previous exchange has been unsuccessful: the buyer delivers the agreed upon payment to the seller if and only if his valuation is $v_b$, while the seller delivers an item which has been paid for if and only if his valuation is $v_s$. Conjecturing that an equilibrium of this form exists,

\footnote{As a result, the belief of $B$ at the start of period $n$ regarding the values of $V_{s,n+j}$ does not depend upon $n$. Further, the only reasonable beliefs are those resulting from the actual distribution from which $n_s$ is determined. Equivalent statements must be true for}
conditions must be determined under which no type of B and no type of S has a unilateral incentive to deviate from the proposed equilibrium strategies.

3.1 Choice of Seller

Consider the choice of S in an arbitrary period n, during which B has chosen to send the agreed upon payment of p to S. For any \( n > n_s \), S will clearly want to choose not to deliver the item (since honoring the current agreement results in a payoff of at most \( (p - C) + p \) which is strictly less than the payoff of \( p \) which is realized by not honoring the current agreement). Thus, any strategy of S which calls for honoring the agreement beyond period \( n_s \) is dominated by the strategy which calls for honoring the agreement only in the first \( n_s \) periods (and not honoring the agreement starting in period \( n_s + 1 \)).

Suppose that during period \( n \), S intends to honor the agreement for exactly \( k \) additional periods (that is, choosing to not honor the agreement starting in period \( n + k \)). Presuming that B will honor the agreement during every future period for which his valuation is \( v_b \), the expected additional payoff for S from behaving in this manner can be expressed as

\[
H^k_s = \lambda^k (k(p - v_s) + p) + \sum_{j=1}^{k} (p - v_s)(1 - \lambda^{j})^j (1 - \lambda)
\]

Proposition 1 specifies the optimal behavior of S during Stage 2 of any period \( n \leq n_s \) for which B delivered the agreed upon payment during Stage 1.

**Proposition 1** Consider the choice of a seller of type \( n_s \) during Stage 2 of any period \( n \leq n_s \), after receiving the agreed upon payment of \( p = \alpha v_b + (1 - \alpha) v_s \) during Stage 1 of period \( n \). Presuming that a buyer of type \( n_b \) will choose to send the agreed upon payment only during the first \( n_b \) periods, such a seller maximizes his expected additional payoff by honoring the current agreement if and only if \( \lambda \geq \frac{v_s}{\alpha v_b + (1 - \alpha)v_s} \).

**Proof of Proposition 1.** The result will follow if it can be shown that \( H_{j+1}^s \geq H_j^s \) for each \( j = 0, \ldots, n_s - n \) if and only if \( \lambda \geq \frac{v_s}{\alpha v_b + (1 - \alpha)v_s} \).

Consider \( \frac{H_{j+1}^s - H_j^s}{\lambda - \lambda^j} \), which is equal to

\[
\frac{1}{\lambda^j} \left( \frac{1}{1 - \lambda} \right) \left\{ p(1 - \lambda^{j+2}) - v_s(1 - \lambda^{j+1}) - p(1 - \lambda^{j+1}) + v_s(1 - \lambda^j) \right\}.
\]

the belief of S about the type of B.
This expression simplifies to $p\lambda - v_s$, which is clearly non-negative if and only if $\lambda \geq \frac{v_s}{p}$. Since $p = \alpha v_b + (1 - \alpha)v_s$, this condition is equivalent to 

$$\lambda \geq \frac{v_s}{\alpha v_b + (1 - \alpha)v_s}.$$ 

Q.E.D.

Based upon the analysis thus far, presuming that a buyer of type $n_b$ would choose to honor the agreement only during each of the first $n_b$ periods: a seller of type $n_s$ will intend to honor the agreement up to and including period $n_s$ (and not honor the agreement beginning with period $n_s+1$) if $\lambda \geq \frac{v_s}{\alpha v_b + (1 - \alpha)v_s}$; while a seller of type $n_s$ will choose not to honor the agreement during period 1 if $\lambda < \frac{v_s}{\alpha v_b + (1 - \alpha)v_s}$. The condition under which the seller will honor the agreement can equivalently be expressed as $\alpha v_b + (1 - \alpha)v_s \geq \frac{v_s}{\lambda}$.

