103 Section 3.1, 3.2: Math of Finance continued

- simple interest
- compound interest
- continuous compounded interest

Three ways to compute future value

Simple interest	$A = P(1 + rt)$
Compound interest	$A = P(1 + i)^n$
Continuous compounded interest	$A = Pe^{rt}$

Continuous compounded interest

Example: You deposit $1000 into an account earning 12% for 10 years. The table shows the future value for various compounding periods:

	periods per year	future value	$= \$$
1	yearly	$1000(1.12)^{10}$	$= \$3105.85$
2	semiannually	$1000(1.06)^{20}$	$= \$3207.14$
4	quarterly	$1000(1.03)^{40}$	$= \$3262.04$
12	monthly	$1000(1.01)^{120}$	$= \$3300.39$
365	daily	$1000(1 + .12/365)^{3650}$	$= \$3319.46$

Is there any limit to how large the future value can be?
Yes, the largest future value from more frequent compounding is $3320.12.

Continuous compounded interest

Formula for continuous compounded interest:

$$A = Pe^{rt}$$

r is the annual rate
t is the number of years.

For a rate of 12% for 10 years, the formula gives

$$1000e^{.12\times 10} = 1000e^{1.20} = 3320.12.$$

$$A = Pe^{rt}$$

Problem: You deposit \$12,000 into an account earning 10% compounded continuously. How much will you have at the end of 25 years?

Problem: You deposit \$650 into an account earning 4% compounded continuously. How much will you have at the end of 11 years?

Comparisons

Compound interest	$A = P(1+i)^n$
Continuous compounded interest	$A = Pe^{rt}$

Problem: You deposit \$800 into an account earning 9% interest for 6 years. What is the future value if
a. interest is compounded quarterly?
b. continuously?
c. Which is better?

a.

b.

Finding present value given future value

Example: How much money do you have to invest to have $100,000 in 10 years? The rate is 8% compounded continuously:

In this problem we use $A = Pe^{rt}$ with $r = .08$ and $t = 10$. We know $A = 100,000$ (future value) and want to know P (present value).

$$100,000 = Pe^{.08\times 10} = Pe^{.8} = P \times 2.225541$$

So

$$P = \frac{100,000}{2.225541} = 44932.90$$

Summary: To have a future value of $100,000 in 10 years, you must invest $44,932.90.

Finding present value given future value

Example: How much money do you have to invest to have $45,200 in 5 years? The rate is 6% compounded monthly:

In this problem we use $A = P(1 + i)^n$ with
$r =$
$t =$
$A =$ (future value)

Summary:

Finding present value given future value

Example: How much money do you have to invest to have \$5,100 in 12 years? The rate is 7.5% compounded quarterly:

In this problem we use $A = P(1 + i)^n$ with
$r =$
$t =$
$A =$ (future value)

Summary:

Finding present value given future value

Example: How much money do you have to invest to have \$140,000 in 15 years? The rate is 6% compounded continuously:

In this problem we use $A = Pe^{rt}$ with
$r =$
$t =$
$A =$ (future value)

Summary:

More comparisions

Problem: You are going to invest some money for two years. Bank A offers to give 5% interest for the first year and 15% interest for the second year. Bank B offers to give 10% interest compounded yearly.

Which is better?

More comparisions

Problem: You are going to invest some money for one year. Bank A offers to give 6.2% interest compounded annually. Bank B offers to give 6.1% interest compounded monthly.

Which is better?

The exponential curve

You deposit \$1.00 earning 8% interest compounded continuously for t years. So the amount $A(t)$ you have is a function of the time t the money earns interest.

$$A(t) = (1.00)e^{.08t} = e^{.08t}$$