Review for Final Review

Topics

1. Functions and equations and graphing: linear, absolute value, quadratic, polynomials, rational (first 1/3 of semester)
2. Simple Interest, compounded interest, and continuously compounded interest
3. Limits & Continuity
4. Derivative of definition
5. Basic Derivatives: Linear, Quadratic, polynomials
6. Sum, product, quotient and chain rules
7. Finding Max/Min over intervals
8. **Marginal Revenue, Marginal Cost, Marginal Profit**
9. **Maximize Profit.**
10. **Elasticity of demand and relation to revenue**
11. Solving 2 equations in 2 unknowns: Substitution, elimination, Augmented matrices
12. Adding, subtracting, multiplying matrices.
13. Matrix applications

Lines and linear equations:

Standard	**$Ax + By = C$**
Slope-Intercept Form	**$y = mx + b$**
Point-slope form	**$y - y_1 = m (x - x_1)$**
Horizontal line	**$y = b$**
Vertical line	**$x = a$**

Find the equation of the line through (2,3) with slope -2/5.

Find the equation of the line through (2,3) and (3,-1).

Find the equation of the line through (2,3) and (1,3).

App.s of linear equations (PP Sec. 1)

Price-demand equation is d= 1720-.50p where d is the demand in hundreds.

We are interested in pairs of number d and p (written (d,p)) that satisfy this equation. Such a pair is called a *solution* to the equation.

What is the demand (in thousands) if the price is $1440?

What is the demand if the price is $2500?

If price increases $1 how does demand change?

What price should we charge if we want to sell 500 thousand TVs?

Solve for p.

Function (PP Sec. 1)

Edcon power company charges its residential customers \$14.00 per month plus \$0.10 per kilowatt-hour (KWH) of electricity used. Thus, the monthly cost for electricity is a function of the number of KWHs used. In symbols, let k be the number of KWHs used in a month, and E(k) be the monthly cost for electricity in dollars.

What are the units of measurement for k and for E(k)?

Write the symbolic form for the statement:
The monthly cost for using 800 kilowatt-hours of electricity is \$94.00.

Write the symbolic statement E(660)=80 in words.

Write formula for
E(k)=

What is the domain?

What is the range?

Practice.
Cost, price-demand, Revenue, Profit functions

x = number of units manufactured and sold=independent variable.

Cost Function	$C(x) = a + b\,x$
Price-Demand Function	$p(x) = m - n\,x$
Revenue Function	$R(x) = x\,p(x) = mx-nx^2$
Profit Function	$P(x) = R(x)-C(x)$

Example: x = number of units manufactured and sold
$C(x) = 2x+20$
$p(x)=-2x+16$
$R(x)$ =*Revenue function*=
$P(x)$=*Profit function*=
$R(0)=$
$P(0)=$
$R(3)=$
$P(3)=$
$R(10)=$
$P(10)=$

Revenue vs. Cost

Let C represent cost and R represent revenue. Then one of three things can occur:

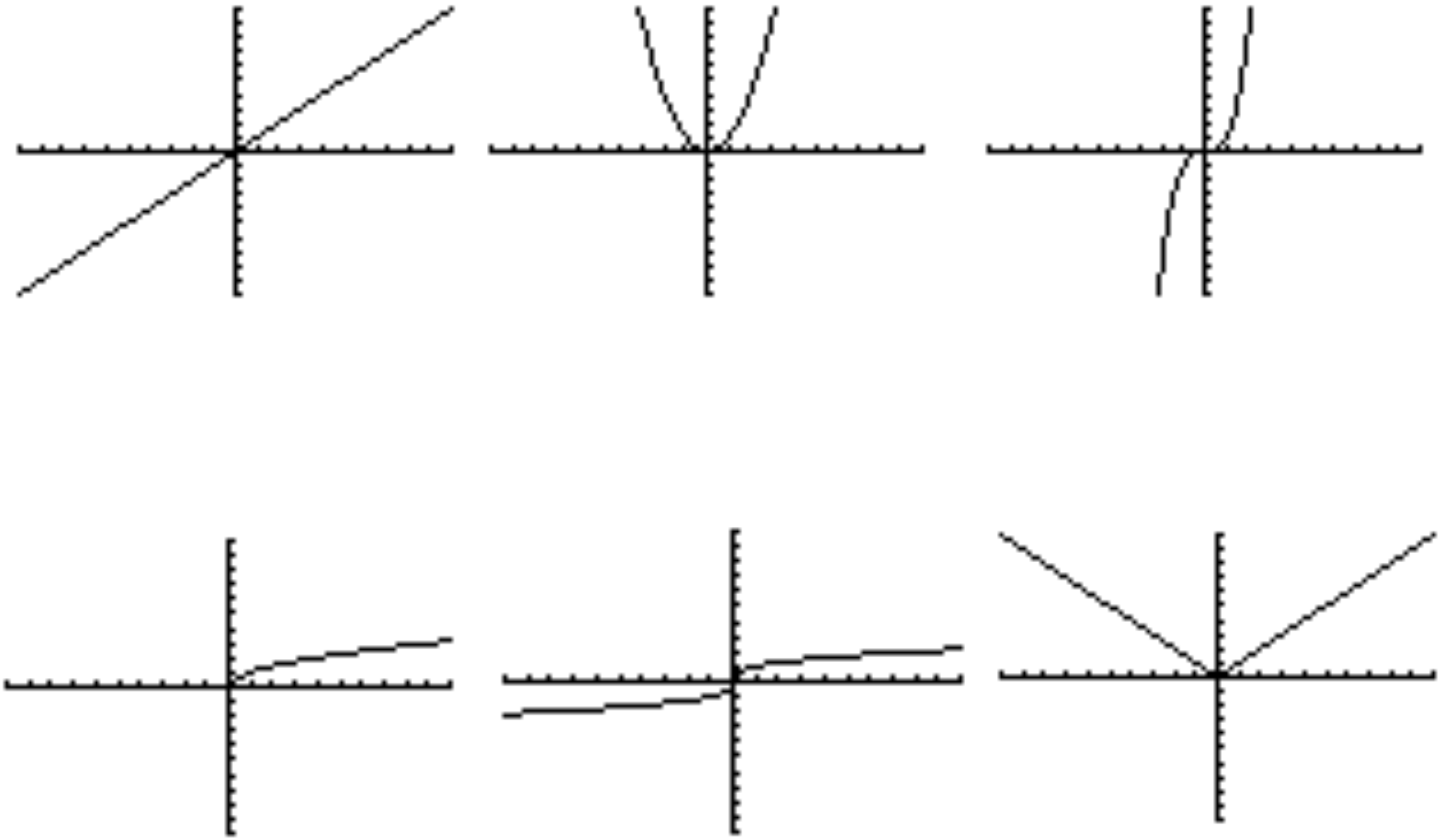
$R > C$ a profit

$R = C$ a break-even point, or

$R < C$ a loss.

Example: x = number of units manufactured and sold
$C(x) = 2x+20$
$p(x)=-2x+16$
$R(x) =-2x^2+16x$
When do we have a profit? A loss? What is the break-even point?

Basic Functions (PP Sec. 3)



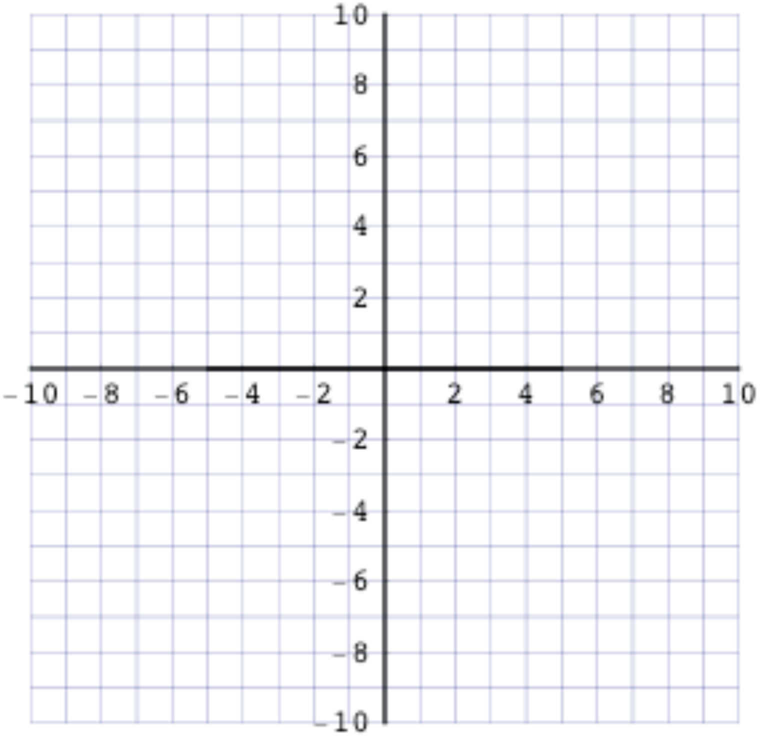
Combined Transformations

(PP Sec. 3)

