4.3 Gauss Jordan Elimination

- Any linear system must have exactly one solution, no solution, or an infinite number of solutions.
- Just as in the 2X2 case:
  - the term **consistent and independent** is used to describe a system with a **unique solution**,
  - **consistent and dependent** is used for a system with an infinite number of solutions.
  - **inconsistent** is used to describe a system with no solution,

Matrix representations of consistent, inconsistent and dependent systems

- The following matrices represent systems of three linear equations in three unknowns illustrate the three different cases:
- Case I : consistent $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{array}\right)$
- Case 2: Inconsistent case: $\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right)$
- Case 3: Dependent system $\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

Reduced row echelon form

A matrix is said to be in **reduced row echelon form** or, more simply, in **reduced form**, if :

1. Each row consisting entirely of zeros is below any row having at least one non-zero element.
2. The leftmost nonzero element in each row is 1.
3. All other elements in the column containing the leftmost 1 of a given row are zeros.
4. The leftmost 1 in any row is to the right of the leftmost 1 in the row above.

Which is i reduced row echelon form?

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 3 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 & 8 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Solving a system using Gauss-Jordan Elimination

Solve:

$x + y - z = -2$

$2x - y + z = 5$

$-x + 2y + 2z = 1$

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Karl Frederick Gauss:

- At the age of seven, **Carl Friedrich Gauss** started elementary school, and his potential was noticed almost immediately. His teacher, Büttner, and his assistant, Martin Bartels, were amazed when Gauss summed the integers from 1 to 100 instantly by spotting that the sum was 50 pairs of numbers each pair summing to 101.

Example 2

$3x - 4y + 4z = 7$

$x - y - 2z = 2$

$2x - 3y + 6z = 5$

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Representation of a solution of a dependent system

$$\left(\begin{array}{ccc|c} 1 & -1 & -2 & 2 \\ 0 & -1 & 10 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Example 3

A company that rents small moving trucks wants to purchase 16 trucks with a combined capacity of 19,200 cubic feet. Three different types of trucks are available: a cargo van with a capacity of 300 cubic feet, a 15-foot truck with a capacity of 900 cubic feet, and a 24-foot truck with a capacity of 1,500 cubic feet. How many of each type of truck should the company purchase?