4.4 Matrices: Basic Operations

- Addition and subtraction of matrices
- Product of a number k and a matrix M
- Matrix Product.

Addition and Subtraction of matrices

- To add or subtract matrices, they must be of the same size mxn.
- To add matrices of the same size, add their corresponding entries.

$$A+B=\left[a_{ij}+b_{ij}\right] \qquad \begin{pmatrix}1 & 2\\ 5 & 2\end{pmatrix}+\begin{pmatrix}4 & 5\\ 9 & 1\end{pmatrix}=$$

- To subtract matrices of the same order, subtract their corresponding entries. The general rule is as follows using mathematical notation:

$$A-B=\left[a_{ij}-b_{ij}\right] \qquad \begin{pmatrix}1 & 2\\ 5 & 2\end{pmatrix}-\begin{pmatrix}4 & 5\\ 9 & 1\end{pmatrix}=$$

More examples:

$$\begin{bmatrix} 4 & -3 & 1 \\ 0 & 5 & -2 \\ 5 & -6 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 3 \\ 6 & -7 & 9 \\ 0 & -4 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -3 & 1 \\ 0 & 5 & -2 \\ 5 & -6 & 0 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 3 \\ 6 & -7 & 9 \\ 0 & -4 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -3 & 1 \\ 0 & 5 & -2 \\ 5 & -6 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 5 \\ 3 & 7 \\ 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 6 & 3 \end{bmatrix} - \begin{bmatrix} -2 & 3 & 1 \end{bmatrix}$$

Scalar Multiplication

- The **scalar product** of a number *k* and a matrix **A** is the matrix denoted by *k***A,** obtained by multiplying each entry of **A** by the number *k* .
- The number *k* is called a **scalar.**

$$kA = \left[ka_{ij} \right]$$

- **Example:**

$$(-1)\begin{bmatrix} -1 & 2 & 3 \\ 6 & -7 & 9 \\ 0 & -4 & 8 \end{bmatrix}$$

More Examples of scalar multiplication & addition:

$$\begin{bmatrix} -1 & 2 & 3 \\ 6 & -7 & 9 \\ 0 & -4 & 8 \end{bmatrix} + 3\begin{bmatrix} 1 & 4 & 2 \\ 0 & 4 & 1 \\ 2 & 5 & -1 \end{bmatrix} =$$

$$(2)\begin{bmatrix} -1 & 2 & 3 \\ 6 & -7 & 9 \\ 0 & -4 & 8 \end{bmatrix} + (0)\begin{bmatrix} 1 & 4 & 2 \\ 0 & 4 & 1 \\ 2 & 5 & -1 \end{bmatrix} =$$

Alternate definition of subtraction of matrices:

- The definition of subtract of two real numbers *a* and *b* is

a – b = a + (-1)b

i.e. *a* plus negative *b*.

- We can define subtraction of matrices similarly:

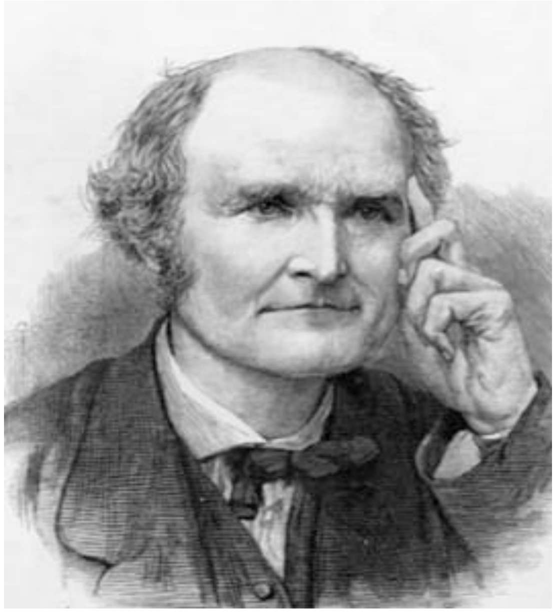
If A and B are two matrices of the same dimensions, then

A – B = A + (-1)B,

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + (-1)\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} =$$

Matrix product

- The method of multiplication of matrices is not as intuitive and may seem strange, although this method is extremely useful in many mathematical applications.
- Matrix multiplication was introduced by an English mathematician named Arthur Cayley (1821-1895) .
- We will see shortly how matrix multiplication can be used to solve systems of linear equations.