Shifts and stretches:

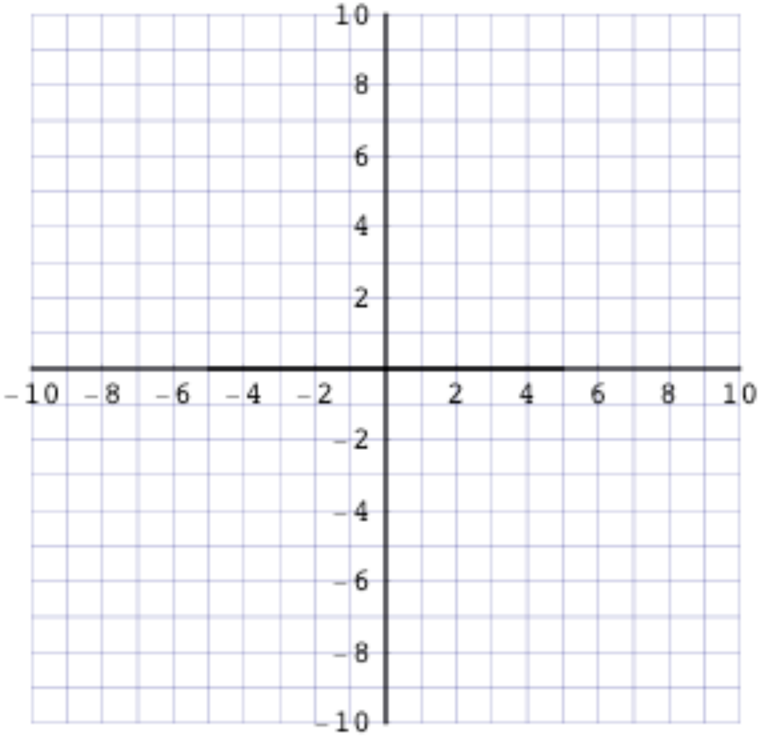
$$y = a\,f(x + h) + k$$

Graph $y = -3x^2 - 12x - 13$



Rational functions (PP Sec. 3)

Graph $y = 2x/(3x-6)$



Piecewise Functions (PP Sec. 3)

The Trussville Utiliities uses the rates shown in Table 2 to compute the monthly cost of natural gas for residential customers. Write a piecewise definition for the cost of consuming *x* CCF of natural gas and graph the function.

Cost per CCF	Per CCF for
$0.7	The first 50 CCF
$0.6	The next 150 CCF
$0.5	All CCF over 200CCF

Interest Problems (PP Sec. 4)

What amount will an account have after 10 years if \$15000 is invested at an annual rate of 4%

A) Compounded Weekly

$$A = P\left(1 + \frac{r}{m}\right)^{mt} =$$

B) Compounded Continuously

$$A = Pe^{rt} =$$

Interest Problems: Book p. 176 # 13-15, 19, 20

At what interest rate will we have to invest in order for an account that started with a principle of $15,000 to have $150000 after 20 years?

A) Compounded Weekly

B) Compounded Continuously

Basic Properties of Continuity:

A function f is continuous at a point x = c if

1. $\lim\limits_{x \to c} f(x)$ exists

2. $f(c)$ exists

3. $f(c) = \lim\limits_{x \to c} f(x)$

The following are all continuous on their domains:

Constant Function: $f(x) = c$

Absolute Value: $f(x) = |x|$

Positive integer power: $f(x) = x^n$

Roots: $f(x) = \sqrt[n]{x} = x^n$ (if n is even then for $x > 0$)