In any period $n \leq n_s$, the seller is essentially faced with the decision of either “attempting to continue the relationship” versus “ending the relationship now.” The expected value of the difference in future payoffs between these two choices is $(1 - \lambda)(-v_s) + \lambda(p - v_s) = \lambda p - v_s$. The seller will “attempt to continue the relationship” (which is done by choosing to honor the agreement), so long as this expression is non-negative.

### 3.2 Choice of Buyer

Consider the choice of $B$ in an arbitrary period $n$, during which exchange has never been unsuccessful during a previous period. For any $n > n_b$, $B$ will clearly not wish to deliver the agreed upon payment (since doing so results in a payoff of at most $-p$ which is strictly less than the payoff of zero which is realized by not sending the payment). Thus, any strategy of $B$ which calls for honoring the agreement beyond period $n_b$ is dominated by the strategy which calls for honoring the agreement only in the first $n_b$ periods (and not honoring the agreement in period $n_b + 1$). Suppose that during period $n$, $B$ intends to honor the agreement for exactly $k$ additional periods (that is, choosing to not deliver the agreed upon payment $p$ starting in period $n + k$). If a seller of type $n_s$ will honor the agreement only during the first $n_s$ periods, the expected additional payoff for $B$ from behaving in this manner is

$$H^k_b = \tau^k k(v_b - p) + \sum_{j=0}^{k-1} (1 - \tau)^j \{ j(v_b - p) - p \}$$

$$= \left( \tau v_b - p \right) \frac{1 - \tau^k}{1 - \tau}.$$

Proposition 2 characterizes the optimal behavior of $B$ during Stage 1 of any period $n \leq n_b$, such that no previous exchange has been unsuccessful.
Proposition 2 Consider the choice of a buyer of type $n_b$ during Stage 1 of any period $n \leq n_b$ such that no previous exchange has been unsuccessful. Presuming that a seller of type $n_s$ will choose to deliver an item which has been paid for only during the first $n_s$ periods, such a buyer maximizes his expected additional payoff by honoring the current agreement if and only if $\tau \geq \frac{\alpha v_b + (1-\alpha) v_s}{v_b}$.

Proof of Proposition 2. The result will follow if it can be shown that $H_{j+1}^b \geq H_j^b$ for each $j = 0, \ldots, n_b - n$ if and only if $\tau \geq \frac{\alpha v_b + (1-\alpha) v_s}{v_b}$. Since $(1 - \tau^j) > (1 - \tau) \tau^j$ for each $j = 0, \ldots, n_b - n$, it is clear that $H_{j+1}^b \geq H_j^b$ if and only if $\tau v_b - p \geq 0$. With $p = \alpha v_b + (1 - \alpha) v_s$, this condition is $\tau \geq \frac{\alpha v_b + (1-\alpha) v_s}{v_b}$. Q.E.D.

From here it follows that, when a seller of type $n_s$ will deliver the item only during each of the first $n_s$ periods: a buyer of type $n_b$ will intend to honor the agreement up to and including period $n_b$ (and not honor the agreement beginning in period $n_b+1$) if and only if $\tau \geq \frac{\alpha v_b + (1-\alpha) v_s}{v_b}$, while a buyer of type $n_b$ will choose not to honor the agreement during period 1 if $\tau < \frac{\alpha v_b + (1-\alpha) v_s}{v_b}$. This condition can equivalently be stated as $\alpha v_b + (1 - \alpha) v_s \leq \tau v_b$.

During any period $n \leq n_b$ the buyer is similarly faced with a decision of either “attempting to continue the relationship” versus “ending the relationship now.” For $B$, the difference between the expected future payoff of these two choices is $(1 - \tau)(-p) + \tau (v_b - p) = \tau v_b - p$. $B$ maximizes his expected future payoff by “attempting to continue the relationship” (that is, by choosing to honor the agreement) so long as this difference is non-negative.

