# Introduction to Functions
# Section 2.1

- Notation
- Evaluation
- Solving
- Unit of measurement



## Introductory Example: Fill the gas tank

Your gas tank holds 12 gallons, but right now you're running on empty. As you pull into the gas station, the engine sputters and dies—the gas tank is completely empty. You pump 12 gallons into the tank and swipe your credit card. How much does it cost?

That depends on the price of gas, of course! But exactly how does it depend on the price?

If the price per gallon is \$3.40, what is the cost to fill the tank?

If the price per gallon is \$4.10, what is the cost to fill the tank?

# Fill the gas tank: function and symbolic form

The symbolic way to state exactly how the cost to fill the tank depends on the price per gallon is to write an algebraic expression for the cost in terms of the price. In our example:

$$C(p) = 12p.$$

Terminology (function): The cost, $C(p)$ to fill the tank is a **function** of the price $p$ per gallon.

The symbolic form of the statement "the cost of filling the tank at a price of \$3.40 per gallon is \$40.80," is $C(3.40) = 40.80$.

Our purpose here is to practice translating statements in words into symbolic form using function notation.



# Fill the gas tank

Here are some other examples from the gas tank situation:

Words: At a price of \$2.00 per gallon, it costs \$24.00 to fill the tank.
Write the symbolic form:

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Words: The cost to fill a 12-gallon tank at a price of $p$ dollars per gallon is 12 times $p$.
Write the symbolic form:

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Words: What is price per gallon if it costs \$45.00 to fill the tank?
Symbolic form: What is $p$ if $C(p) = 45.00$?
Solve $12p = 45.00$ for $p$.

## Evaluate a function

Evaluate the function $C(p) = 12p$ at $p = 3.50$.
Plug $p = 3.50$ into the expression $12p$.
Given the input $p = 3.50$, determine the output $C(p)$.

$C(3.50) = 12 \times 3.50 = \$42.00$

Evaluate the function $C(p) = 12p$ at $p = 2.00$

Evaluate the function $C(p) = 12p$ at $p = 3.00$



## Solve an equation

Given a cost $C(p)$, say $C(p) = \$45.00$, find the price $p$ at which the cost is \$45.00.
Which value of $p$ can you plug into the expression $12p$ so that $12p = 45.00$?
Given the output $C(p) = \$45.00$, what is the input $p$?

If we want to know what price per gallon results in a cost of \$45.00 to fill the tank, we must solve $12p = 45.00$ for $p$:

$$\begin{aligned} 12p &= 45.00 \\ p &= 45.00/12 \\ &= 3.75. \end{aligned}$$

**Summary:** If the cost to fill the tank is \$45.00, then the price per gallon is \$3.75.

## Units of measurement

Gas tank example:
the price of gas $p$ is measured in?
the cost to fill the tank $C(p)$ is measured in?
These are the **units of measurement** in this example.

## Goals:

1. use function notation to make statements about business situations
2. use functional notation to solve problems in business settings
3. summarize the results of our symbolic (algebraic) manipulations into statements in a business setting
4. use proper units of measurement for the functions and statements.



## Electricity costs

Edcon power company charges its residential customers \$14.00 per month plus \$0.10 per kilowatt-hour (KWH) of electricity used. Thus, the monthly cost for electricity is a function of the number of KWHs used. In symbols, let $k$ be the number of KWHs used in a month, and $E(k)$ be the monthly cost for electricity in dollars.

- What are the units of measurement for $k$ and for $E(k)$?
- Write the symbolic form for the statement: The monthly cost for using 800 kilowatt-hours of electricity is \$94.00.

## Electricity costs

Edcon power company charges its residential customers \$14.00 per month plus \$0.10 per kilowatt-hour (KWH) of electricity used. Thus, the monthly cost for electricity is a function of the number of KWHs used. In symbols, let $k$ be the number of KWHs used in a month, and $E(k)$ be the monthly cost for electricity in dollars.

- Write the symbolic statement $E(660) = 80$ in words.
- Write the symbolic form for the statement: The monthly cost for the 101st KWH is \$0.10.
- Write the symbolic form for the statement: The monthly cost for using $k$ KWHs is \$100.00.



Continuing with the electricity costs, we can give a formula for the electricity costs as a function of kilowatt-hours uses as follows:

$$E(k) = 14.00 + 0.10k. \tag{1}$$

The 14.00 in the formula is
This is called the **fixed cost**. The customer also pays \$0.10 per kilowatt-hour used. So $k$ KWHs would cost an additional \$0.10$k$. This is called the **variable cost**.