Polynomial: $f(x) = a_n x^n + \ldots + a_1 x + a_0$

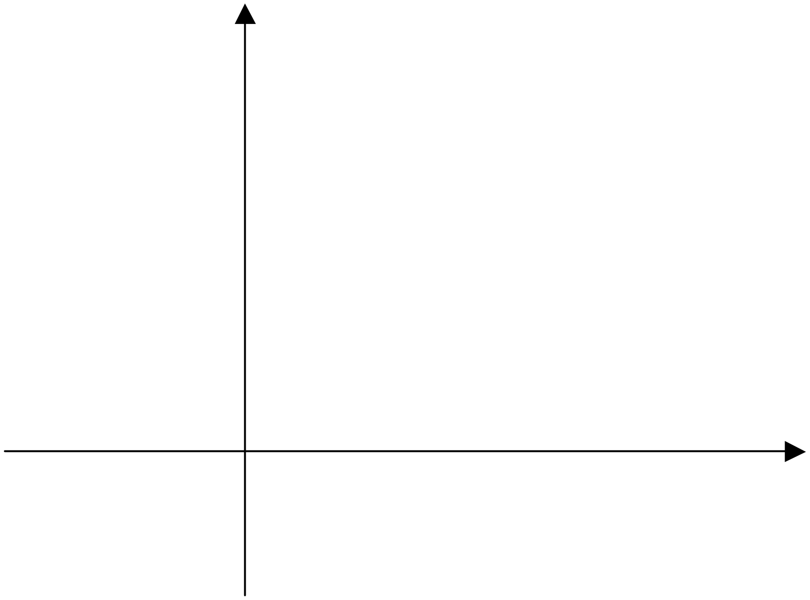
Rational Function: $f(x) = \dfrac{ax + b}{cx + d}$ away from $x = \dfrac{-d}{c}$

Exponential Function: $f(x) = b^x$

Logarithmic Function: $f(x) = \log_b x$ for $x > 0$

Limits and Continuity: p. 626 # 4-9

$$f(x)=\begin{cases} 3x\text{-}2 & \text{if } x<2 \\ 4 & \text{if } 2\le x\le 4 \\ -x+7 & \text{if } x>4 \end{cases}$$