Row by column multiplication

- 1X4 row matrix multiplied by a 4X1 column matrix: Notice the manner in which corresponding entries of each matrix are multiplied:

$$\begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix}\begin{pmatrix} 5 \\ 6 \\ 7 \\ 8 \end{pmatrix} = 1 \cdot 5 + 2 \cdot 6 + 3 \cdot 7 + 4 \cdot 8 = 70$$

$$\begin{pmatrix} 4 & 2 & 3 & 1 \end{pmatrix}\begin{pmatrix} 6 \\ 3 \\ 5 \\ 2 \end{pmatrix} =$$

Row by column multiplication

1Xn row matrix multiplied by a *nX1* column matrix:

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \end{pmatrix} \begin{pmatrix} b_{11} \\ b_{21} \\ \vdots \\ b_{n1} \end{pmatrix} =$$

Revenue of a car dealer

- A car dealer sells four model types: A,B,C,D. On a given week, this dealer sold 10 cars of model A, 5 of model B, 8 of model C and 3 of model D. The selling prices of each automobile are respectively \$12,500, \$11,800, \$15,900 and \$25,300. Represent the data using matrices and use matrix multiplication to find the total revenue.

Solution using matrix multiplication

- We represent the number of each model sold using a row matrix (4X1) and we use a 1X4 column matrix to represent the sales price of each model. When a 4X1 matrix is multiplied by a 1X4 matrix, the result is a 1X1 matrix of a single number.

$$\begin{bmatrix} 10 & 5 & 8 & 3 \end{bmatrix} \begin{bmatrix} 12,500 \\ 11,800 \\ 15,900 \\ 25,300 \end{bmatrix} = \begin{bmatrix} 10(12,500) + 5(11,800) + 8(15,900) + 3(25,300) \end{bmatrix} = \begin{bmatrix} 387,100 \end{bmatrix}$$

Matrix Product

- If **A** is an *m x p* matrix and **B** is a *p x n* matrix, the **matrix product** of *A* and *B* denoted by ***AB*** is an *m x n* matrix whose element in the *ith* row and *jth* column is the real number obtained from the product of the *Ith* row of **A** and the *jth* column of **B.**
- If the number of columns of **A** does **not equal** the number of rows of **B**, the matrix product **AB** is **not defined.**

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{pmatrix} = \begin{pmatrix} 70 & 80 & 90 \\ 158 & 184 & 210 \end{pmatrix}$$

Undefined matrix multiplication

Why is this matrix multiplication not defined?

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{pmatrix} \quad \text{is not defined}$$

More examples:

$$\begin{bmatrix} 3 & 1 & -1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 3 & -5 \\ -2 & 4 \end{bmatrix}$$

Is Matrix Multiplication Commutative?

$$\begin{bmatrix} 1 & 6 \\ 3 & -5 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 & -1 \\ 2 & 0 & 3 \end{bmatrix}$$

Practical application

- Suppose you a business owner and sell clothing. The following represents the number of items sold and the cost for each item: Use matrix operations to determine the total revenue over the two days:
- Monday: 3 T-shirts at $10 each, 4 hats at $15 each, and 1 pair of shorts at $20.
  Tuesday: 4 T-shirts at $10 each, 2 hats at $15 each, and 3 pairs of shorts at $20.

Solution of practical application

- Represent the information using two matrices: The product of the two matrices give the total revenue:

Unit price of each item:

Qty sold of each item on Monday

Qty sold of each item on Tuesday

$$\begin{bmatrix}10 & 15 & 20\end{bmatrix}\begin{bmatrix}3 & 4\\ 4 & 2\\ 1 & 3\end{bmatrix}$$

- Then your total revenue for the two days is =[110   130]
Price Quantity=Revenue

Practical application

- A company with manufacturing plants located in Mass. And Virginia has labor-hour and wage requirements for the manufacture of three types of boats: What is MN?

$$\begin{array}{c} \\ \text{1 man boat} \\ \text{2 man boat} \\ \text{3 man boat} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Cutting Dept.} & \text{Assemb. Dept.} & \text{Package. Dept.} \end{array} \\ \begin{pmatrix} .6 & .6 & .2 \\ 1 & .9 & .3 \\ 1.5 & 1.2 & .4 \end{pmatrix} \end{array} = M$$

$$N = \begin{array}{c} \begin{array}{cc} \text{MA} & \text{VA} \end{array} \\ \begin{pmatrix} 17 & 14 \\ 12 & 10 \\ 10 & 9 \end{pmatrix} \end{array} \begin{array}{c} \\ \text{Cutting Dept.} \\ \text{Assemb. Dept.} \\ \text{Package. Dept.} \end{array}$$