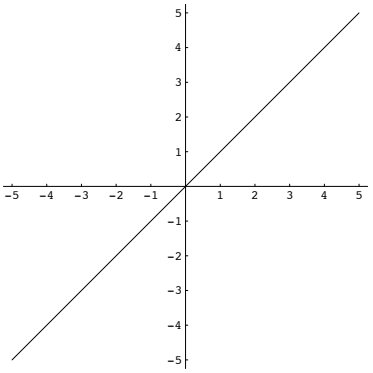
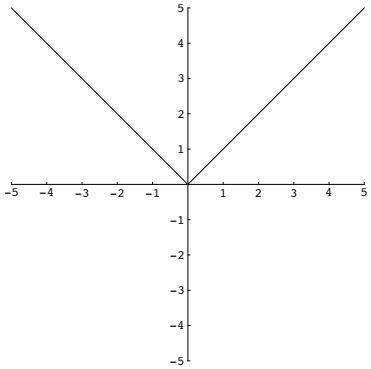
Math 103 Section 2.2: Elementary Functions and Transformations

1. A beginning library of elementary functions
2. Graphs of elementary functions
3. Shifts and stretches
4. Piecewise -defined functions

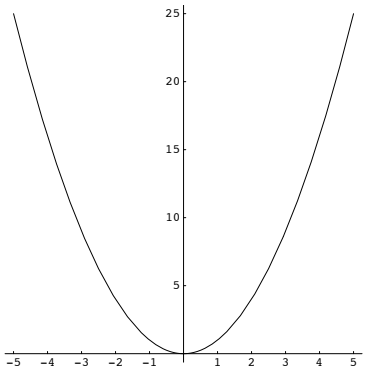
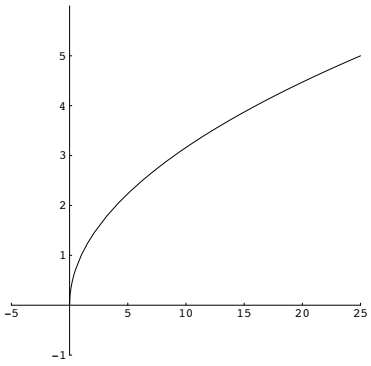
Beginning Library

- identity function $f(x) = x$
- absolute value function $f(x) = |x|$
- square function $f(x) = x^2$
- square-root function $f(x) = \sqrt{x}$
- piecewise defined functions

Identity and Absolute value functions

Identity function	Absolute-value function
Expression $f(x) = x$	Expression $f(x) = \|x\|$ or abs (x)
Domain: all numbers $(-\infty, \infty)$	Domain: all numbers $(-\infty, \infty)$
Range: all numbers $(-\infty, \infty)$	Range: $x \geq 0$, $[0, \infty)$
	

Square and Square-root functions

Square function	Square-root function
Expression: $f(x) = x^2$	Expression: $f(x) = \sqrt{x}$
Domain: all numbers $(-\infty, \infty)$	Domain: $x \geq 0$, $[0, \infty)$
Range: $x \geq 0$, $[0, \infty)$	Range: $x \geq 0$, $[0, \infty)$
	

Transformations

- vertical translations (shift)
  Business shifting up: Suppose x is the number of items you produce and $C(x)$ is the cost to produce x items. If your fixed costs (e.g. rent) increases by \$5, then the cost curve will shift up 5 units.

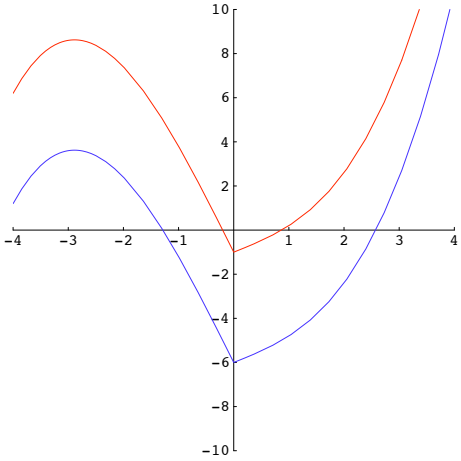
- vertical stretch
  Business stretching up: Suppose x is the number of items you produce and $C(x) = 10x$ is the cost to produce x items. If your variable costs to produce items (e.g. you are taxed on each unit produced) increases by \$2, then the cost curve will be stretched up by a factor of 2.

- horizontal translation (shift)
  Business shifting left: Suppose the units on the x-axis are years starting in 2000 (i.e. $x = 0$ is the year 2000) and $P(x)$ is your profits for the year. In updating your graphs for your manager you want to have the graph "start" with the year 2002. Then you want to ...