$$\lim_{x\to 2^-} f(x) =$$

$$\lim_{x\to 2^+} f(x) =$$

$$\lim_{x\to 2} f(x) =$$

$$\lim_{x\to 4^-} f(x) =$$

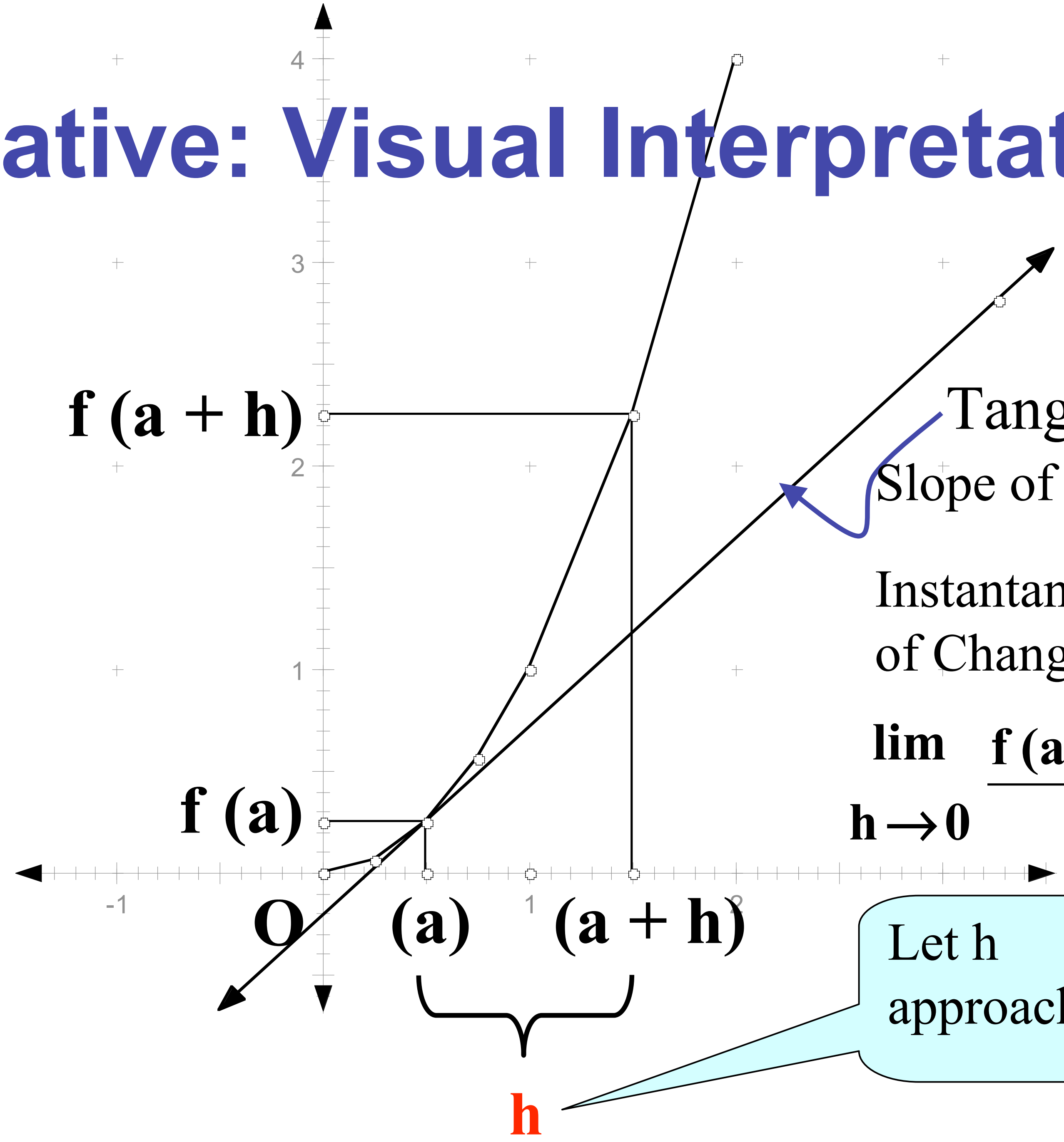
$$\lim_{x\to 4^+} f(x) =$$

$$\lim_{x\to 4} f(x) =$$

$$\lim_{x\to \infty} f(x) =$$

Where is f(x) continuous?
Where is f(x) differentiable?

Derivative: Visual Interpretation



Tangent

Slope of tangent.

Instantaneous Rate of Change.

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Let h approach 0

Example (PP Sec. 6)

Use the definition of the derivative to find *f'(x)* for $f(x) = 3x^2 + 5$

Derivative Rules.

If $f(x) = C$ then $f'(x) = 0$.

If $f(x) = x^n$ then $f'(x) = n\,x^{n-1}$.

If $f(x) = k \bullet u(x)$ then $f'(x) = k \bullet u'(x) = k \bullet u'$.

Sum Rule: If $f(x) = u(x) \pm v(x)$, then $f'(x) = u'(x) \pm v'(x)$.

Prod. Rule: If $f(x) = F(x) \bullet S(x)$, then $f'(x) = F(x) \bullet S'(x) + S(x) \bullet F'(x)$,

Quot. Rule: If $f(x) = T(x) / B(x)$, then $f'(x) = \dfrac{B(x) \cdot T'(x) - T(x) \cdot B'(x)}{[B(x)]^2}$

Chain Rule: If $h(x) = f \circ u(x) = f(u(x))$ then $h'(x) = \left(f'(u(x))\right)\left(u'(x)\right)$

Examples: (PP Sec. 5)

$$f\,(x) = \frac{2x-3}{\left(x-1\right)^2} + 3$$

$$f\,(x) = (3x^5 + 2x - 4)^{1/3}$$

Absolute Max and Min (PP Sec. 5)

On [*0,2*]ind the absolute maximum and minimum value of, $f(x) = x^3 - 6x^2 + 9x + 1$

Marginal Revenue etc…

If x is the number of units of a product sold in some time interval, then:

Total cost = $C(x)$

Marginal cost = $C'(x) \approx C(x+1) - C(x)$

= the additional cost incurred by producing the x+1st unit.

Total revenue = $R(x)$

Marginal revenue = $R'(x) \approx R(x+1) - R(x)$

= the additional revenue earned by producing the x+1st unit.

Total profit = $P(x) = R(x) - C(x)$

Marginal profit = $P'(x) = R'(x) - C'(x) \approx P(x+1) - P(x)$.=

Suppose that for a particular company profits are maximized at production level of 500 units per week. Explain to the sales staff why if production is currently at 450 units per week that you should continue to increase production.