3.3 Perfect Bayesian Equilibrium

Based upon the analysis of the two preceding subsections, a perfect Bayesian equilibrium can be identified for this game.\footnote{A formal statement of a perfect Bayesian equilibrium is a “strategy-belief pair.” In order to simplify the statement of Theorem 1, the corresponding beliefs are not stated. As previously noted, the only reasonable beliefs are those resulting from the actual distributions from which the types are determined. When specifying a perfect Bayesian equilibrium in Theorem 1, it is assumed that the corresponding beliefs of each agent are determined in this manner.}

Theorem 1 There exists a perfect Bayesian equilibrium such that:

i. If $\alpha v_b + (1 - \alpha) v_s \in \left[\frac{v_s}{\alpha}, \tau v_b\right]$ a buyer of type $n_b$ will send the agreed upon payment only during any period $n \leq n_b$ for which the seller has honored
the agreement during every previous period; a seller of type \( n_s \) will deliver an item which has been paid for during any period \( n \leq n_s \).

ii. If \( \alpha_v b + (1 - \alpha) v_s \notin \left[ \frac{v_s}{X}, \tau v_b \right] \): a buyer of type \( n_b \) will never send the agreed upon payment; a seller of type \( n_s \) will never deliver an item.

**Proof of Theorem 1.** First suppose \( \alpha_v b + (1 - \alpha) v_s \notin \left[ \frac{v_s}{X}, \tau v_b \right] \). By Proposition 1 we know that if \( B \) chooses to send the agreed upon payment during each of the first \( n_b \) periods, \( S \) maximizes his expected payoff by only delivering an item which has been paid for during each period \( n \leq n_s \). By Proposition 2 we know that if \( S \) chooses to deliver an item which has been paid for only during each of the first \( n_s \) periods, \( B \) maximizes his expected payoff by sending the agreed upon payment to \( S \) during each of the first \( n_b \) periods. Thus, no type of either individual has a unilateral incentive to deviate from the proposed strategies.

If instead \( \alpha_v b + (1 - \alpha) v_s \in \left[ \frac{v_s}{X}, \tau v_b \right] \), then either \( \alpha_v b + (1 - \alpha) v_s < \frac{v_s}{X} \) or \( \alpha_v b + (1 - \alpha) v_s > \tau v_b \). First suppose \( \alpha_v b + (1 - \alpha) v_s < \frac{v_s}{X} \). In this case (by Proposition 1), \( B \) can reason that even if he were to honor the agreement, \( S \) would choose not to honor the agreement. Thus, \( B \) is better off not delivering payment during Stage 1. Next consider \( \alpha_v b + (1 - \alpha) v_s > \tau v_b \). In this case (by Proposition 2), we see that even if a seller with valuation \( v_s \) would honor the agreement, \( B \) would maximize his expected payoff by not honoring the agreement. Thus, once again, \( B \) will choose not to deliver payment during Stage 1. Q.E.D.

From Theorem 1, if \( \alpha_v b + (1 - \alpha) v_s \notin \left[ \frac{v_s}{X}, \tau v_b \right] \) exchange will be completely unsuccessful in every period. This outcome is inefficient whenever \( \min \{ n_b, n_s \} > 0 \), a direct result of the fact that in this case there are units which are not traded for which the valuation of \( B \) exceeds that of \( S \).

