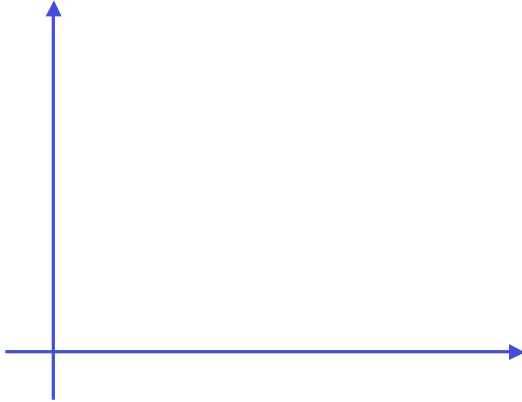


## WARM UP EXERCISE

Find the absolute maximum/minimum for the following function on  $[0,10,000]$ .

$$R(x) = 10x - .001x^2$$



1

## §12.5&6 Absolute Maxima and Minima and Maximizing Profit

**The student will learn about:**

- **absolute maxima,**
- **absolute minima, and**
- **Application to Maximizing profit: Profit is maximized when marginal revenue equals marginal cost.**

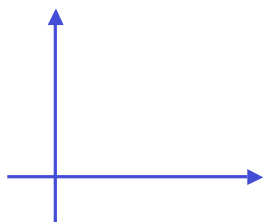
2

## Absolute Maxima and Minima.

Definition:  $f(c)$  is an absolute maxima of  $f$  if  $f(c) \geq f(x)$  for all  $x$  in the domain of  $f$ .

Definition:  $f(c)$  is an absolute minima of  $f$  if  $f(c) \leq f(x)$  for all  $x$  in the domain of  $f$ .

Find the absolute minimum and maximum value of,



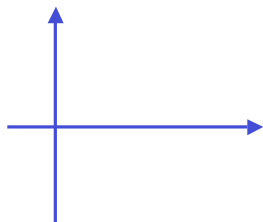
$$f(x) = .5(x - 3)^2 + 2$$

What do you notice about  $f'(x)$  at the max/min?

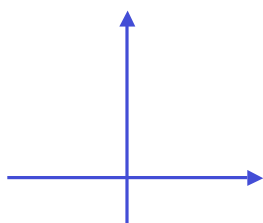
3

## Absolute Maxima and Minima.

Find the absolute minimum and maximum value of:



$$f(x) = -.5(x - 3)^2 + 2$$



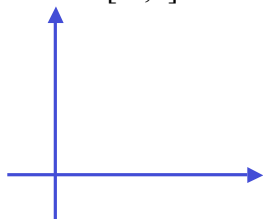
$$f(x) = (x + 1)(x)(x - 1)$$

4

## Absolute Maxima and Minima.

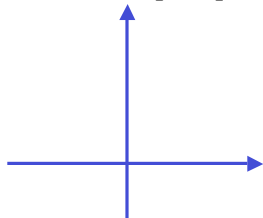
Find the absolute minimum and maximum value of  $f(x)$  on the interval  $[-1,1]$ :

$$f(x) = \frac{2x - 1}{x - 2}$$



Find the absolute minimum and maximum value of  $f(x)$  on the interval  $[-1,1]$ :

$$f(x) = (x + 1)(x)(x - 1)$$



What do you notice about  $f'(x)$  at the max/min?

5

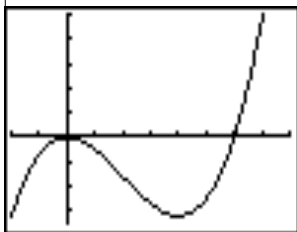
## Extreme Value Theorem

Theorem 1. Extreme Value Theorem.

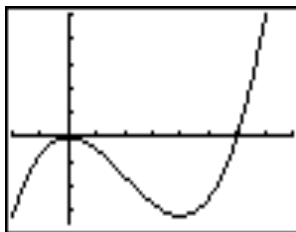
A function  $f$  that is continuous on a closed interval  $[a, b]$  has both an absolute maximum value and an absolute minimum value on that interval.

Find the absolute minimum and maximum value of  $f(x)$  on the intervals below:

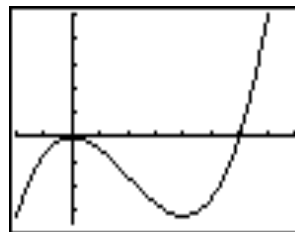
$$f(x) = x^3 - 6x^2$$



$[-1,5]$



$[-1,7]$



$[1,4]$

6

## Steps in finding absolute maximum and minimum values

Definition. The values of  $x$  in the domain of  $f$  where  $f'(x) = 0$  or where  $f'(x)$  does not exist are called the critical values of  $f$ .

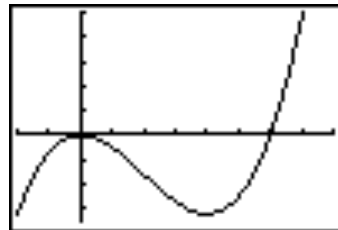
Theorem 2. Absolute extrema (if they exist) must always occur at critical values of the derivative **or** at end points.

- Check to make sure  $f$  is continuous over  $[a, b]$ .
- Find the critical values of  $f$  in the interval  $[a, b]$ .
- Evaluate  $f$  at the end points  $a$  and  $b$  and at the critical values found in step  $b$ .
- The absolute maximum of  $f(x)$  on  $[a, b]$  is the largest of the values found in step  $c$ .

7

## Example 2

Use algebra to find the **absolute** maximum and **absolute** minimum value on  $[-1, 7]$  of,  $f(x) = x^3 - 6x^2$



8

## Maximize Revenue

An office supply company sells  $x$  mechanical pencils per year at  $\$p$  per pencil.

The price demand equation for these pencils is  $p = 10 - .001x$ .

What price should the company charge for these pencils to maximize their revenue?

What is the maximum revenue?

9

## Maximize Profit

An office supply company sells  $x$  mechanical pencils per year at  $\$p$  per pencil.

The price demand equation for these pencils is  $p = 10 - .001x$ .

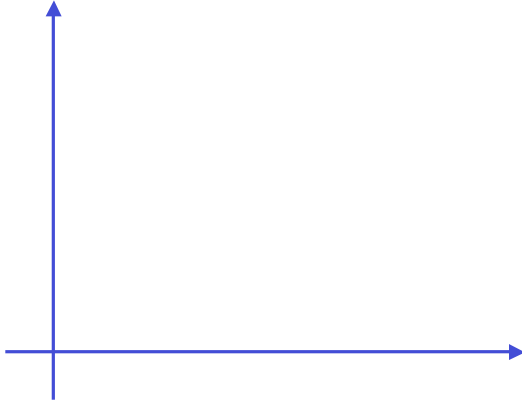
Suppose further that the total annual cost of manufacturing  $x$  mechanical pencils is  $C(x) = 5000 + 2x$ .

What is the company's maximum profit?

What should the company charge for each pencil and how many pencils should be produced?

10

## Maximize Profit... work space



11

## Maximize Profit... work space

12