The above formula for $E(k)$ allows us to calculate the month cost for any number of KWHs used.

If the customer uses 200 KWHs, then $k = 200$ and the cost is

$$E(200) =$$

In words,

The monthly cost for using 200 KWHs is \$34.00.

## Use symbolic form to solve problems:

How many KWHs can be used if the monthly cost is \$55.00?
We do not know $k$, the number of KWHs used.
We do know the cost of using $k$ KWHs, $E(k) =$. So...

$$14.00 + 0.10k = 55.00.$$

Now the problem is to solve the above equation using algebra. The answer is $k = 410$.

| **Summary:** If the monthly cost is \$55.00, then the number of KWHs used is 410. |
|---|



## Problems on electricity costs

$$E(k) = 14.00 + 0.10k$$

- Compute the monthly cost for using 500 KWHs and write a summary statement in words using the proper units.

- Compute the monthly cost for using 600 KWHs and write a summary statement in words using the proper units.

- How many KWHs can be used if the monthly cost is \$50.00? Translate this problem into symbolic form as an algebra problem.

# Functions: Part 2
# Section 2.1

- Definition
- Graphs
- General cost and demand functions



## Graphing: point-by-point

Sketch a graph of $y = 2x - 1$.

Construct a table of values that satisfy the equation for the function.

| $x$ | $y$ |
|---:|---:|
| $-2$ | |
| 0 | |
| 1/2 | |
| 1 | |

## Function Notation

$$y = 2x - 1 \qquad f(x) = 2x - 1$$

$$\begin{aligned} f(x) &= 2x - 1 \\ f(2) &= \\ f(-1) &= \\ f(0) &= \\ f(2/3) &= \end{aligned}$$



# Definition of a Function

**Examples:**

For each sale of $x$ items there corresponds a revenue.

To each legal automobile driver, there is a driver's license number.

To each student in Math 103, there corresponds a grade in the course.

To each number $x$, there corresponds $2x$, its double.

**Definition:** A function is a rule that produces a correspondence between two sets of objects (usually numbers) such that each object in the first set (called the domain of the function) corresponds to exactly one object in the second set (called the range of the function).

# Functions Determined by Equations
# Graphs of Functions

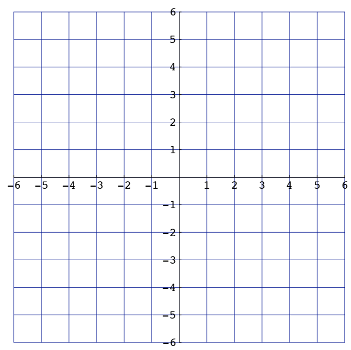
Both the domain and the range are sets of numbers.

$x$ is usually used for the number in the domain

$y$ is usually used for the number in the range.

**Examples:**

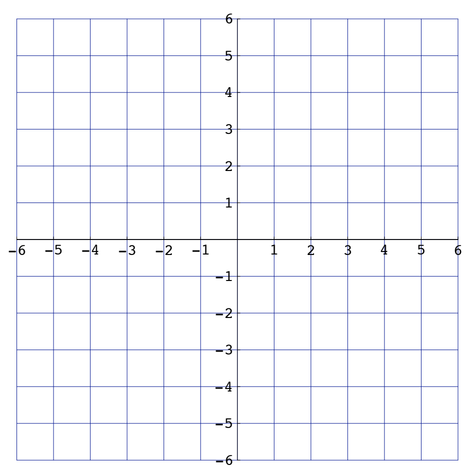
$2x - y = 1$





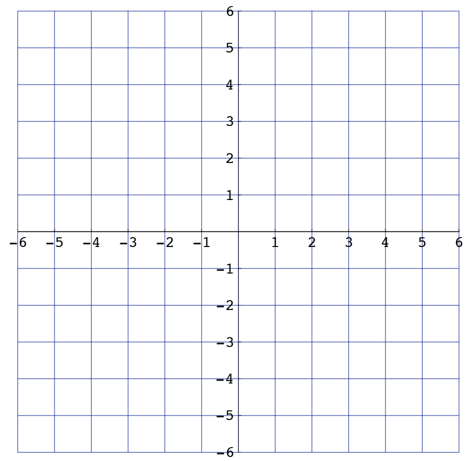
# Functions Determined by Equations
# Graphs of Functions

**Example:** $xy = 1$



# Functions Determined by Equations
# Graphs of Functions

**Example:** $x^2 + y^2 = 25$






# Function Notation

| $x, y$ equation | function notation |
|---|---|
| $y = -5x + 3$ | $f(x) = -5x + 3$ |
| $y = 2x^2 + 4x - .5$ | $g(x) =$ |
| $xy = 1$ | $h(x) =$ |

