# 4.2 Systems of Linear equations and Augmented Matrices

1. It is impractical to solve more complicated linear systems by hand.
2. Computers and calculators now have built in routines to solve larger and more complex systems. Matrices, in conjunction with graphing utilities and or computers are used for solving more complex systems.
3. In this section, we will develop certain matrix methods for solving two by two systems.

# Matrices

- A <u>matrix</u> is a rectangular array of numbers written within brackets.
- Example with three rows and three columns:
- The subscripts give the "address" of each entry of the matrix. For example the entry $a_{23}$ is found in the second row and third column
- Find $a_{21}$ in the following matrices if it exists.
- The size of a matrix is written *n x m* when the matrix has *n* rows and *m* columns.
- What sizes are these matrices?

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 \\ 5 & 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix}, \begin{pmatrix} 4 & 3 & 7 & 23 \end{pmatrix}, \begin{pmatrix} 4 & 8 \\ 2 & 33 \\ 6 & 5 \\ 7 & 3 \\ 2 & 1 \end{pmatrix}$$

# Matrix solutions of linear systems

- Can represent a linear system of equations using an augmented matrix: a matrix which stores the coefficients and constants of the linear system
- Can manipulate the augmented matrix to obtain the solution of the system.
- Example:

Linear system:

x +3y=5

2x – y=3

Associated augmented matrix:

$$\left[\begin{array}{cc|c} 1 & 3 & 5 \\ 2 & -1 & 3 \end{array}\right]$$

- In general:

$$a_{11}x_1 + a_{12}y_1 = k_1$$
$$a_{21}x_1 + a_{22}x_2 = k_2$$

$$\left[\begin{array}{cc|c} a_{11} & a_{12} & k_1 \\ a_{21} & a_{22} & k_2 \end{array}\right]$$

# Operations that Produce Row-Equivalent Matrices:

- 1. Two rows are interchanged ~~ change order of equations

$$R_i \leftrightarrow R_j$$

- 2. A row is multiplied by a nonzero constant ~~ multiply both sides of an equation by a constant.

$$kR_i \rightarrow R_i$$

- 3. A constant multiple of one row is added to another row ~~ replace one equation by the sum of it with another.

$$kR_j + R_i \rightarrow R_i$$

# Solve using Augmented matrix:

Solve

x +3y=5

2x – y=3

1. Augmented system
2. Eliminate 2 in 2nd row by row operation
3. Divide row two by -7 to obtain a coefficient of 1.
4. Eliminate the 3 in first row, second position.
5. Read solution from matrix

- $$\left[\begin{array}{cc|c} 1 & 3 & 5 \\ 2 & -1 & 3 \end{array}\right]$$

$$-2R_1 + R_2 \rightarrow$$

$$\left[\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & -7 & -7 \end{array}\right]$$

$$R_2 / -7 \rightarrow R_2 \rightarrow$$

$$\left[\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & 1 & 1 \end{array}\right]$$

$$-3R_2 + R_1 \rightarrow R_1 \rightarrow$$

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array}\right] \rightarrow x = 2, y = 1; (2,1)$$

# Solving a system using augmented matrix methods

$x+2y=4$

$x+(1/2)y=4$

# Solving a system using augmented matrix methods

$10x - 2y = 6$

$-5x + y = -3$

# Another example

- Solve
$$5x - 2y = -7$$
$$y = \frac{5}{2}x + 1$$

- Rewrite second equation :

# Summary:

Given a system of equations ax+by=c and ex+gy = f, the possible forms of the "reduced" augmented matrix are:

$$\begin{pmatrix} 1 & 0 & m \\ 0 & 1 & n \end{pmatrix}$$

Form 1: Unique solution *x=m, y=n* (consistent and independent).

$$\begin{pmatrix} 1 & r & m \\ 0 & 0 & 0 \end{pmatrix}$$

Form 2: Infinitely many solutions x=m-ry, y (consistent and dependent)

$$\begin{pmatrix} 1 & r & m \\ 0 & 0 & n \end{pmatrix}$$

Form 3: No solution (inconsistent)