WARM UP EXERCSE

Roots, zeros, and x-intercepts.

$f(x) = x^2 - 25$

$f(x) = x^2 + 25$

$f(x) = x^3 - 25x$

$f(x) = \text{polynomial}, f(a) = 0 \Rightarrow f(x) = (x - a)g(x)$

§ 2-3 Polynomials and Rational Functions

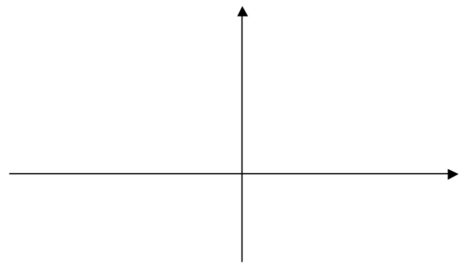
Students will learn about:

- Polynomial functions
  - Behavior & graphs
  - Root approximation
- Rational functions:
  - Behavior & graphs

Examples

$f(x) = (x-3)^3 + 2$

$=$



$f(x) = x^3 - 4x$

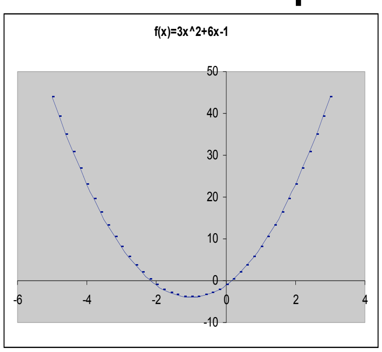
$f(x) = x^4 - 6x^2$

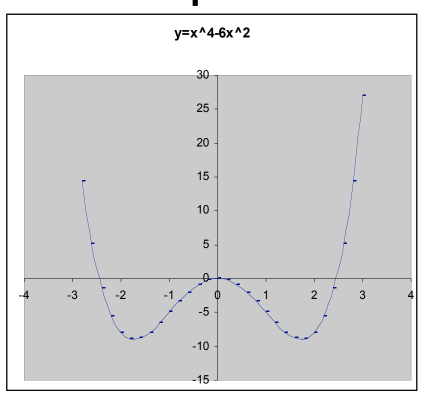
$f(x) = x^5 - 5x^3 + 4x + 1$

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x^1 + a_0$

Behavior as x gets big?
How many intercepts?
How many turning points?

Graphs of examples

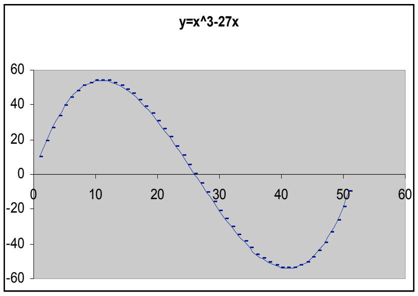


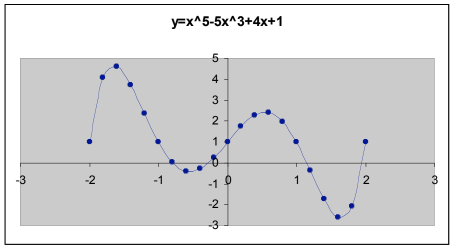


Number of intercepts?
Number of turning points?
Behavior as x gets big?
Behavior as x goes to negative infinity?

How do your answers change if we shift these left or right?
How do your answers change if we shift these up or down?

Graphs of examples





Number of intercepts?
Number of turning points?
Behavior as x gets big?
Behavior as x goes to negative infinity?

How do your answers change if we shift these left or right?
How do your answers change if we shift these up or down?

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Examples

x goes to infinity?

$f(x) = x^3 - 2$

$h(x) = (x)^2(x-1)$

$g(x) = (x-1)(x-2)(x-3)$

$j(x) = (x-1)(x^2+1)$

x gets to negative infinity?

$f(x) = x^3 - 2$

$h(x) = (x)^2(x-1)$

$g(x) = (x-1)(x-2)(x-3)$

$j(x) = (x-1)(x^2+1)$

How many intercepts?

$f(x) = x^3 - 2$

$h(x) = (x)^2(x-1)$

$g(x) = (x-1)(x-2)(x-3)$

$j(x) = (x-1)(x^2+1)$

How many turning points?

$f(x) = x^3 - 2$

$h(x) = (x)^2(x-1)$

$g(x) = (x-1)(x-2)(x-3)$

$j(x) = (x-1)(x^2+1)$

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In General

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x^1 + a_0$$

Degree n EVEN:
Behavior as x goes to infinity?
Behavior as x gets to negative infinity?

How many intercepts? Between 0 and n
How many turning points? Between 1 and $n-1$

Degree n Odd:
Behavior as x goes to infinity?
Behavior as x gets to negative infinity?

How many intercepts?
How many turning points?

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Rational Function Examples

Graph the following:

$$f(x) = \frac{1}{x}$$

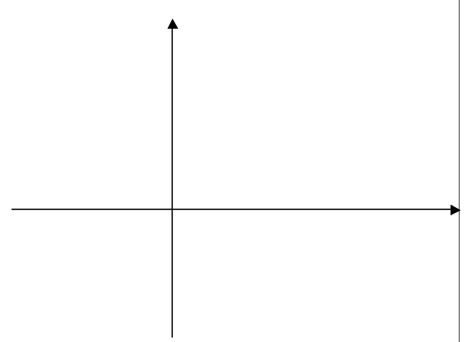
Domain
Range

$\lim_{x \to \infty} f(x) =$

$\lim_{x \to -\infty} f(x) =$

$\lim_{x^+ \to 2} f(x) =$

$\lim_{x^- \to 2} f(x) =$



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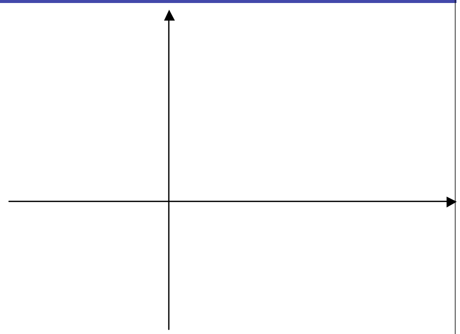
Rational Function Examples

Graph the following:

$g(x) = \frac{1}{x} + 3 =$



Domain

Range

$\lim_{x \to \infty} f(x) =$

$\lim_{x \to -\infty} f(x) =$

Remark: Limits to infinity in general:

$\lim_{x \to \infty} \frac{ax+b}{cx+b} = \lim_{x \to \infty} \frac{ax}{cx+b} + \frac{b}{cx+b} =$

How about as x approaches 0?