Maximize Profit (PP Sect. 7)

An office supply company sells x shredders per year at \$$p$ per shredders.

The price demand equation for these shredders is $p = 9 - x$.

Suppose further that the total annual cost of manufactureing x shredders is

$C(x) = 1 + 3x$.

What is the company's maximum revenue?

What is the company's maximum profit?

What should the company charge for each shredders and how many shredders should be produced?

Graph revenue and cost together.

Price sensitivity of demand: Elasticity

The Elasticity of demand if $x = f(p)$ is

$$E(p) = -\frac{pf'(p)}{f(p)} = -\frac{\text{relative rate of change of demand}}{\text{relative rate of change of price}}$$

E(p)	Demand	Interpretation
0<E(p)<1	Inelastic	
E(p)>1	Elastic	
E(p)=1	Unit	

Elasticity (PP Sec. 7)

The Elasticity of demand if $x = f(p)$ is

$$E(p) = -\frac{pf'(p)}{f(p)} = -\frac{\text{relative rate of change of demand}}{\text{relative rate of change of price}}$$

$$\left(\text{relative of change of demand}\right) = -E(p)\left(\text{relative rate of change of price}\right)$$

Find E(p) when x=f(p) = 300(10-p).

E(8) = 8/(40-8)=8/32=1/4 =.25 <1.

At p=$8: a price increase of 5% will create a demand decrease of

E(5)

E(2)

Terminology: p. 267 # 3-14, 16,40

- A consistent linear system is one that has one or more solutions.
  - A) If a consistent system has exactly one solution then the system is said to be independent. E.g.
  - What can you say about the slopes of the lines in this case?
  - B) if a consistent system has more than one solution, then the system is said to be dependent. E.g.
  - What can you say about the slopes of the lines in this case?
- An inconsistent linear system is one that has no solutions.
  - E.g.
  - What can you say about the slopes of the lines in this case?

Supply and demand example:

- A company manufactures and sells q things per week. Suppose that when the price is \$36 the quantity supplied weekly is 1 thing and when the price is \$40 per thing the quantity supplied weekly is 2 things.
- Find the supply and demand equation in the form: $p=aq+b$.
- If the demand function is p=-10q+ 60. Find the equilibrium price and quantity.
- Find the augmented matrix that represents this system of equations you solved above.
- Remark: Other questions here could be: Graph the lines. Look also at problems 61-66 in 4.4 homework problems in text (I would use easier numbers).

Which is reduced row echelon form?

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 1 & 5 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 3 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 1 & 8 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 0 & 1 & 3 & 4 \\ 1 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 3 & 4 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

More Examples of scalar multiplication & addition:

$$\begin{bmatrix} -3 & 0 & 1 \\ 2 & 1 & 3 \\ 0 & 5 & -2 \end{bmatrix} + (-2)\begin{bmatrix} 1 & 4 & 2 \\ 0 & 4 & 1 \\ 2 & 5 & -1 \end{bmatrix} =$$

$$(3)\begin{bmatrix} -1 & 2 & 3 \\ 6 & -7 & 9 \\ 0 & -4 & 8 \end{bmatrix} + (-1)\begin{bmatrix} 1 & 4 & 2 \\ 0 & 4 & 1 \\ 2 & 5 & -1 \end{bmatrix} =$$

Matrix Multiplication:

$$\begin{bmatrix} 3 & 1 & -1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 6 & 1 \\ -5 & 2 \\ 4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 1 \\ -5 & 2 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 3 & 1 & -1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 3 & 1 & -1 \\ 2 & 0 & 3 \end{bmatrix}$$

Practical application(PP 8.7,8)

- A company with manufacturing plants located in Mass. And Virginia has labor-hour and wage requirements for the manufacture of three types of boats: What is MN?

$$\begin{array}{cccc} & \text{Cutting Dept.} & \text{Assemb. Dept.} & \text{Package. Dept.} \\ \text{1 man boat} & .6 & .6 & .2 \\ \text{2 man boat} & 1 & .9 & .3 \\ \text{3 man boat} & 1.5 & 1.2 & .4 \end{array} = M$$

$$N = \begin{array}{ccc} \text{MA} & \text{VA} & \\ 17 & 14 & \text{Cutting Dept.} \\ 12 & 10 & \text{Assemb. Dept.} \\ 10 & 9 & \text{Package. Dept.} \end{array}$$