If \( \alpha_v b + (1 - \alpha) v_s \in \left[ \frac{v_s}{X}, \tau v_b \right] \), one of two possible outcomes will occur. If \( n_b \leq n_s \), \( B \) will choose to terminate the relationship weakly before \( S \) would choose to do so. Thus, exchange is completely successful for every unit up to and including \( n_b \) and completely unsuccessful for every unit beyond \( n_b \). This outcome is efficient since every unit for which the valuation of \( B \) exceeds the valuation of \( S \) is transferred from \( S \) to \( B \). If instead \( n_s < n_b \), \( S \) will choose to terminate the relationship strictly before \( B \) would choose to do so. Exchange is: completely successful during every period up to and including \( n_s \), partially unsuccessful during period \( n_s + 1 \) (during which \( B \) delivers payment to \( S \), but \( S \) does not deliver the item to \( B \)), and completely unsuccessful in period \( n_s + 2 \) and beyond. It should be noted that this outcome is still efficient since every unit for which the valuation of \( B \) exceeds the valuation of \( S \) is
transferred from $S$ to $B$. In either instance, when $\alpha v_b + (1-\alpha) v_s \in \left[ \frac{v_s}{s}, \tau v_b \right]$ exchange is completely successful for $n = \min \{n_b, n_s\}$ periods.

4 Successful Exchange

From Theorem 1, exchange is completely successful during the first $n = \min \{n_b, n_s\}$ periods so long as $\alpha v_b + (1-\alpha) v_s \in \left[ \frac{v_s}{s}, \tau v_b \right]$. In order to see precisely when this will be the case, this condition is examined in greater detail.

Letting $f(\alpha, v_s, v_b) = \frac{\alpha v_b}{\alpha v_b + (1-\alpha) v_s}$ and $g(\alpha, v_s, v_b) = \frac{\alpha v_b + (1-\alpha) v_s}{v_b}$, exchange will be successful\(^9\) so long as the following conditions hold:

$$\lambda \geq f(\alpha, v_s, v_b)$$

\((1)\)

and

$$\tau \geq g(\alpha, v_s, v_b).$$

\((2)\)

Recall that $0 < v_s < v_b$ and $\alpha \in (0,1)$. From here it is clear that $f(\alpha, v_s, v_b) \in (0,1)$. Further,

$$\frac{\partial f(\alpha, v_s, v_b)}{\partial v_s} = \frac{\alpha v_b}{[\alpha v_b + (1-\alpha) v_s]^2} > 0,$$

and

$$\frac{\partial f(\alpha, v_s, v_b)}{\partial v_b} = \frac{-\alpha v_s}{[\alpha v_b + (1-\alpha) v_s]^2} < 0,$$

Similarly, it is clear that $g(\alpha, v_s, v_b) \in (0,1)$. Further,

$$\frac{\partial g(\alpha, v_s, v_b)}{\partial v_s} = \frac{1-\alpha}{v_b} > 0,$$

$$\frac{\partial g(\alpha, v_s, v_b)}{\partial v_b} = \frac{-(1-\alpha) v_s}{v_b^2} < 0,$$

and

$$\frac{\partial g(\alpha, v_s, v_b)}{\partial \alpha} = \frac{v_b - v_s}{v_b} > 0.$$

\(^9\)In this context, “successful” means completely successful for the first $n = \min \{v_b, v_s\}$ periods and either completely or partially unsuccessful in every subsequent period.

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Note that both \( f(\alpha, v_s, v_b) \) and \( g(\alpha, v_s, v_b) \) do not depend upon either \( \lambda \) or \( \tau \). From here it is clear that, all other factors constant, exchange is more likely to be successful if either \( \lambda \) is larger or \( \tau \) is larger. That is, we arrive at the expected result that exchange is more easily sustained if the probability of future interaction (on either side of the transaction) is larger.

It is also clear that, all other factors constant, exchange is more likely to occur successfully if either: \( v_s \) is smaller (since \( \frac{\partial f(\alpha, v_s, v_b)}{\partial v_s} > 0 \) and \( \frac{\partial g(\alpha, v_s, v_b)}{\partial v_s} > 0 \)) or \( v_b \) is larger (since \( \frac{\partial f(\alpha, v_s, v_b)}{\partial v_b} < 0 \) and \( \frac{\partial g(\alpha, v_s, v_b)}{\partial v_b} < 0 \)). Thus, if the potential gains from trade on a single unit (that is, \( v_b - v_s \)) are larger, exchange is more easily sustained.

The impact of \( \alpha \) is not as straightforward. Note that \( \frac{\partial f(\alpha, v_s, v_b)}{\partial \alpha} < 0 \) and \( \frac{\partial g(\alpha, v_s, v_b)}{\partial \alpha} > 0 \). As a result, Condition 1 is more easily satisfied for larger values of \( \alpha \), while Condition 2 is more easily satisfied for smaller values of \( \alpha \). In order for exchange to possibly be successful, it must be that \( \frac{\tau v_b}{\lambda} \leq v_b \). When this is the case, \( f(0, v_s, v_b) = 1 > \lambda \) and \( f(1, v_s, v_b) = \frac{v_s}{v_b} < \lambda \). As a result, (since \( \frac{\partial f(\alpha, v_s, v_b)}{\partial \alpha} < 0 \)) there exists a unique value of \( \alpha \in (0, 1) \) such that \( f(\alpha, v_s, v_b) = \lambda \). Let \( \alpha^* \) denote this value. Also note that when \( \frac{\tau v_b}{\lambda} \leq v_b \), we have \( v_s < \tau v_b \). Thus, \( g(0, v_s, v_b) = \frac{v_s}{v_b} < \tau \) and \( g(1, v_s, v_b) = 1 > \tau \). As a result, (since \( \frac{\partial g(\alpha, v_s, v_b)}{\partial \alpha} > 0 \)) there exists a unique value of \( \alpha \in (0, 1) \) such that \( g(\alpha, v_s, v_b) = \tau \). Let \( \alpha^\tau \) denote this value. Specifically, \( \alpha = \left(1 - \frac{\lambda}{\tau} \right) \frac{v_s}{v_b - v_s} \) and \( \alpha^\tau = \frac{\tau v_b - v_s}{v_b - v_s} \). So long as \( \frac{\tau v_b}{\lambda} \leq v_b \), it follows that \( \alpha \leq \alpha^\tau \). Exchange can be sustained so long as \( \alpha \in [\alpha, \alpha^\tau] \).

### 4.1 Net Gains from Honoring the Agreement

When called upon to act, an agent will choose to honor the current agreement if his expected net gains from doing so are positive. Recall that when \( B \) must decide upon a course of action, his expected net gains from honoring the agreement are \( \tau v_b - p \). Similarly, at the time when \( S \) must choose his action, his expected net gains from honoring the agreement are \( \lambda p - v_s \). If bargaining during Stage 0 leads to a price \( p \notin \left[\frac{v_s}{\lambda}, \tau v_b\right] \), \( B \) will choose not to send the agreed upon payment during Stage 1. As such, the equilibrium expected net gains from interaction are essentially zero for each agent. Letting \( \pi_b \) and \( \pi_s \) represent the equilibrium net gains for the buyer and seller respectively, we have that: \( \pi_b \) is equal to \( (\tau - \alpha)v_b - (1 - \alpha)v_s \) if \( \alpha v_b + (1 - \alpha)v_s \in \left[\frac{v_s}{\lambda}, \tau v_b\right] \) and equal to zero otherwise; and \( \pi_s \) is equal to \( \lambda \alpha v_b + [\lambda(1 - \alpha) - 1]v_s \) if \( \alpha v_b + (1 - \alpha)v_s \in \left[\frac{v_s}{\lambda}, \tau v_b\right] \) and equal to zero otherwise. The expected net gains of each agent are illustrated in Figure 1, each as a function of \( \alpha \).

Since the decision by each agent of whether or not to honor the agreement is directly based upon the net gains of the agent, it is worthwhile to see how
\( \pi_b \) and \( \pi_s \) behave as the parameters of the model change. It is easily seen that (holding all other factors constant) \( \pi_b \) and \( \pi_s \) will each weakly increase as: \( \lambda \) increases, \( \tau \) increases, \( v_s \) decreases, or \( v_b \) increases.

Again, the effect of \( \alpha \) is not as straightforward. An increase in \( \alpha \) will lead to an increase in \( \lambda \alpha v_b + [\lambda(1-\alpha) - 1] v_s \) and a decrease in \( (\tau - \alpha) v_b - (1-\alpha) v_s \). Thus, if trade can successfully occur both before and after the change in \( \alpha \) then: an increase in \( \alpha \) will lead to an increase in \( \pi_s \) and a decrease in \( \pi_b \). However, consider an increase in \( \alpha \) such that initially \( \alpha \in (\underline{\alpha}, \overline{\alpha}] \), but then \( \alpha > \overline{\alpha} \). As a result of such a change, \( \pi_s \) will decrease from some positive level down to zero. Recalling that \( \alpha \) is an index of the relative bargaining strength of the seller, we arrive at the counterintuitive result that the seller may actually become worse off with more relative bargaining power (that is, a larger value of \( \alpha \)).

Similarly, a decrease in \( \alpha \) will lead to an increase in \( (\tau - \alpha) v_b - (1-\alpha) v_s \) and a decrease in \( \lambda \alpha v_b + [\lambda(1-\alpha) - 1] v_s \). As a result, if trade can successfully occur both before and after the change in \( \alpha \) then: a decrease in \( \alpha \) will lead to an increase in \( \pi_b \) and a decrease in \( \pi_s \). However, consider a decrease in \( \alpha \) such that initially \( \alpha \in [\underline{\alpha}, \overline{\alpha}) \), but then \( \alpha < \underline{\alpha} \). As a result of such a change, \( \pi_b \) will decrease from some positive level down to zero. Recalling that \( \alpha \) is an index of the relative bargaining strength of the seller, we arrive at the similarly counterintuitive result that the buyer may actually become worse off with more relative bargaining power (that is, a smaller value of \( \alpha \)). Graphically, these observations can be seen from Figure 1.

5 Free Disposal of Bargaining Advantage

Suppose that there exists some “true” value of \( \alpha \in (0, 1) \), denoted \( \alpha_T \). \( \alpha_T \) can be thought of representing the relative bargaining advantage of \( S \) when each agent bargains to the best of his own ability. From the results thus far, exchange will successfully occur when \( \alpha_T \in [\underline{\alpha}, \overline{\alpha}] \). Equivalently, as stated in Theorem 1, exchange can be sustained so long as \( \alpha_T v_b + (1 - \alpha_T) v_s \in [\frac{v_s}{\lambda}, \tau v_b] \). In this section it is argued that exchange will always successfully occur whenever \( \frac{v_s}{\lambda} \leq \tau v_b \), so long as each agent can “freely dispose of his own relative bargaining advantage.”

It would be reasonable to suppose that an individual would be able to bargain at a level less than his full potential. When such action is undertaken by a single agent, the relative bargaining powers of each will change. If the seller chooses to bargain at less than his full potential (while the buyer bargains at his full potential), the relative bargaining power of the seller will decrease; if the buyer chooses to bargain at less than his full potential (while
the seller bargains at his full potential), the relative bargaining power of the seller will increase.

An agent should be willing to continually relinquish his own relative bargaining advantage if and only if doing so does not decrease his own payoff. Since \( \alpha \) is a relative index of bargaining power, each agent should be able to individually “veto” any incremental change in \( \alpha \). Let \( \alpha^* \) denote the value of \( \alpha \) that the buyer and seller mutually “choose” when each agent can continually surrender his own relative bargaining advantage, beginning at \( \alpha_T \). After the agents choose \( \alpha^* \), exchange will either occur or not occur according to the conditions previously determined.