**Evaluation of a function:**

$$f(3) =$$

$$f(-2) =$$

$$g(1) =$$

$$h(4) =$$

$$h(1/4) =$$

$$g(a) =$$

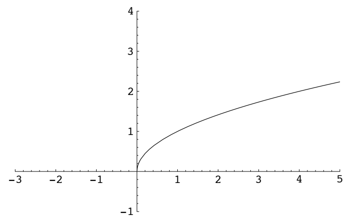
$$g(a+1) =$$

# Finding the Domain of a Function

Let $f(x)$ be a function given by an equation. Sometimes $f(x)$ doesn't make sense for certain values of $x$.

**Example:** $f(x) = \sqrt{x}$.

What's $f(-3)$? Not defined.



Domain of $f(x)$:

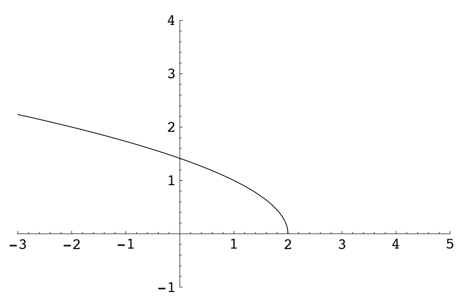
Inequality notation: $x \geq 0$

Interval notation: $[0, +\infty)$



# Finding the Domain of a Function **Example:**

$f(x) = \sqrt{2 - x}$



Domain of $f(x)$:

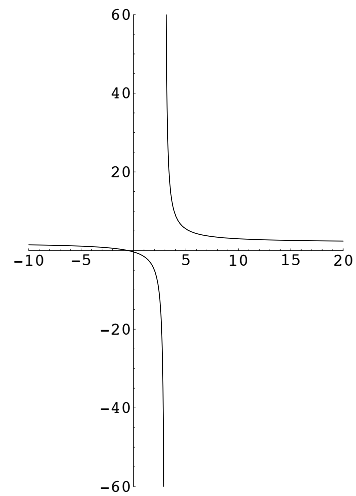
Inequality notation:

Interval notation

# Finding the Domain of a Function

**Example:** $f(x) = \dfrac{2x+1}{x-3}$

Domain of $f(x)$:






# Cost, Price-Demand, Revenue, and Profit Functions

$x$ is the number of units sold (independent variable)

**Cost Function:** $C(x)$ is the cost to produce $x$ units.

There is a fixed cost, $a$ and a cost per unit, $b$.

$C(x) = a + bx$

**Price-demand Function:** $p(x)$ is the price per unit. The price per unit decreases as the number of units produced increases.

$p(x) = m - nx$

**Revenue Function:** $R(x) = xp(x) = x(m - nx) = xm - nx^2$

Total revenue is number sold times the price per unit.

**Profit Function:**

$P(x) = R(x) - C(x)$

Profit is revenue minus cost.

## Cost, Price-Demand, Revenue, and Profit Functions

**Example:** A publishing company is preparing to market a new book on turnip gardening. The author gets a flat fee of \$5000 for writing the book. It costs \$8200 to to typeset the book plus \$18 per book to print and bind the book.

Fixed costs are \$5000 + \$ 8200 = \$13,200.
Cost per unit is \$18.

$C(x) =$

The publisher believes that the price-demand function is
$p(x) = 140.00 - .10x$

That is, starting at a sales price of \$140 per copy, the price decreases by \$.10 for each additional copy sold.

So

$R(x) =$

and

$P(x) = R(x) - C(x) =$

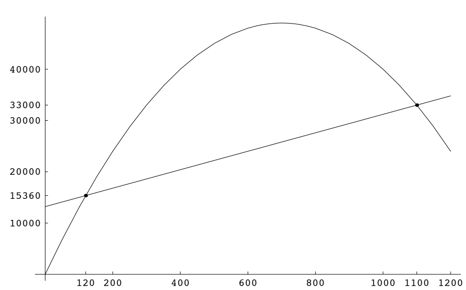
## Cost, Price-Demand, Revenue, and Profit Functions

$C(x) = 13200 + 18x$

$p(x) = 140.00 - .10x$

$R(x) = x(140 - .10x) = 140x - .10x^2$

$P(x) = R(x) - C(x) = -13200 + 122x - .10x^2$




**Break-even:** How many units must be produced to break-even?

Solve $C(x) = R(x)$