- reflections (lab sessions)

Vertical shift

Vertical shift 5 units up



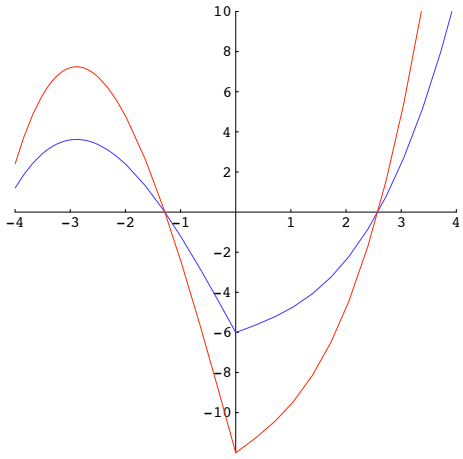
The graph of $f(x)$ is blue (dark line).

The graph of $f(x) + 5$ is red (light line).

The vertical distance between the curves is 5.

Vertical stretch

Vertical stretch by a factor of 2:



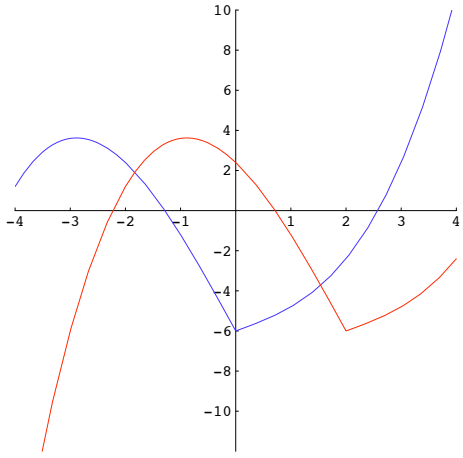
The graph of $f(x)$ is blue (dark line).

The graph of $2f(x)$ is red (light line).

The vertical distance from the x-axis of the graph of $2f(x)$ is twice that of $f(x)$.

Horizontal shift

Horizontal shift two units to the right



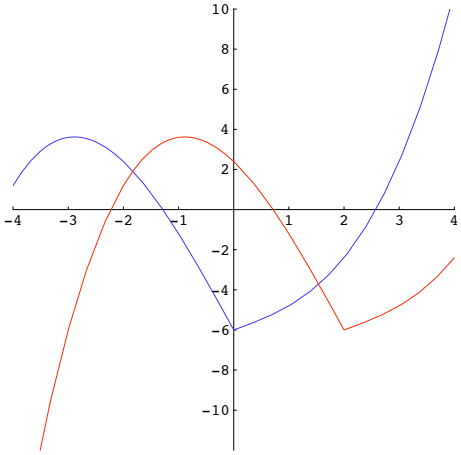
The graph of $f(x)$ is blue (dark).

The graph of $f(x-2)$ is red (light).

The horizontal distance between the curves is 2.

Horizontal shift

Horizontal shift two units to the left



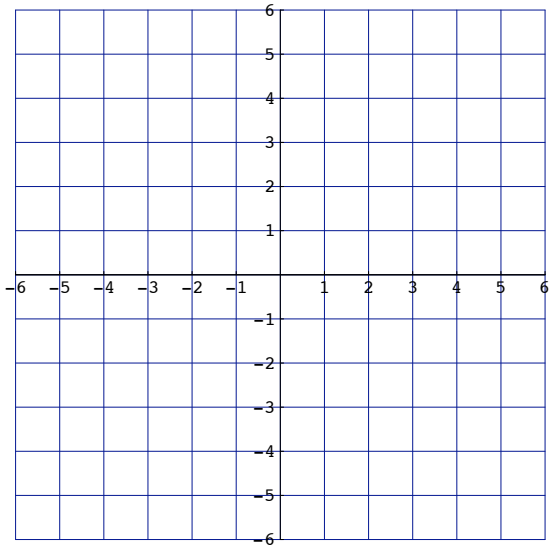
The graph of $h(x)$ is red (light).

The graph of $h(x+2)$ is blue (dark).

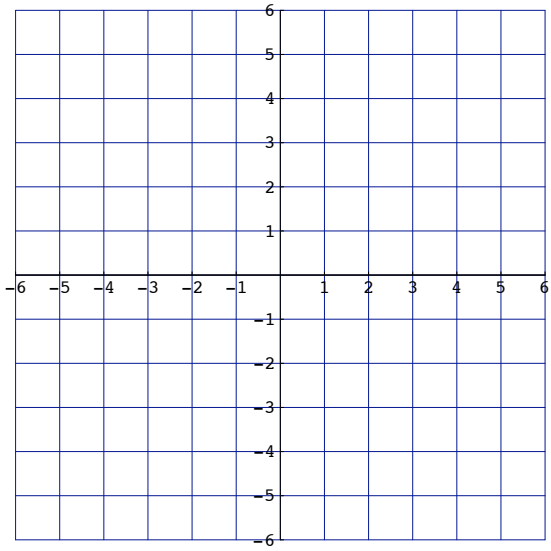
The horizontal distance between the curves is 2.

Practice: Graph these functions

$y = 2|x|$



$y = |x + 3|$



Practice: Each function corresponds to geometric description

$f(x-5)$	horizontal shift 5 units to the right
$f(x)+7$	
$3f(x)$	
$f(x-3)-1$	
	vertical shift 2 units up
	vertical shrink by a factor of $1/2$
	horizontal shift 4 units to left

Piecewise defined functions, an example

A car rental agency charges $30 per day (or partial day) or $150 per week, whichever is least. What is the rental cost $C(x)$ for x days?