Recall that the equilibrium expected net gains of each agent are illustrated in Figure 1. Note that:

\( \pi_s \) is strictly increasing in \( \alpha \) for \( \alpha \in (\alpha, \overline{\alpha}) \) and \( \pi_b \) is strictly decreasing in \( \alpha \) for \( \alpha \in (\underline{\alpha}, \overline{\alpha}) \). As a result, if \( \alpha_T \in [\underline{\alpha}, \overline{\alpha}] \): \( S \) would veto any decrease in \( \alpha \) and \( B \) would veto any increase in \( \alpha \). Therefore, for \( \alpha_T \in [\underline{\alpha}, \overline{\alpha}] \), we would expect \( \alpha^* = \alpha_T \). When \( \alpha^* \in [\underline{\alpha}, \overline{\alpha}] \) exchange will successfully occur.

Now suppose \( \alpha_T < \underline{\alpha} \). \( B \) would be willing to relinquish his relative bargaining advantage up to \( \alpha = \underline{\alpha} \). \( S \) will not object to any incremental change over this range, since \( \pi_s = 0 \) for every \( \alpha \leq \underline{\alpha} \). Thus, when \( \alpha_T < \underline{\alpha} \) we would realize \( \alpha^* = \underline{\alpha} \), which again results in successful exchange.

Finally consider \( \alpha_T > \overline{\alpha} \). \( S \) is willing to relinquish his relative bargaining advantage down to \( \alpha = \overline{\alpha} \). \( B \) will not object to any incremental change in this interval, since \( \pi_s = 0 \) for every \( \alpha \geq \overline{\alpha} \). Thus, when \( \alpha_T > \overline{\alpha} \) the agents will choose \( \alpha^* = \overline{\alpha} \), which also results in exchange successfully occurring.

Note that in any case, the degree to which an agent is willing to relinquish his relative bargaining advantage does not depend upon his true type. As a result, an agent never reveals any information about his type by his “choice” of \( \alpha \). Further, because of the stationary nature of the framework, the agents will choose the same value of \( \alpha^* \) in each period. The following proposition specifies \( \alpha^* \) as a function of \( \alpha_T \) when \( \frac{v_s}{\lambda} \leq \tau v_b \).

**Proposition 3** Suppose \( \frac{v_s}{\lambda} \leq \tau v_b \). If \( \alpha_T \in [\underline{\alpha}, \overline{\alpha}] \), then \( \alpha^* = \alpha_T \); if \( \alpha_T < \underline{\alpha} \), then \( \alpha^* = \underline{\alpha} \); if \( \alpha_T > \overline{\alpha} \), then \( \alpha^* = \overline{\alpha} \).

**Proof of Proposition 3.** This result follows immediately from the preceding discussion. \( Q.E.D. \)

The following theorem follows from the discussion thus far.

**Theorem 2** Suppose each agent can freely dispose of his own relative bargaining advantage. If \( \frac{v_s}{\lambda} > \tau v_b \) exchange will be completely unsuccessful.
during every period. If \( \frac{v_s}{\lambda} \leq \tau v_b \) exchange will be completely successful during the first \( n = \min \{ n_s, n_b \} \) periods.

**Proof of Theorem 2.** If \( \frac{v_s}{\lambda} > \tau v_b \) there is no price for which \( B \) and \( S \) would both honor the agreement. Thus, even when each agent can freely dispose of his own relative bargaining advantage, exchange cannot be sustained.

It already has been argued that when \( \frac{v_s}{\lambda} \leq \tau v_b \), \( \alpha^* \in [\lambda, \pi] \). As a result, exchange is completely successful during each of the first \( n = \min \{ n_s, n_b \} \) periods. Q.E.D.

Thus (supposing \( \frac{v_s}{\lambda} \leq \tau v_b \)), whenever \( \alpha_T \) is either “too large” or “too small” for exchange to occur, one agent will relinquish enough of his own relative bargaining advantage so that exchange will successfully occur.

If \( \alpha_T < \lambda \), the buyer will relinquish a portion of his relative bargaining advantage and as a result agree to pay a higher price on each unit than he could otherwise bargain for. The motivation for doing so is that at this higher price the seller now has just enough of an incentive to honor the agreement during Stage 2 of any period for which his valuation is \( v_s \). After relinquishing a portion of his relative bargaining power in this fashion, the equilibrium expected net gains for \( B \) will have increased from zero up to \( \pi_b^* = (\tau - \lambda) v_b - (1 - \lambda) v_s = \tau v_b - \frac{v_s}{\lambda} \) (which is non-negative when \( \tau v_b \geq \frac{v_s}{\lambda} \)).

