Math 103 Section 2.3: Quadratic Functions and Parabolas

- Parabolas

- Quadratic equations and functions

- Graphs of quadratic functions

- Applications
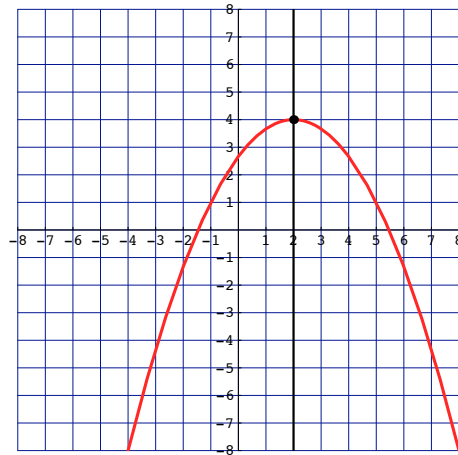
Quadratic Functions and Expressions

A quadratic function has two forms:

- $f(x) = ax^2 + bx + c$ (standard form)

- $f(x) = a(x - h)^2 + k$ (vertex-axis form)

The graph of a quadratic function is a parabola. It is easy to graph a quadratic function if it is expressed in the vertex-axis form.

Graphing a quadratic function
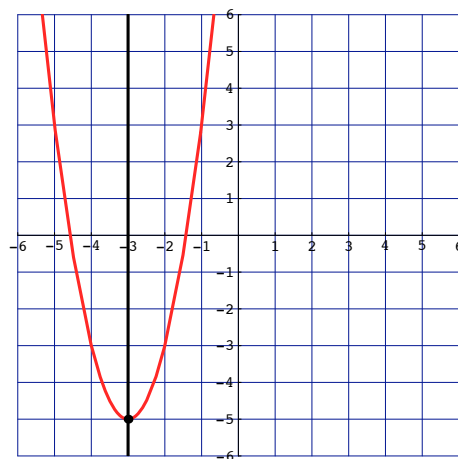
$$y = -\frac{1}{3}(x-2)^2 + 4$$

The vertex is the point at $(2, 4)$

The axis of symmetry is the vertical line $x = 2$

Graphing a quadratic function

$$y = 2(x+3)^2 - 5$$

The vertex is the point at

The axis of symmetry is the vertical line

Graphing a quadratic function
The general case

$$y = a(x - h)^2 + k$$

The vertex is the point at (h, k)
The axis of symmetry is the vertical line
If $a > 0$, the parabola opens upward.
If $a < 0$, the parabola opens downward.

Questions:

- Does the graph of $f(x) = 5(x - 1)^2 + 8$ open upward or downward?

- Does the graph of $f(x) = \frac{3}{7}(x + 2)^2 + 9$ open upward or downward?

- What is the equation for the axis of symmetry for the graph of $f(x) = 5(x - 1)^2 + 8$?

- What are the coordinates of the vertex of the graph of $f(x) = \frac{3}{7}(x + 2)^2 + 9$?

Completing the square

How to change the standard form for the function into the vertex-axis form.
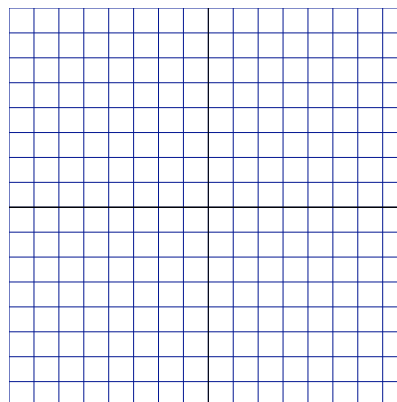
Example: $f(x) = x^2 - 6x + 10$

Change into vertex-axis form.