Fill in the charges for the values of x:

x	1.0	2.0	2.6	3.0	3.1	4.0	4.2	5.0	6.0	7.0	7.1
C(x)											

Example from business continued:

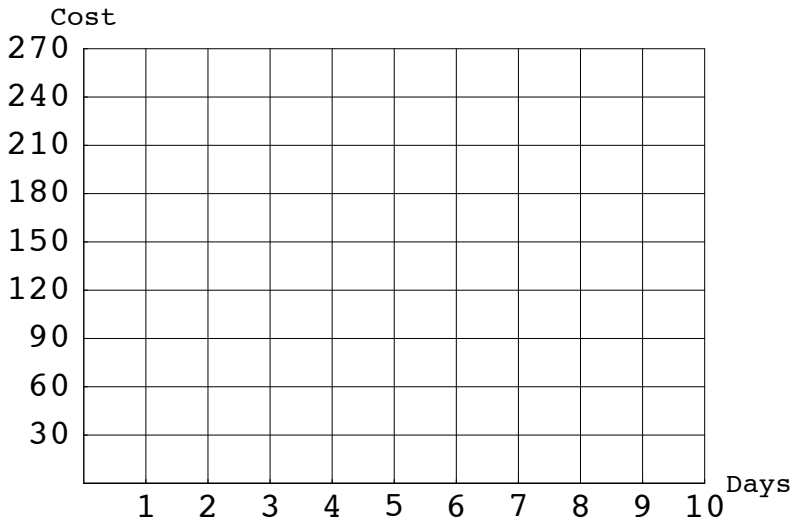
A car rental agency charges $30 per day (or partial day) or $150 per week, whichever is least. What is the rental cost $C(x)$ for x days?

Fill in the charges for the values of x:

x	1.0	2.0	2.6	3.0	3.1	4.0	4.2	5.0	6.0	7.0	7.1
C(x)	30	60	90	90	120	120	150	150	150	150	180

Example from business

A car rental agency charges \$30 per day (or partial day) or \$150 per week, whichever is least. Graph the cost function $C(x)$.



Example from business $T(x)$ is the tax on taxable income of x.

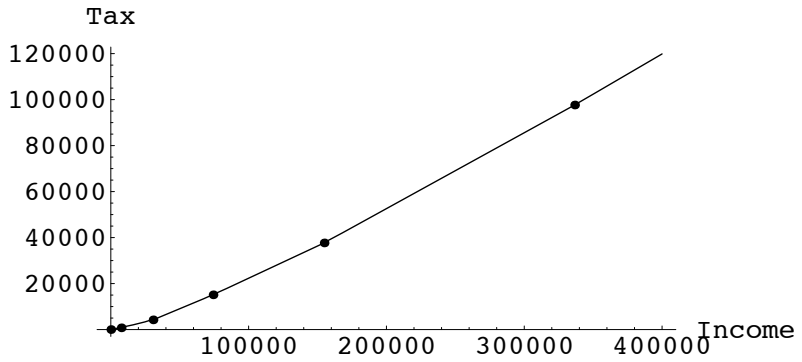
The federal income tax rate is

Between	But Not Over	Base Tax	Rate	Of the Amount Over
\$0	\$7,550	0	10%	\$0.00
\$7,550	\$30,650	\$755.00	15%	\$7,550
\$30,650	\$74,200	\$4,220.00	25%	\$30,650
\$74,200	\$154,800	\$15,107.50	28%	\$74,200
\$154,800	\$336,550	\$37,675.50	33%	\$154,800
\$336,550		\$97,653.00	35%	\$336,550

If you have a taxable income of $x = \$110,000$, your tax is

$$\begin{aligned} T(110,000) &= \text{Base Tax} + (\text{Rate} \times \text{Amount Over}) \\ &= 15,107.50 + [.28 \times (110,000 - 74,200)] \\ &= 15,107.50 + [.28 \times 35,800] \\ &= 15,107.50 + 10,024.00 \\ &= 25,131.50 \end{aligned}$$

The graph of $T(x)$:



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The equations for $T(x)$:

Between	But Not Over	Base Tax	Rate	Of the Amount Over
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$154,800	$336,550	$37,675.50	33%	$154,800
$336,550		$97,653.00	35%	$336,550

For income between $74,200 and $154,800:
Line 4 in the table.
$74200 \leq x \leq 154800$:

The equations for $T(x)$:

Between	But Not Over	Base Tax	Rate	Of the Amount Over
$0	$7,550	0	10%	$0.00
$7,550	$30,650	$755.00	15%	$7,550
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$154,800	$336,550	$37,675.50	33%	$154,800
$336,550		$97,653.00	35%	$336,550

For income between $30,650 and $74,200:
Line 3 in the table.
$30650 \leq x \leq 74200$: