Three ways to compute future value

Simple interest \( A = P(1 + rt) \)

Compound interest \( A = P(1 + i)^n \)

Continuous compounded interest \( A = Pe^{rt} \)

These formulas can also be used to compute the present value required to attain a given future value.

**Example:** What present value \( P \) is required for a future value \( F \) of $4,000? Interest is compounded semiannually for 5 years at a rate of 8%.

Solve the equation for \( P \):

\[
4000 = P(1 + 0.08/2)^{10} \\
= P(1.48024) \\
P = 4000/1.48024 \\
= 2702.26
\]

**Summary:** A present value of $2702.26 is required for a future value of $4000 if interest is compounded semiannually for 5 years at a rate of 8%.
Solving the future/present value formula for time $t$

Example: Use the graph to solve the equation for the number of years $t$:

$$3000 = 1000e^{(.10)t}$$

![Graph showing future value and years]

Solving the future/present value formula for time $t$

Use logarithms graph to solve the equation for the number of years $t$:

$$3000 = 1000e^{(.10)t}$$

$$e^{(.10)t} = 3$$
$$\ln(e^{(.10)t}) = \ln(3)$$
$$(.10)t = \ln(3)$$
$$t = \ln(3)/.10$$
$$t = 10.98612$$

Summary: It takes 10.98612 years for a present value of $1000 to grow to a future value of $3000 at a rate of 10% compounded continuously.
Solving the future/present value formula for time $t$

Use logarithms graph to solve the equation for the number of years $t$:

$$5000 = 1200e^{(.08)t}$$

Solving the future/present value formula for time $t$

Use logarithms graph to solve the equation for the number of years $t$:

$$5000 = 1200e^{(.08)t}$$

$$2500 = 1000e^{(.09)t}$$

$$3600 = 1000e^{(.05)t}$$
Two facts about the natural logarithm, ln:

\[ \ln(e^x) = x \]  \hspace{1cm} (1)
\[ \ln(a^x) = x \ln(a) \]  \hspace{1cm} (2)

<table>
<thead>
<tr>
<th>Fact (1):</th>
<th>Fact (2):</th>
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</thead>
<tbody>
<tr>
<td>( \ln(e^{.03}t) = .03t )</td>
<td>( \ln((1.02)^n) = n \ln(1.02) )</td>
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<tr>
<td>( \ln(e^{.09}t) = .09t )</td>
<td>( \ln((1.10)^n) = n \ln(1.10) )</td>
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<tr>
<td>( \ln(e^{.06}t) = )</td>
<td>( \ln((1.045)^n) = n \ln(1.045) )</td>
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<tr>
<td>( \ln(e^{.10}t) = .10t )</td>
<td>( \ln((1.01)^n) = n \ln(1.01) )</td>
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How to use Fact (2):

Compound Interest Formula: \( A = P(1 + i)^n \)

**Problem:** Deposit $100 into an account earning 4.5% interest compounded annually. How many years will it take to have a future value of $200? Solve for \( n \):

\[ 200 = 100(1 + .045)^n. \]

\[ (1.045)^n = 2 \]
\[ \ln((1.045)^n) = \ln(2) \]
\[ n \ln(1.045) = \ln(2) \]
\[ n = \frac{\ln(2)}{\ln(1.045)} \]
\[ n = \approx 15.747302 \]

**Summary:** It takes 15.75 years to have a future value of $200 if a present value of $100 earns 4.5% interest compounded annually.


How to use Fact (2):

Compound Interest Formula: \( A = P(1 + i)^n \)

**Problem:** Deposit $100 into an account earning 9% interest compounded semiannually. How many years will it take to have a future value of $200? Solve for \( n \):

\[
200 = 100(1 + 0.045)^n.
\]

This is the same equation as in the previous slide. The answer is still \( n = 15.747302 \), but it must be interpreted differently. \( n \) is the number of compounding periods. \( n \) isn’t always the number of years. In this example, interest is compounded semiannually. So 15.747302 periods is \( 15.747302/2 = 7.873651 \) years.

**Summary:** It takes 7.87 years to have a future value of $200 if a present value of $100 earns 9% interest compounded semiannually.


**Example:** The present value is $200. Interest is compounded quarterly at a rate of 10%. How many years does it take for a future value of $500?

Compound interest formula: \( A = P(1 + i)^n \)

\( i = 0.10/4 = 0.025 \) Solve for \( n \):

\[
200(1.025)^n = 500
\]

\[
(1.025)^n = 2.5
\]

\[
\ln((1.025)^n) = \ln(2.5)
\]

\[
n \ln(1.025) = \ln(2.5)
\]

\[
n = \ln(2.5)/\ln(1.025)
\]

\[
n = 37.107890
\]

There are 37.108 periods. Each period is a quarter (of a year). So that’s 37.108/4 = 9.277 years.

**Summary:** It takes 9.277 years to have a future value of $500 if a present value of $200 earns 10% compounded quarterly.
Problem: The present value is $1200. Interest is compounded monthly at a rate of 8%. How many years does it take for a future value of $2000?

Problem: The present value is $1800. Interest is compounded quarterly at a rate of 12%. How many years does it take for a future value of $3200?