Similarly, if \( \alpha_T > \pi \), the seller will relinquish a portion of his relative bargaining advantage and as a result agree to accept a lower price on each unit than he could otherwise bargain for. The seller will agree to this lower price so that the buyer has just enough incentive to honor the agreement during Stage 1 of any period for which his valuation is \( v_b \). As a result of agreeing to this lower price the equilibrium expected net gains for \( S \) will have increased from zero up to \( \pi_s^* = \lambda \pi (v_b - v_s) - (1 - \lambda) v_s = \lambda \tau v_b - v_s \) (which is non-negative when \( \tau v_b \geq \frac{v_s}{\lambda} \)).

For the sake of comparison, consider a traditional atomic transaction in which exchange is simultaneous. In such an environment, the parties again begin by agreeing upon a price \( p = \alpha_T v_b + (1 - \alpha_T) v_s \). Each agent must still choose whether or not to honor the agreement. If both honor the agreement, exchange occurs between the two parties at the agreed upon price \( p \); if either chooses not to honor the agreement, no transaction occurs and each party realizes a payoff of zero. In contrast to the case of sequential exchange, the buyer is no longer exposed to the possibility that the seller will choose not to deliver an item which has been paid for. For any \( p \in (v_s, v_b) \): \( B \) has a weakly dominant strategy of honoring the agreement so long as his valuation is \( v_b \) and \( S \) has a weakly dominant strategy of honoring the agreement so
long as his valuation is $v_s$ (further, it is weakly dominant for $B$ to not honor the agreement when his valuation is zero and it is weakly dominant for $S$ to not honor the agreement when his valuation is $C$). Note that when the valuation of $B$ is $v_b$ and the valuation of $S$ is $v_s$, exchange will successfully take place for any $p \in (v_s, v_b)$. As a result, neither agent ever has anything to gain by relinquishing any portion of his own relative bargaining advantage. This directly follows from the observations that in this case: the per period payoff of the seller is strictly increasing in $p$ over the entire range from $p = v_b$ up to $p = v_s$, while the per period payoff of the buyer is strictly decreasing in $p$ over the entire range from $p = v_b$ up to $p = v_s$.

6 Conclusion

The issue of whether or not exchange is sustainable for non-simultaneous transactions in which the buyer is exposed to the possibility that the seller will not deliver the item has been addressed. Repeated interaction of this nature between a single buyer and a single seller has been considered. It is assumed that each agent has incomplete information about the true type of his potential trading partner. Thus, neither party knows precisely when the interaction will no longer be beneficial.

During each period the interaction between the buyer and seller occurs in three distinct stages as follows: the agents bargain with each other in order to determine a mutually beneficial selling price; the buyer chooses whether to pay the seller or not; the seller chooses whether to deliver the item or not. Conditions under which successful exchange can be sustained have been specified. Further, it is shown that if an agent is able to freely dispose of a portion of his own relative bargaining advantage, he may choose to do so. Specifically, in some instances the buyer may optimally agree to pay a higher per unit price than he could otherwise bargain for, while in other instances the seller may agree to accept a lower price than he could otherwise bargain for. In these cases, exchange will successfully occur when it otherwise would not have, as a direct result of this disposal of relative bargaining power. Thus, successful trade is more easily sustained when agents may freely dispose of their own relative bargaining advantage.
References:


FIGURE 1

Expected Net Gain as a Function of $\alpha$

\[
\pi_s|_{\alpha=\bar{\alpha}} = \lambda \left( \tau v_b - \frac{1}{\lambda} v_s \right)
\]

\[
\pi_b|_{\alpha=\bar{\alpha}} = \tau v_b - \frac{1}{\lambda} v_s
\]