$g(1/10) =$

$g(1/100) =$

$g(-1/10) =$

$g(-1/100) =$

$\lim_{x^+ \to 0} f(x) =$ as x approaches 0 from the right $f(x)$ approaches _______

$\lim_{x^- \to 0} f(x) =$ as x approaches 0 from the left $f(x)$ approaches _______

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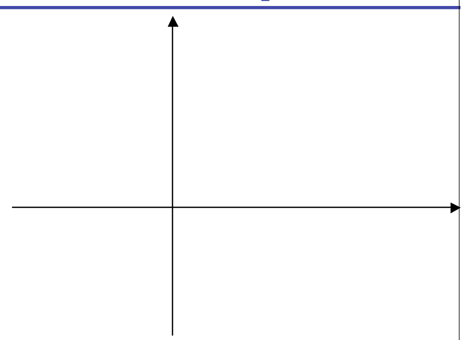
Rational Function Examples

Graph the following:

$f(x) = \frac{1}{x-2}$



Domain

Range

$\lim_{x \to \infty} f(x) =$

$\lim_{x \to -\infty} f(x) =$

How about as x approaches 2?

$f(1.9) =$

$f(1.99)$

$f(2.1) =$

$f(2.01)$

$\lim_{x^+ \to 2} f(x) =$

$\lim_{x^- \to 2} f(x) =$

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Rational Function Examples

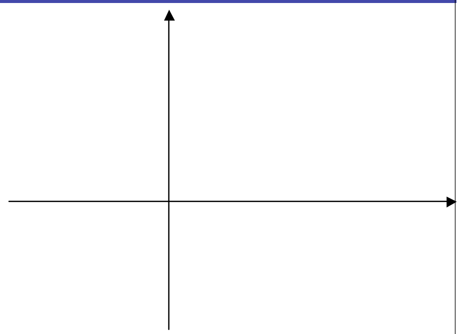
Graph the following:

$$g(x) = \frac{1}{2(x-1)} + 3 =$$

Domain
Range

$\lim_{x\to\infty} f(x) =$

$\lim_{x\to-\infty} f(x) =$



$\lim_{x^+\to} f(x) =$

$\lim_{x^-\to} f(x) =$

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Rational Functions

- Definition: A Rational function is a quotient of two polynomials, *P(x)* and *Q(x)*: *R(x)=P(x)/Q(x)*.

Example: Let $P(x) = x + 5$ and
$Q(x) = x - 2$ then

$$R(x)= \frac{x+5}{x-2}$$

Domain:

Range:
Zeros:

x-intercepts:
y-intercepts:

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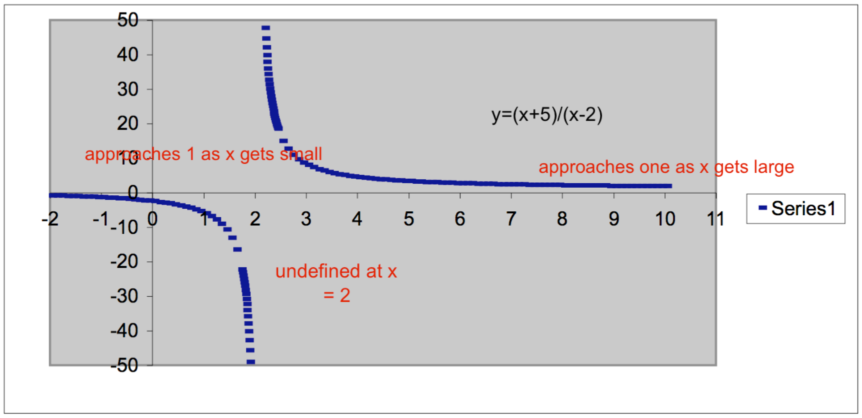
Graph of rational function



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Rational Functions

- Definition: A Rational function is a quotient of two polynomials, *P(x)* and *Q(x): R(x)=P(x)/Q(x). We will focus on R(x)=* $\frac{ax+b}{cx+d}$

Domain:
Don't want cx+d = 0. So…
All real numbers except x=-d/c.
The line x=-d/c is the vertical aspmptote
Range:

$\lim_{x\to -\infty} f(x) = a/c$

$\lim_{x\to \infty} f(x) = a/c$

y=a/c is the horizontal asymptote
Range: All real numbers except y=a/c.

Zeros: Want ax+b=0
so zero at x=-b/a (if a not zero)

x-intercepts: (-b/a,0)
y-intercepts: (0,b/d)

$\lim_{x^+ \to} f(x) =$

$\lim_{x^- \to} f(x) =$

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Example:

$$f(x)=\frac{3x+5}{x+1}$$

Domain:

Range:

$\lim_{x\to-\infty} f(x) =$

$\lim_{x\to\infty} f(x) =$

Zeros:

x-intercepts:

y-intercepts:

$\lim_{x^+\to} f(x) =$

$\lim_{x^-\to} f(x) =$

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