$$\begin{aligned} f(x) &= x^2 - 6x + 10 \\ &= (x^2 - 6x + \quad) + 10 - \\ &= (x - 3)^2 + \end{aligned}$$
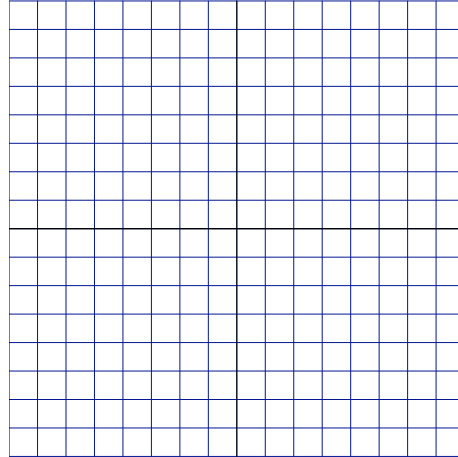
Vertex:

Opens:

Completing the square

Problem: Change to vertex-axis form by completing the square:
$f(x) = x^2 + 4x - 5$
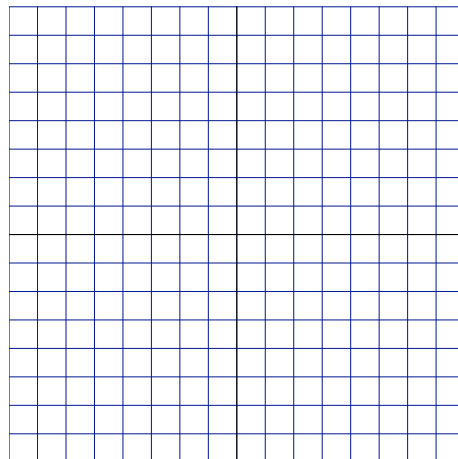
$f(x) \ =$

Vertex:

Opens:

Completing the square

Problem: Change to vertex-axis form by completing the square:
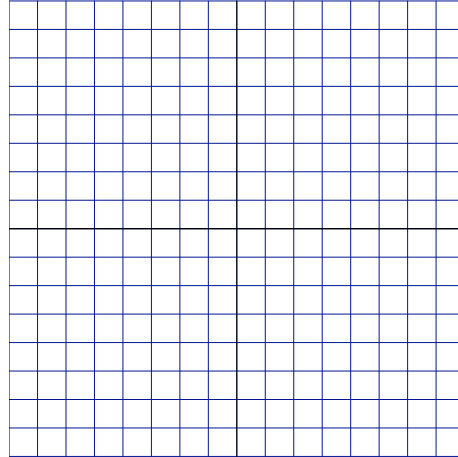$f(x) = -x^2 - 10x + 1$

$f(x) \ =$

Vertex:

Opens:

Completing the square

Problem: Change to vertex-axis form by completing the square:
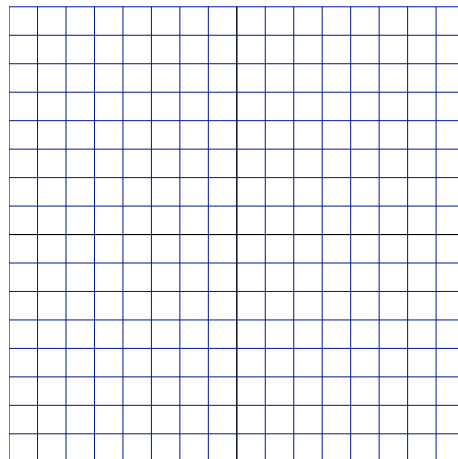$f(x) = 3x^2 + 6x + 1$

$f(x) =$

Vertex:

Opens:

Completing the square

Problem: Change to vertex-axis form by completing the square:
$f(x) = 5x^2 - 30x + 11$

$f(x) =$

Vertex:

Opens:

The quadratic formula

The general quadratic function:

$$f(x) = ax^2 + bx + c$$

The quadratic formula tells you the solutions to $f(x) = 0$, which is the same as locating the x-intercepts on the graph:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Example: Solve

$$2x^2 - 5x - 3 = 0,$$

for x.

$$a = 2, \quad b = -5, c = -3$$

$$x = \frac{5 \pm \sqrt{5^2 - 4(2)(-3)}}{2(2)} = \frac{5 \pm \sqrt{49}}{4} = \frac{5 \pm 7}{4}.$$

So $x = 3$ and $x = 1/2$ are the solutions.

The quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Example: Solve

$$x^2 - 5x - 5 = 0,$$

for x.

$$a = \qquad , \quad b = \qquad , c =$$